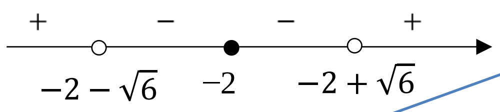
H2 Mathematics (9758)
Chapter 4 Equations and Inequalities
Assignment Suggested Solutions

1 **2020Promo/NYJC/Q4**

(i) Using an algebraic method, solve the inequality $\dfrac{6}{2-4x-x^2} \leq 1$, leaving your answers in exact form. [4]

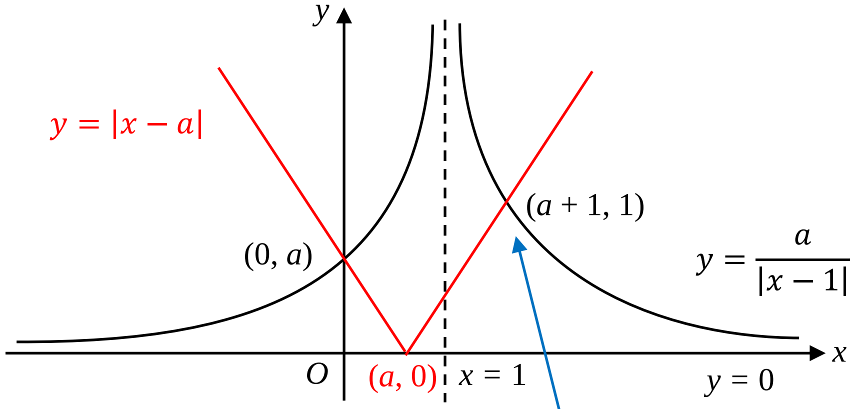
(ii) Hence, solve $\dfrac{6}{2-4\ln x-(\ln x)^2} \leq 1$. [3]

Q1	**Solution**
(i)	$\dfrac{6-(2-4x-x^2)}{2-4x-x^2} \leq 0$ Bring everything to one side $\dfrac{x^2+4x+4}{2-4x-x^2} \leq 0$ $\dfrac{(x+2)^2}{x^2+4x-2} \geq 0$ $\dfrac{(x+2)^2}{(x+2)^2-6} \geq 0$ $\dfrac{(x+2)^2}{(x+2+\sqrt{6})(x+2-\sqrt{6})} \geq 0$ Factorise $\dfrac{(x+2)^2}{\left[x-\left(-2-\sqrt{6}\right)\right]\left[x-\left(-2+\sqrt{6}\right)\right]} \geq 0$ Solve using number line  **Note:** the inequality to solve is ≥ 0, so $x = -2$, satisfies the inequality $x < -2-\sqrt{6}$ or $x = -2$ or $x > -2+\sqrt{6}$
(ii)	From **(i)**, replace x with $\ln x$. $\ln x < -2-\sqrt{6}$ or $\ln x = -2$ or $\ln x > -2+\sqrt{6}$ $\therefore 0 < x < e^{-2-\sqrt{6}}$ or $x = e^{-2}$ or $x > e^{-2+\sqrt{6}}$ **<u>Alternatively,</u>** $\therefore 0 < x < 0.0117$ or $x = 0.135$ or $x > 1.57$

2 **2020/EJC/I/Q4 (modified)**

(i) On the same axes, sketch the graphs of $y=\dfrac{a}{|x-1|}$ and $y=|x-a|$, where a is a constant such that $0<a<1$. It is given that the two graphs intersect exactly twice. [3]

(ii) Hence, solve the inequality $|x-a|\le\dfrac{a}{|x-1|}$. [3]

Q2	**Solution**
(i)	 Finding intersection points: $\dfrac{a}{\lvert x-1\rvert}=\lvert x-a\rvert$ Since $x>1>a$, $\dfrac{a}{x-1}=(x-a)$ Notice from the graph drawn, the 2nd intersection point occurs after x = 1 $(x-1)(x-a)=a$ $x^2-(a+1)x=0$ $x\left[x-(a+1)\right]=0$ $x=0$ or $x=a+1$ When x = a + 1, y = 1
(ii)	Hence from the graph, $\lvert x-a\rvert\le\dfrac{a}{\lvert x-1\rvert} \quad\Rightarrow\quad 0\le x\le a+1, x\neq 1$ **<u>Alternatively,</u>** $\Rightarrow\quad 0\le x<1$ or $1<x\le a+1$

3 2019/HCI/I/Q1

The number of units, $D(x)$ of a particular product that people are willing to purchase per week in city A at a price \$$x$ is given by the function $D(x)=\dfrac{40320}{\mathrm{g}(x)}$, where $\mathrm{g}(x)$ is a quadratic polynomial in x. The following table shows the number of units people are willing to purchase at different prices.

x	5	8	10
$D(x)$	384	224	168

Find the number of units of the product that people are willing to purchase at a price of \$18. [4]

Q3	Solution
	$D(x)=\dfrac{40320}{\mathrm{g}(x)}$ $\Rightarrow \mathrm{g}(x)=\dfrac{40320}{D(x)}$ $ax^2+bx+c=\dfrac{40320}{D(x)},\ a,b,c\in\mathbb{R}$ Given $5^2a+5b+c=\dfrac{40320}{384}=105$ -- (1) $8^2a+8b+c=\dfrac{40320}{224}=180$ -- (2) $10^2a+10b+c=\dfrac{40320}{168}=240$ -- (3) Using GC, $a=1,\ b=12,\ c=20$ When x = 18, $D(18)=\dfrac{40320}{18^2+12(18)+20}=72$ units