2020 JC2 H2 PRELIMS Paper 1's Suggested Solutions

<u>1:</u>

(i)

$$\mathrm{f}(x)=\frac{2x+3}{3x+5}$$
$$=\frac{2}{3}-\frac{1}{3(3x+5)}$$
$$=\frac{2}{3}+\frac{-\frac{1}{3}}{3x+5}$$

(ii) $y=\frac{1}{x} \rightarrow y=\frac{1}{9}\left(\frac{1}{x}\right)=\frac{1}{9x}$: scale by a factor of $\frac{1}{9}$ parallel to the y-axis

Or scale by a factor of $\frac{1}{9}$ parallel to the x-axis

$\rightarrow y=\frac{1}{9\left(x+\frac{5}{3}\right)}$: translate by $\frac{5}{3}$ in the negative x-direction

$\rightarrow y=-\frac{1}{9\left(x+\frac{5}{3}\right)}$: reflect about the x-axis

$\rightarrow y=\frac{2}{3}-\frac{1}{9\left(x+\frac{5}{3}\right)}$: translate by $\frac{2}{3}$ in the positive y-direction

2.

(i) $w = 1 + \mathrm{i} = \sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{4}\right)}$

$$\frac{z^n}{w^*} = \frac{2^n \mathrm{e}^{\mathrm{i}\left(\frac{n\pi}{8}\right)}}{\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4}\right)}}$$

$$= 2^{n-\frac{1}{2}} \mathrm{e}^{\mathrm{i}\left(\frac{n\pi}{8}+\frac{\pi}{4}\right)}$$

(ii) For $\frac{z^n}{w^*}$ to be real and negative,

$$\arg\frac{z^n}{w^*} = \pi \pm 2k\pi, \quad \text{where } k \in \mathbb{Z}$$

$$\frac{n\pi}{8} + \frac{\pi}{4} = \pi \pm 2k\pi$$

$$\frac{n}{8} + \frac{1}{4} = 1 \pm 2k$$

$$n = 6 \pm 16k$$

Smallest $n = 6,\ 22$

Q3:

Since all coefficients are real, $z=-1-\mathrm{i}$ is also a root.

A quadratic factor
$=\left[z-(-1+\mathrm{i})\right]\left[z-(-1-\mathrm{i})\right]$
$=\left[z+1-\mathrm{i}\right]\left[z+1+\mathrm{i}\right]$
$=(z+1)-(\mathrm{i})^2$
$=z^2+2z+2$

$2z^3+5z^2+pz+q=\left(z^2+2z+2\right)\left(2z+d\right)$
$2z^3+5z^2+pz+q=2z^3+4z^2+4z+dz^2+2dz+2d$
$\quad=2z^3+\left(4+d\right)z^2+\left(4+2d\right)z+2d$

Comparing the coefficients of z^2, z and constant terms,
$4+d=5\Rightarrow d=1$
$4+2d=p\Leftrightarrow \quad p=6$
$2d=q\Leftrightarrow \quad q=2$

Alternative:
$2(-1+\mathrm{i})^3+5(-1+\mathrm{i})^2+p(-1+\mathrm{i})+q=0$
$4-6\mathrm{i}-p+p\mathrm{i}+q=0$
$(4-p+q)+\mathrm{i}(p-6)=0+\mathrm{i}0$
Comparing the real and imaginary parts,
$4-p+q=0 \qquad p=6$
$q=p-4$
$q=2$

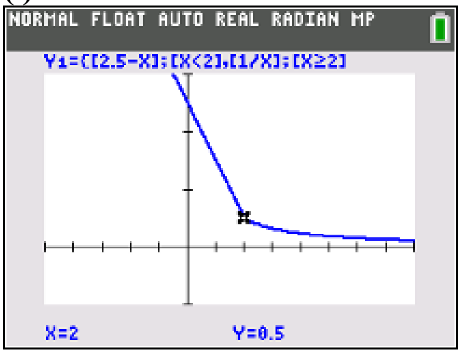
Last factor is $(2z+d)=(2z+1)$.

Therefore, the real root is $-\frac{1}{2}$.

All the other roots are $z=-1\pm\mathrm{i},-\frac{1}{2}$.

Q4:

(i)



Horizontal asymptote ($y = 0$) and y-intercept $\left(0, \dfrac{5}{2}\right)$

(ii) Since every horizontal line cuts the graph at most once. Therefore, f is a 1-1 function and hence f has an inverse function.

When $x < 2$, let $y = \dfrac{5}{2} - x$

$$x = \frac{5}{2} - y$$

$$\mathrm{f}^{-1}(x) = \frac{5}{2} - x \ , \ x > \frac{1}{2}$$

When $x \geq 2$, let $y = \dfrac{1}{x}$

$$x = \frac{1}{y}$$

$$\mathrm{f}^{-1}(x) = \frac{1}{x} \ , \ 0 < x \leq \frac{1}{2}$$

Hence,

$$\mathrm{f}^{-1}(x) = \begin{cases} \dfrac{1}{x}, & x \in \mathbb{R}, 0 < x \leq \dfrac{1}{2} \\ \dfrac{5}{2} - x, & x \in \mathbb{R}, x > \dfrac{1}{2} \end{cases}$$

(iii) Since the range of $\mathrm{f}^{-1} = \mathbb{R}$ or $(-\infty, \infty)$ $\not\subset$ of Domain of $\mathrm{g} = [2, \infty)$, gf^{-1} does not exist.

Q5:

$$A = 2\pi r^2 + 2\pi rh = 16200 \Rightarrow h = \frac{8100}{\pi r} - r$$

$$V = \frac{2}{3}\pi r^3 + \pi r^2\left(\frac{8100}{\pi r} - r\right)$$

$$= -\frac{1}{3}\pi r^3 + 8100r$$

$$\frac{\mathrm{d}V}{\mathrm{d}r} = -\pi r^2 + 8100$$

At maximum V, $\frac{\mathrm{d}V}{\mathrm{d}r} = 0$.

$$-\pi r^2 + 8100 = 0$$

$$\pi r^2 = 8100$$

$$r^2 = \frac{8100}{\pi}$$

$$r = \sqrt{\frac{8100}{\pi}}, \text{ since } r > 0$$

Maximum $V = -\frac{1}{3}\pi\frac{8100}{\pi}\sqrt{\frac{8100}{\pi}} + 8100\sqrt{\frac{8100}{\pi}}$

$$= \sqrt{\frac{8100}{\pi}}\left(-\frac{1}{3}\pi\frac{8100}{\pi} + 8100\right)$$

$$= \frac{90}{\sqrt{\pi}}(5400)$$

$$= \frac{486000}{\sqrt{\pi}}$$

$$\frac{\mathrm{d}^2V}{\mathrm{d}r^2} = -2\pi < 0$$

Hence, V is maximum.

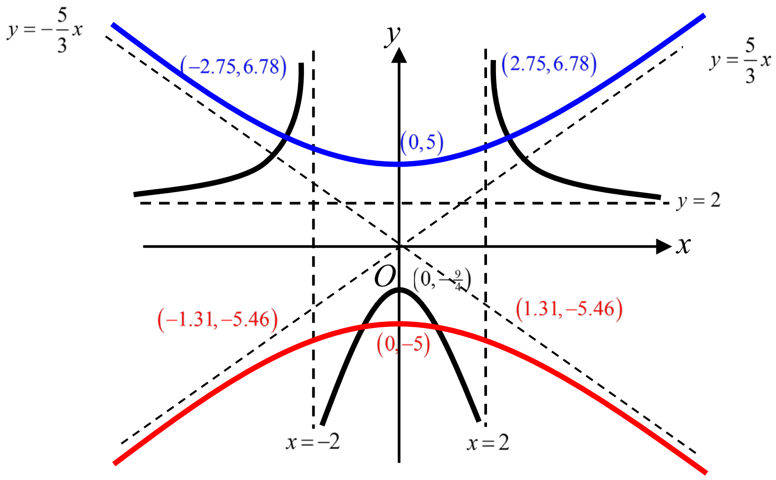
<u>Q6:</u>

(i) $\dfrac{y^2}{25}-\dfrac{x^2}{9}=1$

$$\frac{y^2}{25}=1+\frac{x^2}{9}$$

$$y^2=25\left(1+\frac{x^2}{9}\right)$$

$$y=\pm 5\sqrt{1+\frac{x^2}{9}}$$



Asymptotes $x=-2,\ x=2, y=2,\ y=\dfrac{5}{3}x$ and $y=-\dfrac{5}{3}x$

Turning point/Intercept $\left(0,-\frac{9}{4}\right)$

Vertices $(0,5),\ (0,-5)$

$(-1.31,-5.46),\ (1.31,-5.46),(-2.75,6.78),\ (2.75,6.78)$

(ii) From the graph,
$x \le -2.75$ or $-2<x<2$ or $x \ge 2.75$

Q7:

(i)
$$y^3+y^2+2y=x^2-3x$$
$$3y^2\frac{dy}{dx}+2y\frac{dy}{dx}+2\frac{dy}{dx}=2x-3$$
$$3y^2\frac{d^2y}{dx^2}+\frac{dy}{dx}6y\frac{dy}{dx}+2y\frac{d^2y}{dx^2}+\frac{dy}{dx}2\frac{dy}{dx}+2\frac{d^2y}{dx^2}=2$$
$$3y^2\frac{d^2y}{dx^2}+6y\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2+2\frac{d^2y}{dx^2}=2$$

When $x=0$, $y^3+y^2+2y=0$
$$y\left(y^2+y+2\right)=0$$
$$y\left(\left(y+\frac{1}{2}\right)^2-\frac{1}{4}+2\right)=0$$
$$y\left(\left(y+\frac{1}{2}\right)^2+\frac{7}{4}\right)=0$$
$$\Rightarrow y=0 \quad \text{since} \quad \left(\left(y+\frac{1}{2}\right)^2+\frac{7}{4}\right)\neq 0$$

When $x=0$, $3y^2\frac{dy}{dx}+2y\frac{dy}{dx}+2\frac{dy}{dx}=-3$
$$2\frac{dy}{dx}=-3$$
$$\frac{dy}{dx}=-\frac{3}{2}$$

When $x=0$, $3y^2\frac{d^2y}{dx^2}+6y\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2+2\frac{d^2y}{dx^2}=2$
$$2\left(\frac{dy}{dx}\right)^2+2\frac{d^2y}{dx^2}=2$$
$$\left(-\frac{3}{2}\right)^2+\frac{d^2y}{dx^2}=1$$
$$\frac{d^2y}{dx^2}=1-\frac{9}{4}$$
$$\frac{d^2y}{dx^2}=-\frac{5}{4}$$

$\frac{dy}{dx}=-\frac{3}{2}$ and $\frac{d^2y}{dx^2}=-\frac{5}{4}$.

$$y = 0 + -\frac{3}{2}x + \frac{\left(-\frac{5}{4}\right)}{2!}x^2 + \ldots$$

$$y = -\frac{3}{2}x - \frac{5}{8}x^2 + \ldots$$

(ii) $\dfrac{1}{2+y} = \dfrac{1}{2}\dfrac{1}{\left(1+\frac{y}{2}\right)}$

$$\frac{1}{2}\left(1+\frac{y}{2}\right)^{-1} = \frac{1}{2}\left(1-\left(-\frac{y}{2}\right)\right)^{-1}$$

$$= \frac{1}{2}\left(1+\left(-\frac{y}{2}\right)+\left(-\frac{y}{2}\right)^2 + \ldots\right)$$

$$= \frac{1}{2} - \frac{y}{4} + \frac{y^2}{8} + \ldots$$

$$= \frac{1}{2} - \frac{1}{4}\left(-\frac{3}{2}x - \frac{5}{8}x^2 + \ldots\right) + \frac{1}{8}\left(-\frac{3}{2}x - \frac{5}{8}x^2 + \ldots\right)^2 + \ldots$$

$$= \frac{1}{2} - \frac{1}{4}\left(-\frac{3}{2}x - \frac{5}{8}x^2 + \ldots\right) + \frac{1}{8}\left(\frac{9}{4}x^2\right) + \ldots$$

$$= \frac{1}{2} + \frac{3}{8}x + \frac{5}{32}x^2 + \frac{9}{32}x^2 + \ldots$$

$$= \frac{1}{2} + \frac{3}{8}x + \frac{7}{16}x^2 + \ldots$$

Q8:

(i) $x = 3t^2$, $y = -6t$

$\frac{dx}{dt} = 6t$, $\frac{dy}{dt} = -6$,

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

$= \frac{-6}{6t} = -\frac{1}{t}$

At point P, gradient of tangent = $-\frac{1}{p}$.

Equation of the tangent to curve C at P :

$$y - (-6p) = -\frac{1}{p}(x - 3p^2),$$
$$y = -\frac{1}{p}x + 3p - 6p,$$
$$y = -\frac{1}{p}x - 3p.$$

At point P, gradient of normal = $-\frac{1}{\text{gradient of tangent}} = -\frac{1}{-\frac{1}{p}} = p$.

Equation of the normal to curve C at P :

$$y - (-6p) = p(x - 3p^2),$$
$$y = px - 3p^3 - 6p.$$

(ii) When the tangent at P meets the x-axis, $y = 0$

$0 = -\frac{1}{p}x - 3p$,

$\frac{1}{p}x = -3p$,

$x = -3p^2 \quad \Rightarrow \quad T(-3p^2,\ 0)$

When the normal at P meets the x-axis, $y = 0$

$0 = px - 3p^3 - 6p$,

$px = 3p^3 + 6p$

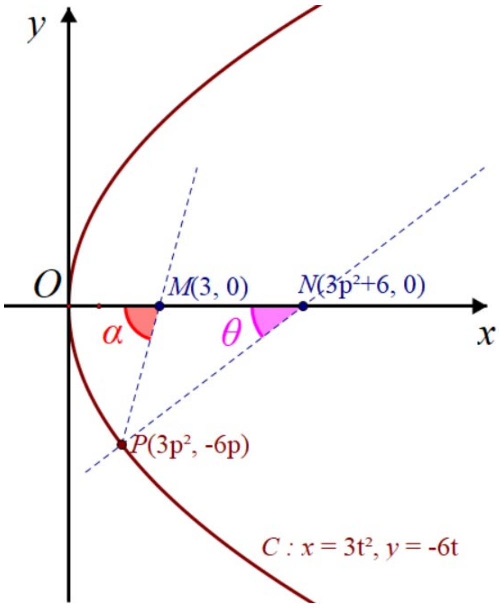
$\because p \neq 0$, $x = 3p^2 + 6 \quad \Rightarrow \quad N(3p^2 + 6,\ 0)$

$\therefore M = \left(\frac{1}{2}\left[-3p^2 + \left(3p^2 + 6\right)\right],\ 0\right)$

$= (3,\ 0)$, which is independent of p (shown)

(iii) Proof :

For any value of p where $0 < p < 1$,
let the acute angle PN makes with the x-axis be θ, and
let the acute angle PM makes with the x-axis be α.



$$\text{Length } MN = (3p^2+6)-3$$
$$= 3p^2+3$$

$$\text{Length } PM = \sqrt{\left(3-3p^2\right)^2+\left(0-(-6p)\right)^2}$$
$$= \sqrt{\left(9-18p^2+9p^4\right)+\left(36p^2\right)}$$
$$= \sqrt{9+18p^2+9p^4}$$
$$= \sqrt{\left(3+3p^2\right)^2}$$
$$= 3+3p^2 \quad \because\ 3+3p^2 \geq 0$$

$\therefore$ Length PM = Length MN

ΔMPN is an isosceles triangle.

$\Rightarrow\ \measuredangle MPN = \measuredangle MNP = \theta$

The acute angle α that PM makes with the x-axis

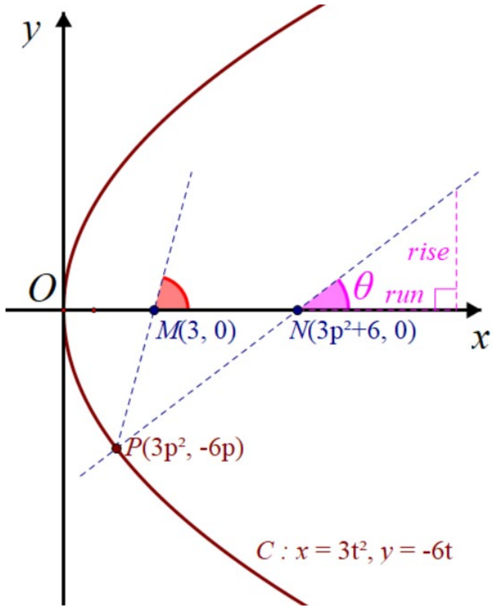
$= \measuredangle MPN + \measuredangle MNP$ (ext. $\measuredangle$ of Δ = sum of opp. int. $\measuredangle$s)

$= \theta + \theta$

$= 2\theta$ (proven).

<u>Alternative proof</u> :

For any value of p where $0 < p < 1$, let the acute angle PN makes with the x-axis be θ.



$\because$ Gradient of PN is given by $\frac{\text{rise}}{\text{run}} = \tan\theta$,
and the gradient of PN (the normal at P) is also p [as found in part **(i)(b)**], $\therefore$ $\tan\theta = p$.

$\because$ $M(3,\ 0)$ and $P(3p^2,\ -6p)$,

$$\therefore \text{ Gradient of } PM = \frac{0-(-6p)}{3-3p^2}$$

$$= \frac{6p}{3(1-p^2)} = \frac{2p}{1-p^2} > 0 \qquad \because p^2 < 1 \text{ and } p > 0$$

$$= \frac{2\tan\theta}{1-(\tan\theta)^2} \qquad \because p = \tan\theta$$

$$= \tan 2\theta$$

$\therefore$ The acute angle PM makes with the x-axis is always 2θ , which is twice the angle PN makes with the x-axis, for any value of p where $0 < p < 1$. (Proven)

Q9:

(i)

$$\text{LHS} = \frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$= x\left[\frac{1}{2}\left(a^2-x^2\right)^{-\frac{1}{2}}(-2x)\right]+\sqrt{a^2-x^2}+a^2\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\left(\frac{1}{a}\right)$$

$$=-x^2\left[\frac{1}{\sqrt{a^2-x^2}}\right]+\sqrt{a^2-x^2}+\frac{a^2}{\sqrt{a^2-x^2}}$$

$$=\left(a^2-x^2\right)\left[\frac{1}{\sqrt{a^2-x^2}}\right]+\sqrt{a^2-x^2}$$

$$=\sqrt{a^2-x^2}+\sqrt{a^2-x^2}$$

$$=2\sqrt{a^2-x^2}\quad\text{(shown)}$$

(ii) Area = $\int_0^2\sqrt{\frac{16-x^2}{16}}\,\mathrm{d}x=\frac{1}{4}\int_0^2\sqrt{16-x^2}\,\mathrm{d}x$

$$=\frac{1}{4(2)}\int_0^2 2\sqrt{16-x^2}\,\mathrm{d}x$$

$$=\frac{1}{8}\left[x\sqrt{4^2-x^2}+4^2\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$$

$$=\frac{1}{8}\left(2\sqrt{4^2-2^2}+4^2\sin^{-1}\left(\frac{2}{4}\right)\right)$$

$$=\frac{1}{8}\left(2\sqrt{12}+4^2\sin^{-1}\left(\frac{1}{2}\right)\right)=\frac{\sqrt{3}}{2}+\frac{\pi}{3}\ \text{units}^2$$

(iii) Volume = $\pi\int_0^2\frac{1}{16}\left(16-x^2\right)\mathrm{d}x$

$$=\frac{\pi}{16}\left[16x-\frac{x^3}{3}\right]_0^2$$

$$=\frac{11\pi}{6}\ \text{units}^3$$

(b) $y=x^2$ and $x^2+4y^2=4$ intersect at $(0.93956, 0.88278)$

Volume $=\pi\int_0^{0.88278}y\,\mathrm{d}y+\pi\int_{0.88278}^{1}4-4y^2\,\mathrm{d}y$

$= 1.39$ units^3

Q10:

(i)(a)

Month, n	Amount at start of month	Amount at end of month
1 (i.e. January 2020)	\$300	$\$\,300(1.003)$
2 (i.e. February 2020)	$\$\,300(1.003)+300$	$\$\,300(1.003)^2+300(1.003)$
3	$\$\,300(1.003)^2+300(1.003)+300$	$\$\,300(1.003)^3+300(1.003)^2+300(1.003)$
$\vdots$	$\vdots$	$\vdots$
n	$\$\,300(1.003)^{n-1}+\ldots+300(1.003)+300$	$\$\,300(1.003)^n+300(1.003)^{n-1}+\ldots+300(1.003)$

Total amount in the account at the start of the nth month

$$=300(1.003)^{n-1}+\ldots+300(1.003)+300$$
$$=300\left(1+1.003+\ldots+1.003^{n-1}\right)$$
$$=300\left(\frac{1\left(1.003^n-1\right)}{1.003-1}\right)$$
$$=\$100000\left(1.003^n-1\right)\quad(\text{shown})$$
$$\therefore A=100000$$

(i)(b) Start of 1 January 2021 $\Rightarrow n=13$

Total amount in the account at the start of 1 January 2021

$$=\$100000\left(1.003^{13}-1\right)$$
$$=\$3970.978$$
$$\approx\$3970.98\quad(\text{to 2 d.p.})$$

(ii) Amount of bonus earned at the end of the 20th month

$$=\$0.01k+(20-1)(0.01k)$$
$$=\$0.2k$$

(iii)

Month, n	Amount at start of month	Amount at end of month
1 (i.e. January 2020)	$\$k$	$\$\,k+0.01k$
2 (i.e. February 2020)	$\$\,k+0.01k+k$	$\$\,k+0.01k+k+0.02k=2k+0.01k(1+2)$
3	$\$\,2k+0.01k(1+2)+k$	$\$\,3k+0.01k(1+2+3)$
$\vdots$	$\vdots$	$\vdots$
n	$\$\,(n-1)k+0.01k(1+2+\ldots+n-1)+k$	$\$\,nk+0.01k(1+2+\ldots+n)$

Total amount in the account at the start of the nth month

$$= (n-1)k + 0.01k(1+2+\ldots+n-1)+k$$
$$= nk + 0.01k\left(\frac{n-1}{2}\right)(1+n-1)$$
$$= nk + 0.01nk\left(\frac{n-1}{2}\right)$$
$$= nk(1+0.005n-0.005)$$
$$= nk(0.005n+0.995)$$
$$= 0.005nk(n+199) \quad (\text{shown})$$
$$\therefore B = 199$$

(iv) 2 January 2040 $\Rightarrow n = 241$

$$100000\left(1.003^{241}-1\right) = 0.005(241)k(241+199)$$
$$k = 199.618$$
$$\approx 200 \quad (\text{to nearest whole number})$$

Alternative (where *n* = 240)

2 January 2040 $\Rightarrow n = 240$

$$100000\left(1.003^{240}-1\right) = 0.005(240)k(240+199)$$
$$k = 199.74$$
$$\approx 200 \quad (\text{to nearest whole number})$$

(v) $0.005n(200)(n+199) \geq 150000$

Using GC,

n	Amount at the start of nth month	Amount at the end of nth month
299	\$148902	\$149500
300	\$149700	\$150300
301	\$150500	\$151102

The total amount in the account for EduPlan Blessing is first over \$150000 on 31 Dec 2044.

Q11:

(i) $E\begin{pmatrix}4\\0\\3\end{pmatrix}, F\begin{pmatrix}4\\12\\4\end{pmatrix}, I\begin{pmatrix}1\\12\\7\end{pmatrix}$

$\overrightarrow{EF}=\begin{pmatrix}4\\12\\4\end{pmatrix}-\begin{pmatrix}4\\0\\3\end{pmatrix}=\begin{pmatrix}0\\12\\1\end{pmatrix}, \overrightarrow{FI}=\begin{pmatrix}1\\12\\7\end{pmatrix}-\begin{pmatrix}4\\12\\4\end{pmatrix}=\begin{pmatrix}-3\\0\\3\end{pmatrix}=3\begin{pmatrix}-1\\0\\1\end{pmatrix}$ or $\overrightarrow{EI}=\begin{pmatrix}-3\\12\\4\end{pmatrix}$

$\begin{pmatrix}0\\12\\1\end{pmatrix}\times\begin{pmatrix}-1\\0\\1\end{pmatrix}=\begin{pmatrix}12\\-1\\12\end{pmatrix}$

$\mathbf{r}\cdot\begin{pmatrix}12\\-1\\12\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}\cdot\begin{pmatrix}12\\-1\\12\end{pmatrix}=48+36$

$12x-y+12z=84$

(ii) $12x-y+12z=84$ and $36x+y-12z=-36$

From GC,

$x=1$

$y=-72+12z$

$z=z$

$l:\mathbf{r}=\begin{pmatrix}1\\-72\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\12\\1\end{pmatrix}, \lambda\in\mathbb{R}$

Therefore, the cartesian equation for line *l*: $x=1, \dfrac{y+72}{12}=z$

Alternative 1,

$\overrightarrow{HI}=\overrightarrow{EF}=\begin{pmatrix}0\\12\\1\end{pmatrix}$

$l:\mathbf{r}=\begin{pmatrix}1\\12\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\12\\1\end{pmatrix}, \lambda\in\mathbb{R}$

Therefore, the cartesian equation for line *l*: $x=1, \dfrac{y-12}{12}=z-7$

<u>Alternative 2</u>,

$\overrightarrow{OH}=\begin{pmatrix}1\\0\\6\end{pmatrix}$

$l:\mathbf{r}=\begin{pmatrix}1\\0\\6\end{pmatrix}+\lambda\begin{pmatrix}0\\12\\1\end{pmatrix},\lambda\in\mathbb{R}$

Therefore, the cartesian equation for line l: $x=1,\dfrac{y}{12}=z-6$

(iii) $\overrightarrow{OM}=\begin{pmatrix}4\\6\\\frac{7}{2}\end{pmatrix}$.

$\overrightarrow{FD}=\begin{pmatrix}0\\0\\3\end{pmatrix}-\begin{pmatrix}4\\12\\4\end{pmatrix}=\begin{pmatrix}-4\\-12\\-1\end{pmatrix},\overrightarrow{GM}=\begin{pmatrix}4\\6\\\frac{7}{2}\end{pmatrix}-\begin{pmatrix}0\\12\\4\end{pmatrix}=\begin{pmatrix}4\\-6\\-\frac{1}{2}\end{pmatrix}$

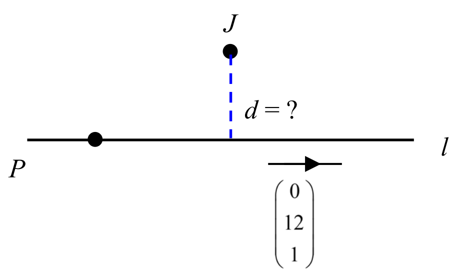
$l_1:\mathbf{r}=\begin{pmatrix}4\\12\\4\end{pmatrix}+s\begin{pmatrix}-4\\-12\\-1\end{pmatrix},s\in\mathbb{R}$, $l_2:\mathbf{r}=\begin{pmatrix}0\\12\\4\end{pmatrix}+t\begin{pmatrix}4\\-6\\-\frac{1}{2}\end{pmatrix},t\in\mathbb{R}$

$$\begin{pmatrix}4-4s\\12-12s\\4-s\end{pmatrix}=\begin{pmatrix}4t\\12-6t\\4-\frac{t}{2}\end{pmatrix}\Rightarrow\begin{matrix}4s+4t=4\\12s-6t=0\\s-\frac{t}{2}=0\end{matrix}$$

From GC, $s=\dfrac{1}{3},t=\dfrac{2}{3}$

The coordinates of J is $\left(\dfrac{8}{3},8,\dfrac{11}{3}\right)$

(iv) Let P be $(1,12,7)$



$$\overrightarrow{PJ} = \begin{pmatrix} \frac{8}{3} \\ 8 \\ \frac{11}{3} \end{pmatrix} - \begin{pmatrix} 1 \\ 12 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} \\ -4 \\ -\frac{10}{3} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 5 \\ -12 \\ -10 \end{pmatrix}$$

$$d = \frac{\left| \overrightarrow{PJ} \times \begin{pmatrix} 0 \\ 12 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} 0 \\ 12 \\ 1 \end{pmatrix} \right|}$$

$$= \frac{\left| \frac{1}{3}\begin{pmatrix} 5 \\ -12 \\ -10 \end{pmatrix} \times \begin{pmatrix} 0 \\ 12 \\ 1 \end{pmatrix} \right|}{\sqrt{145}}$$

$$= \frac{1}{3} \frac{\left| \begin{pmatrix} -108 \\ 5 \\ -60 \end{pmatrix} \right|}{\sqrt{145}}$$

$= 3.4228$

$=3.42 > 3.2$ Therefore, not long enough for this additional cable.