Anglo - Chinese School
(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2017
YEAR THREE EXPRESS
ADDITIONAL MATHEMATICS
PAPER 1

Friday **6 October 2017** **1 hour 30 minutes**

Additional Materials: Answer Paper (8 Sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 Given that $f(x) = x^3 + ax^2 + 2x + 2$ for all values of *x*.

(a) Find the value of a if $f(x)$ has a remainder of -1 when divided by $(x+1)$. [2]

(b) Find the quotient when the expression $x^3 + ax^2 + 2x + 2$ is divided by $(x+1)$. [2]

2 The total resistance of two resistors in an electric circuit is given by the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.
Given that $R = \sqrt{3} - 1$ and $R_1 = \frac{1}{3}\left(\sqrt{3}+1\right)$, find the value of R_2 in the form $a + b\sqrt{3}$ where *a* and *b* are integers. [4]

3 **(i)** Find the coordinates of the point of intersection of $y = \frac{3}{x}$ and $y = 2x^{\frac{1}{4}}$ for $x > 0$. [2]

(ii) Sketch the graph of $y = 2x^{\frac{1}{4}}$ for $x \geq 0$ and $y = \frac{3}{x}$ for $x > 0$ on the same diagram, indicating the point of intersection. [3]

4 The variables *x* and *y* are related by the equation $x - q = \frac{px}{y}$, where *p* and *q* are constants.
When *y* is plotted against $\frac{y}{x}$, a straight line graph is obtained. The line has gradient 3 and it passes through $(1, 5)$.

(i) Find the value of *p* and of *q*. [4]

(ii) Find the value of *y* when $2x - y = 0$. [2]

5 **(a)** Find the range of values of x for which $(6 - x)(1 + x) \geq -8$. [3]

(b) Find the range of values of m for which the curve $y = mx^2 - 8$ meets the line $y + 5m = 4x$. [4]

6 Solve the following equations.

(a) $e^{x} = 8 - 16\, e^{-x}$. [3]

(b) $2\log_9\left(2x^2\right) - \log_3\left(4 - x\right) = \dfrac{1}{\log_9 3}$. [4]

7 A circle, C_1, has equation $x^2 + y^2 - 2x - 6y + 6 = 0$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

The points $A(4, 11)$ and $B(8,\ 7)$ lie on a second circle C_2 whose centre lies on the line $x = 4$.

(ii) Find the equation of C_2. [4]

(iii) Determine, with working, whether C_1 and C_2 intersect. [2]

8 Given the function $f(x) = 3\sin 2x - 2$,

(i) state the period and the amplitude of the function, [2]

(ii) solve the equation $3\sin 2x - 2 = 0$ for $0° \le x \le 180°$, [3]

(iii) sketch the graph of $f(x) = 3\sin 2x - 2$ for $0° \le x \le 180°$, showing the x – and y – intercepts clearly. [2]

9 **(a)** Given that $f(x) = x^2\left(2x - 9\right) + 2$,

(i) prove that $\left(2x - 1\right)$ is a factor, [1]

(ii) solve the equation $x^2\left(2x - 9\right) + 2 = 0$, [3]

(iii) hence, or otherwise, find the values of θ between 0° and 360° inclusive such that $2\sin^2\theta(\sin\theta - 4) + \cos^2\theta = -1$. [3]

(b) If $\sec\theta = \dfrac{13}{5}$ and $180^\circ < \theta < 360^\circ$, find without the use of a calculator, the value of

(i) $\sin\theta$ **(ii)** $\tan\left(180^\circ - \theta\right)$. [4]

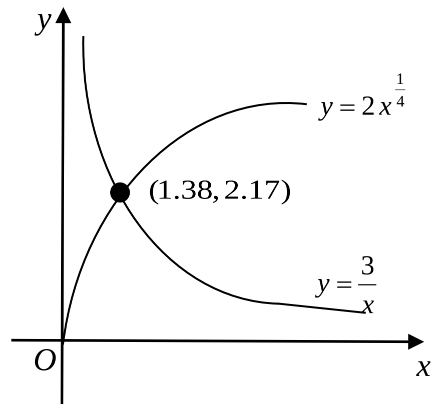
END OF PAPER 1

Answers:

1a. $a=0$ 1b. x^2-x+3

2. $R_2 = 2+\sqrt{3}$

3i. $(1.38, 2.17)$ 3ii.



4i. $p=2; q=3$ 4ii. $y=8$

5a. $-\frac{2}{5} \le m \le 2$ 5b. $-2 \le x \le 7$

6a. $x=1.39$ 6b. $x=2.55$ or -7.05

7i. centre of $C_1(1,3)$; radius of $C_1 = 2$ units

7ii. $(x-4)^2+(y-7)^2=4^2$

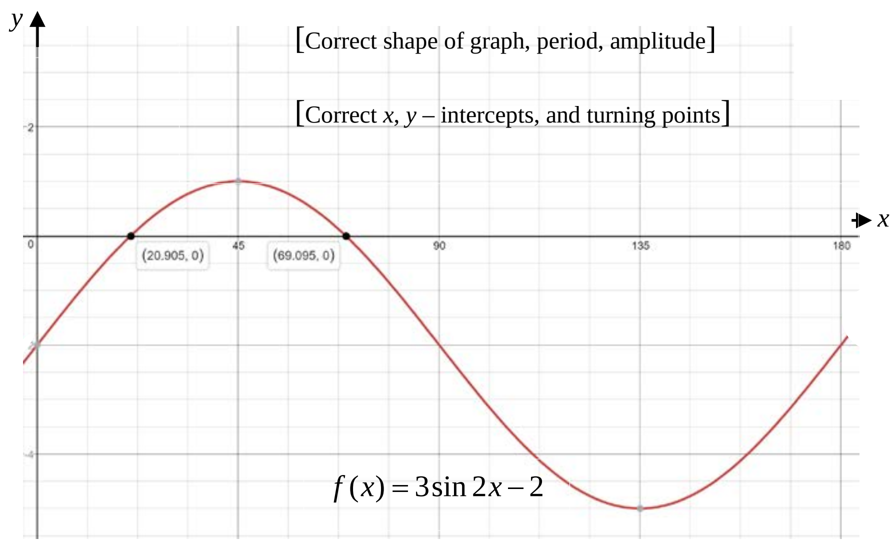
7iii. Distance between the centres of C_1 and C_2 = $\sqrt{(4-1)^2+(7-3)^2}=5$ units

Since $r_1 + r_2 = 2+4>5$ $\therefore C_1$ and C_2 intersects.

8i. Period = 180° Amplitude = 3

8ii. $x=20.9^\circ$, 69.1°

8iii.



9aii. $x=\frac{1}{2}$ or 4.45 or -0.449

9aiii. $\theta =^\circ 30^\circ, 150^\circ, 206.7^\circ, 333.3$

9bi $\sin\theta = -\frac{12}{13}$

9bii $\tan(180^\circ-\theta) = -\tan\theta = \frac{12}{5}$