Topic 1 - Measurement

Guiding Questions

- How are the standards for measurements established?
- Why is uncertainty inherent in all measurements?
- How can uncertainties be estimated? How can they be reduced if necessary?
- Why does the uncertainty of a measurement matter?
- How is the skill of making estimates of physical quantities useful and how can this skill be developed?

Contents

- Physical quantities & SI Units
- Scalars and vectors
- Errors and uncertainties

Learning Outcomes

Students should be able to:

(a) recall the following base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).
(b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.
(c) use SI base units to check the homogeneity of physical equations.
(d) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)*.
(e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).
(f) make reasonable estimates of physical quantities included within the syllabus.
(g) distinguish between scalar and vector quantities, and give examples of each.
(h) add and subtract coplanar vectors.
(i) represent a vector as two perpendicular components.
(j) show an understanding of the distinction between systematic errors (including zero errors) and random errors.
(k) show an understanding of the distinction between precision and accuracy.
(l) assess the uncertainty in a derived quantity by addition of actual, fractional or percentage uncertainties or numerical substitution (a rigorous statistical treatment is not required).

1 Physical Quantities and SI Units

Any scientifically measurable quantities are called physical quantities.
e.g. temperature, force, current, pressure, mass, energy, etc.

All physical quantities consist of a **numerical value** and a **unit**.

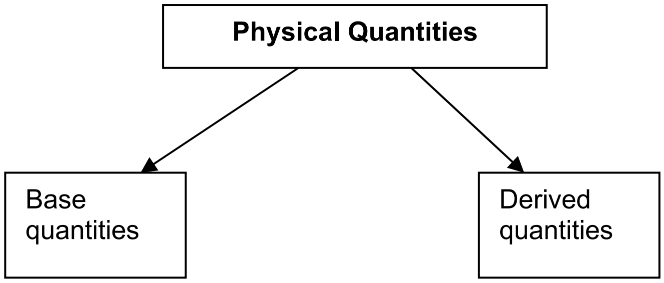
e.g. mass, m = 65.0 kg

Physical Quantity | Numerical value | unit

The Summary of some Quantities and Units used in the A-Level examination is given in Appendix 1.

In very much the same way that languages have developed in various parts of the world, different units of measurements have evolved. Just as languages can be translated from one to another, units of measurement can be converted between systems. For standardisation, it is much better to have just one system of units. Scientists use the **Système International (SI)** which is based on the metric system.

LO(a) recall the following base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).



Physical quantities are divided into *base* quantities and *derived* quantities.

Base quantities are the seven physical quantities of the SI system by which all other physical quantities are defined.

They are: mass, length, time, temperature, amount of substance, electric current and luminous intensity.

The SI base units are a choice of seven well-defined units which by convention are regarded as dimensionally independent: kilogram (kg), metre (m), second (s), ampere (A), kelvin (K), mole (mol). Candela (cd) is not in the syllabus.

LO (b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units'[1] as appropriate.

<u>Base units</u> are the seven units of the SI system, related to the base quantities, whose magnitude is defined without referring to other units.

<u>Derived units</u> are units that are derived from base units and can be expressed in terms of products and quotients of base units.

<u>Derived quantities</u> are physical quantities that are derived from base quantities and can be expressed in terms of products and quotients of base quantities.

E.g. The physical quantity speed, v, may be expressed in terms of the quantities distance x and time t by the defining equation $v = x/t$. x and t are base quantities, with SI base units metre m and second s respectively. Speed v is then a derived quantity, with the derived unit of speed metre-per-second, m s^{-1}.

Some derived units are given names for convenience.

Quantity	Derived Units	Special name	Symbol
Volume	$\text{m} \times \text{m} \times \text{m} = \text{m}^3$	–	–
Velocity	$\text{m} \div \text{s} = \text{m s}^{-1}$	–	–
Force	$\text{kg} \times (\text{m} \div \text{s} \div \text{s}) = \text{kg m s}^{-2}$	newton	N
Work done	$\text{kg m s}^{-2} \times \text{m} = \text{kg m}^2 \text{ s}^{-2}$	joule	J
Power	$(\text{kg m}^2 \text{ s}^{-2}) \div \text{s} = \text{kg m}^2 \text{ s}^{-3}$	watt	W

Some quantities are **<u>dimensionless / unitless</u>**, such as:

1. all numbers, e.g. 2, $\frac{1}{2}$, π, e
2. trigonometrical functions, e.g. sine, cosine, tangent
3. all logarithmic functions, e.g. $\log_x$, ln
4. powers, e.g. $10^{\frac{x}{y}}$, the ratio $\frac{x}{y}$ must be unit-less. If x has a unit, then the unit of y must have the same unit as that of x.
5. Unit-less physical constants: e.g. refractive index of glass, relative density of a liquid.

<u>Practice 1:</u>

Derived quantity	Obtained from	Derived unit	Special name
Density	mass / volume	kg m^{-3}	-
Frequency			
Pressure			
Charge			
Potential difference			

LO (c) Use SI base units to check the homogeneity of physical equation.

An equation is <u>homogeneous</u> if the base units of all the terms in the equation are the same.

Every term on both sides of the equal sign of an equation should have the same units, for the equation to be called *dimensionally consistent* or *homogeneous*. This is just plain common sense, as when $Z = X + Y$, we expect all quantities, Z, X and Y to represent the same item.

Consider the equation: $\boldsymbol{s = ut + \frac{1}{2}at^2}$

Unit of s = m

Unit of ut = (unit of velocity) × (unit of time) = $\text{m s}^{-1} \times \text{s} = \text{m}$

Unit of $\frac{1}{2}at^2 = \text{m s}^{-2} \times \text{s}^2 = \text{m}$

Since the units of every term on both sides of the equation are the same, the equation is homogeneous.

Next consider the equation: $\boldsymbol{v = u^2 + 2as^2}$

Unit of v = m s^{-1}
Unit of u^2 = (unit of velocity)^2 = (m s^{-1})^2 = m^2 s^{-2}
Unit of as^2 = m s^{-2} $\times$ m^2 = m^3 s^{-2}

The equation is not homogeneous.

Checking the homogeneity of an equation using base units is a powerful way of establishing if the physical equation is reasonable. It narrows the numerous combinations that may exist.

A physically correct equation must always be homogeneous. However, a homogeneous equation need not be physically correct. Why?

There are two basic reasons:

(1) The value of the dimensionless factor may be incorrect.

e.g. $E = 3mv^2$ where E = kinetic energy

The coefficient 3 is incorrect! The value should be $\frac{1}{2}$ instead.

(2) Missing or extra terms that may have the same unit.

e.g. $E = \frac{1}{2}mv^2 + mgh$ where E = kinetic energy

There is an extra term *mgh,* which happens to have the same unit as kinetic energy. This is an extra term.

Practice 2:

The period T of a simple pendulum is thought to depend on its length l, its mass m and the acceleration due to gravity g according to the equation $T = k\, l^x\, m^y\, g^z$, where k is a dimensionless constant (i.e. a constant with no unit). Determine the indices x, y and z.

Note that it is not possible to deduce the value of *k* through this analysis.

LO(d) show an understanding and use the conventions for labelling graph axes and table columns.

Conventions for Labelling Table Columns

– All readings or measurements should be tabulated in vertical columns;
For example

***V* / V**	***I* / mA**	***t* / s**	**a / m s^{-2}**	***m* / kg**	**lg *(m* / kg*)***
3.0	0.30	2.0	5.0	2.500	0.3979

– For the column headings, use the standard notation of "**quantity / unit**"; it means "**quantity divided by unit**".

For example
Writing t = 2.0 s as t / s = 2.0,

the expression becomes a pure number. The column with t / s thus consists of just pure numbers with no units.

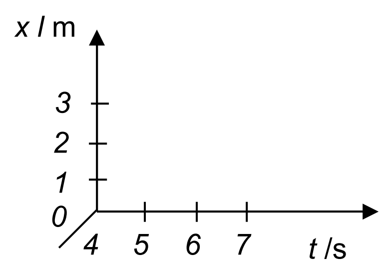
– The unit should be written in the index form, e.g. use m s^{-2} and not m / s^2.
– For columns that involve logarithms, either lg or ln, the unit of the variable must be stated.

For example, lg(m / kg), ln(T / s).

It should be noted that after taking lg or ln, the resulting values have no units.

Conventions for Labelling Graph Axes

– when plotting graphs, both axes must be labelled with the physical quantities and their associated units (e.g. x / m and t / s). Therefore, there is no need to write unit to every number label on the axes.



We will cover these in greater detail during practical lessons.

LO(e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).

10^n	Prefix	Symbol	Name	Decimal equivalent
10^{-12}	pico	p	Trillionth	0.000,000,000,001
10^{-9}	nano	n	Billionth	0.000,000,001
10^{-6}	micro	μ	Millionth	0.000,001
10^{-3}	milli	m	Thousandth	0.001
10^{-2}	centi	c	Hundredth	0.01
10^{-1}	deci	d	Tenth	0.1
10^{0}	-	-	One	1
10^{3}	kilo	k	Thousand	1,000
10^{6}	mega	M	Million	1,000,000
10^{9}	giga	G	Billion	1,000,000,000
10^{12}	tera	T	Trillion	1,000,000,000,000

Note:
- Symbols are case-sensitive. Use “k” for kilo, not “K”.
- There is no space between prefix and the unit, e.g. write “km” not “k m”.

LO(f) make reasonable estimates of physical quantities within the syllabus.

All estimates are to be stated to 1 significant figure only, unless stated otherwise.

Mass	**Approximately (kg)**	**Scientific notation (kg)**
Ping pong ball	0.003	3×10^{-3}
An apple	0.2	2×10^{-1}
Adult Asian man	70	7×10^{1}
Car	1000 - 1500	1×10^{3}
SBS bus	20,000	$2. \times 10^{4}$
Earth	6×10^{24}	6×10^{24}
Length	**Approximately (m)**	**Scientific notation (m)**
Diameter of hair	0.0002 (0.2 mm)	2×10^{-4}
Finger	0.07 (7.0 cm)	7×10^{-2}
Width of a car	2	2×10^{0}
Football field	100	1×10^{2}
Radius of Earth	6×10^{6}	6×10^{6}
Speed	**Approximately (m s^{-1})**	**Scientific notation (m s^{-1})**
Fastest runner	10	1×10^{1}
Sound in air	300	3×10^{2}
Light in vacuum (c)	3×10^{8}	3×10^{8}
α-particle	0.1 c	3×10^{7}
β-particle	0.8 c	2×10^{8}
Temperature	**Approximately (K)**	**Scientific notation (K)**
Room	300	3×10^{2}
Candle flame	1000	1×10^{3}
Blue Bunsen flame	3000	3×10^{3}

2 Scalars and vectors

LO (g) distinguish scalar and vector quantities and give examples of each.

- Scalar is a quantity that has <u>magnitude, not direction</u>. It is completely specified by its numerical value and unit.

- Scalar quantities can be added or subtracted using rules of algebra.

  e.g. 10 kg + 12 kg = 22 kg

- Vector is a quantity having both <u>magnitude</u> and <u>direction</u>. It must be specified with its value, unit and direction.

  e.g.: 2.5 m s^{-1} due East

  direction

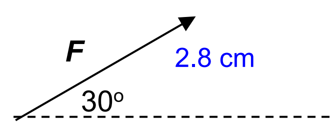
  magnitude

  In print, a vector is often denoted by a letter in bold type, e.g. force ***F***. The magnitude is indicated as | ***F*** |.

- A vector can be represented with an arrow whose length is proportional to the magnitude of the vector, is correctly orientated with respect to a reference direction.

Scale: 2 cm represents 1 m s^{-1}.

The velocity is 2.45 m s^{-1} due East.



Scale 1 cm: 5 N

The force F is 14 N, in a direction 30° to the horizontal.

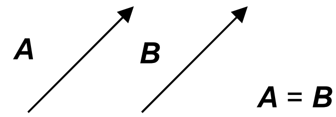
The table below shows a list of scalar and vector quantities:

Scalar quantity
distance
speed
temperature
energy
power
mass
density
pressure
volume
time

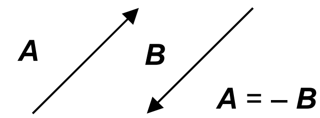
Vector quantity
displacement
velocity
acceleration
force
momentum
weight
moment
torque
electric field
magnetic flux density

LO (h) add and subtract coplanar vectors

Equal vectors
Vectors are equal if they have the *same magnitude and direction*.



Negative vector
The negative of a vector has the *same magnitude but opposite direction*.



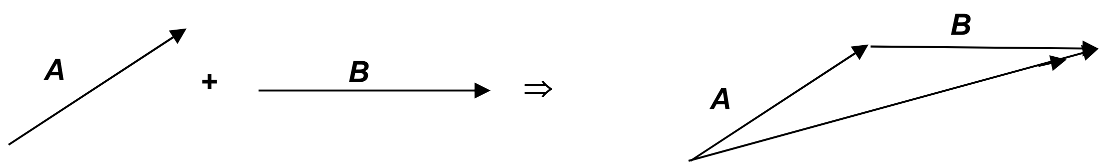
Coplanar vectors are vectors that lie on the same plane (2-dimension).

Unlike scalar quantities, vector quantities cannot be added or subtracted using algebra. Instead a *vector diagram* is needed.

(1) Vector Addition

a. *<u>Triangle method</u>*

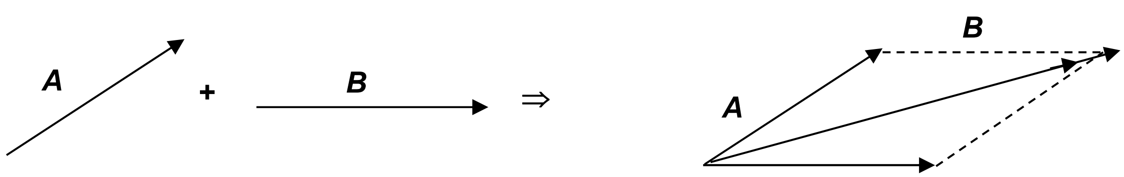
Stack the vectors such that one vector has its tail placed at the tip of the previous vector.



The resultant vector is represented by the line with a *<u>double arrow</u>* directed from the tail of the first vector to the tip of the last vector.

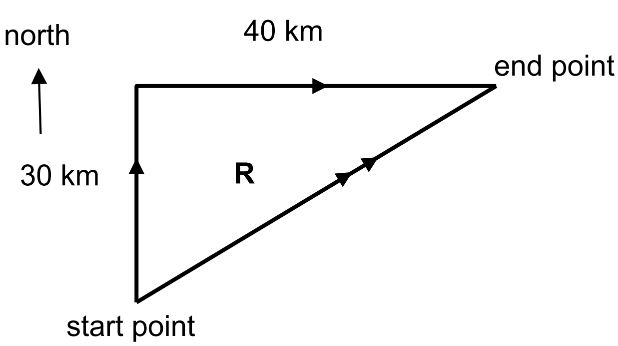
b. *<u>Parallelogram method.</u>*

Put the vectors to be added 'tail to tail'. Complete the parallelogram. The resultant vector is the diagonal from the tail of the two vectors to the other vertex of the parallelogram.

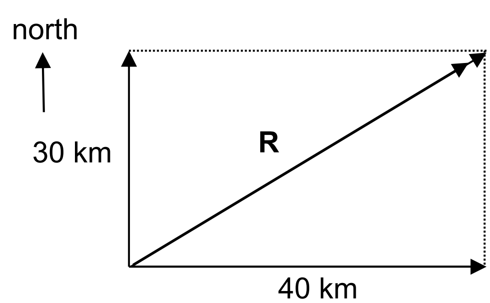


Example

Evaluate the resultant displacement of a ship which travels 30 km due north then 40 km due east.



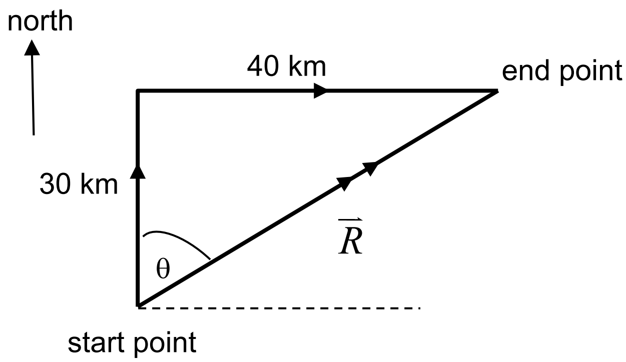
OR



<u>Methods to determine the resultant magnitude and direction</u>

1) Draw the vector to scale, measure the resultant magnitude using a ruler, and angle (for direction) using protractor. OR

2) (a) Use Pythagoras Theorem, trigonometric function to solve for right angle triangle.
(b) Use Sine Rule, Cosine Rule for irregular triangle.



Worked Solution:

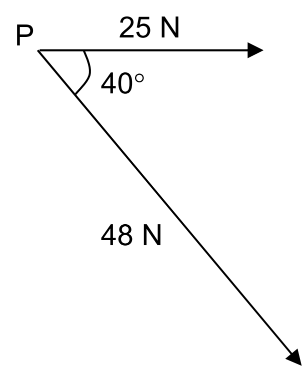
R = $\left|\vec{R}\right| = \sqrt{40^2 + 30^2}$ = 50 km

$\theta = \tan^{-1}(40/30) = 53.1^\circ$

The resultant displacement is 50 km, 53.1° East of North.

Practice 3:

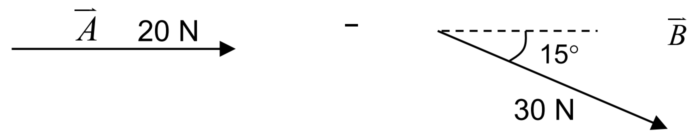
Two forces act at a point P as shown below. Determine (magnitude and direction of) the resultant of these two forces.



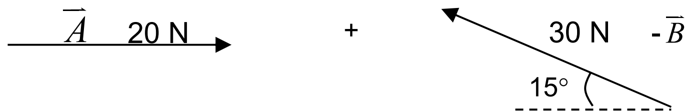
(2) Vector subtraction

$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$.

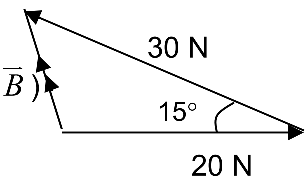
Example



i.e.



Resultant = $(\vec{A} - \vec{B})$



Check
using cosine rule that $|\vec{A} - \vec{B}|$ = 12 N,
using sine rule that it is 139° to the horizontal.

Vector subtraction can come about when we want to determine the *change* in a certain physical quantity.

A change in a physical quantity $\Delta\vec{Q}$ = final value $\vec{Q}_f$ – initial value $\vec{Q}_i$

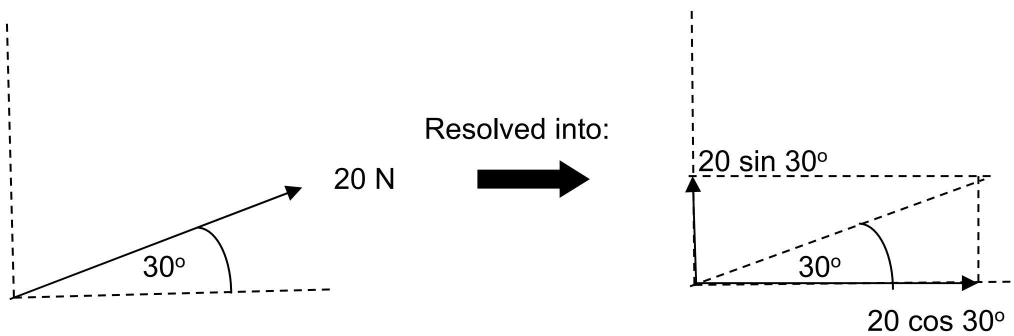
Practice 4:

An object is moving at 5.0 m s^{-1} due east. Its direction changes to due south with a speed of 7.5 m s^{-1}. Determine the *change* in the velocity.

LO(i) represent a vector as two perpendicular components

Since two vectors can be added to give a resultant vector, any vector can be broken up (or resolved) into two vectors or components. It is more convenient to resolve a vector into two mutually-perpendicular components through the use of trigonometry and Pythagoras theorem. Mutually-perpendicular vectors are independent of each other.

Example



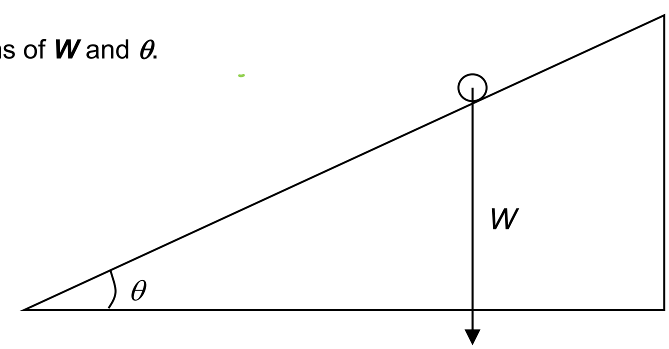
Each of the vectors above is resolved into two perpendicular components. A vector can be resolved into infinite pairs of perpendicular components. The choice of directions depends on the problem at hand.

Practice 5:

An object rests on the plane of an inclined slope as shown. The weight ***W*** acts vertically down. Draw components of the weight

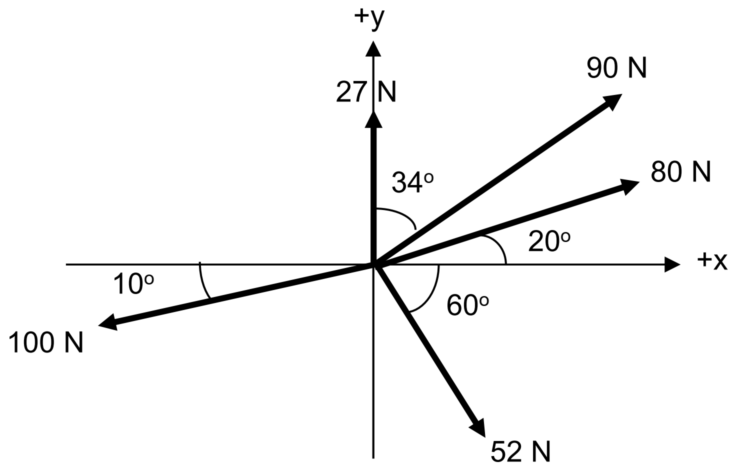
(a) parallel to the slope
(b) perpendicular (normal) to the slope.

Label the magnitude of the two components in terms of ***W*** and θ.



Practice 6 (step-by-step guide to finding the resultant force)

The 5 forces shown act on an object. Find the resultant force due to them.



- Resolve the vectors into two mutually perpendicular components:

Vector/N	**x-component /N (+ →)**	**y-component /N (+↑)**
100		
27		
90		
80		
52		
Resultant		

1. Draw vector diagram to show the resultant vector

2. Compute the magnitude of the resultant vector

3. Indicate the direction with an angle with reference to a certain direction on the diagram and calculate the angle.

4. Write a complete statement to specify the magnitude and direction of the resultant vector.

3 Errors and Uncertainties

Uncertainty is the range of values on both sides of a measurement in which the actual value of the measurement is expected to lie.

Uncertainties in measured quantities arise from:

- limitations of the observer;
- limitations of the measuring instrument used,
- limitations of the method used (the experimental design).

3.1 Experimental Errors

Error is the difference between the measured value and the 'true value'.

For example, if the accepted value of acceleration due to gravity *g* at a certain location is 9.8 m s^{-2}. If an experimental determination yields a result of 9.9 m s^{-2}, the error is +0.1 m s^{-2}.

3.2 Types of Experimental Errors

LO(j) show an understanding of the distinction between systematic errors (including zero errors) and random errors.

In assessing errors, whether human or instrumental, there are two types of errors:
(1) systematic errors and (2) random errors

Systematic errors are present when the measured values produced errors of the <u>same magnitude and sign.</u>

They cannot be eliminated by averaging. However, they can be eliminated by careful design of an experiment, and good experimental techniques. They have the same magnitude and sign.

Examples of Systematic Errors:

Error Sources	Descriptions	Corrections
– Due to apparatus	• zero errors on the scales of instruments • poor calibration of instruments	• correct all measured readings by negating the error accordingly. • calibrate the instrument properly before experiment
– Due to poor experiment-al technique	• consistent parallax error which affect all the readings in the same way, for instance, taking readings off a scale from a fixed angle.	• adopt the correct way to take reading: ensure that the *line of sight* is *perpendicular* to the measuring scale.
– Due to external factors	• *background radiation* causes the count rate of your radioactive sample to be consistently higher than the true reading.	• Take the external factor(s) into account and adjust all readings appropriately. For instance, measure the count rate of background radiation and subtract it from the readings.

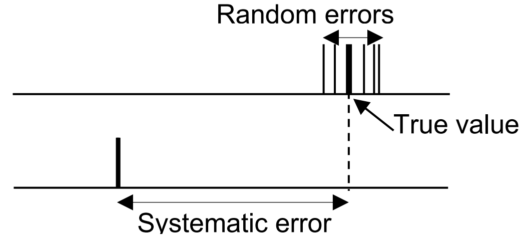
(2) Random errors are present when the measured values produced errors of different magnitudes and signs. These readings are scattered about the mean value with no fixed pattern.

Examples of Random Errors:

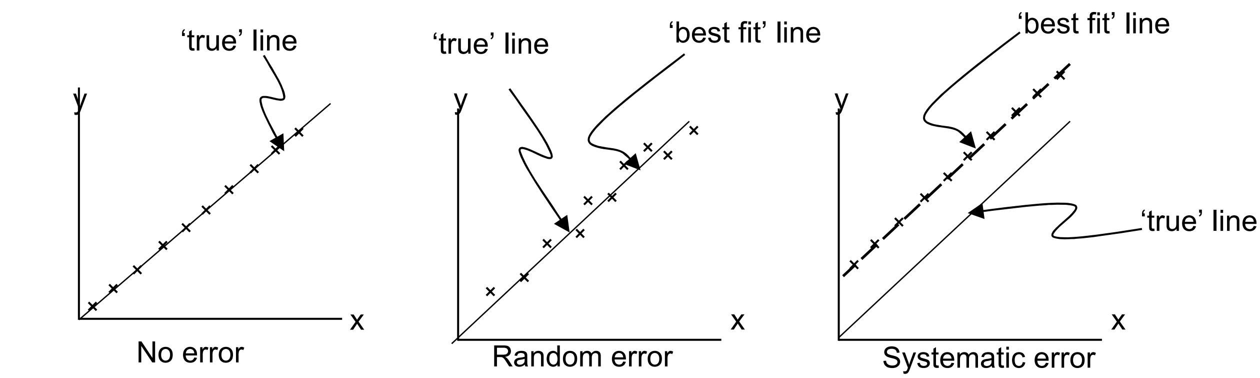
Error Sources	Descriptions	Methods to reduce error
– Due to inability of observer to repeat his action precisely.	• inconsistent reaction time when using stop watch to measure the period of an oscillation.	• To reduce the random errors, take the time of 20 oscillations then find the mean period.
– Due to environmental conditions like pressure, temperature.	• fluctuations in the measurement (of length, time, current, etc.)	• Random errors can be reduced by taking average of repeated readings.
– Due to the limited sensitivity of instruments.	• Precision of metre rule: 1 mm vernier calliper: 0.1 mm micrometer screw gauge: 0.01 mm	• To measure the thickness of a coin with only a metre rule, the random errors can be reduced by determining the mean thickness of a stack of 10 identical coins. (Use a micrometer screw gauge if it is available).

Distinguish between systematic and random errors using Illustrations.

The figure shows a spread of readings caused by random errors; these are approximately centred about a mean value which coincides with the *true value*. If a systematic error is present, the mean value will be shifted.

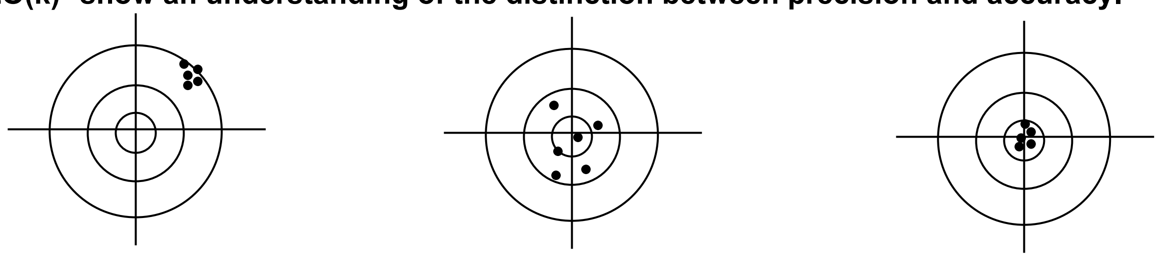


The effects of random errors and of systematic errors appear in graphs as illustrated below.



3.3 Precision and Accuracy

LO(k) show an understanding of the distinction between precision and accuracy.



Good precision,
Poor accuracy
(small random error, but large systematic error!)

Good accuracy,
Poor precision
(large random error, small systematic error)

Good precision and accuracy
(small random error, small systematic error)

The **precision** of a measurement is <u>how close</u> the experimental values are <u>to each other</u>.

Precision is also a term used to describe the level of uncertainty in an instrument's scale. Good precision means the readings are mostly very *close to each other*, and is <u>associated with small random errors</u>.

<u>Accuracy</u> is the <u>closeness</u> of a reading on an instrument to the <u>true value</u> of the <u>quantity being measured</u>.

Good accuracy means the reading or the mean of a set of readings is very close to the true value, and is <u>associated with small systematic errors</u>.

Illustration:

The first few decimal places of the true value for the mathematical constant π are 3.142, and the accepted value for the speed of light in a vacuum is 2.99792458×10^8 m s^{-1}.

Thus, 3.14 is an ***accurate*** value for π to three digits ***precision***, and 3.0×10^8 m s^{-1} is an ***accurate*** value for the speed of light in a vacuum to two digits ***precision***.

Practice 7

Student A carried out a series of experiments to obtain the value of the Earth's gravitational acceleration ***g***. He was very careless and did not take into account the mass of the mass holder. This resulted in all recorded values of the mass being smaller than the actual mass. He obtained the following values of ***g***:

Reading/ m s^{-2}	10.33	10.32	10.31	10.30	10.29

Given that the actual value of ***g*** = 9.81 m s^{-2}, comment on the accuracy and precision for this experiment.

Practice 8

Student B measured the gravitational constant ***g*** a number of times and got the following results

Reading/ m s^{-2}	**12.20**	**9.81**	**7.42**	**9.99**	**9.63**

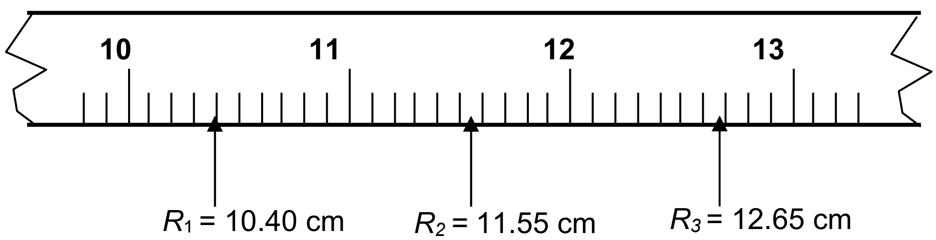
How do these results compare to those of Student A in worked example 1?

3.4 Absolute and Relative (Fractional and Percentage) Uncertainties

3.4.1 Absolute Uncertainty in a scale reading

- A scale reading is the single determination of a value at one point on a measuring scale.

- Generally a scale reading can be estimated to *half* of the smallest division on a measuring scale (Not always true: this depends on the instruments used. More details will be taught during the Practical Sessions).



- The (actual or absolute) *uncertainty* in the reading of an instrument is thus taken as *half the smallest division*. In the above illustration, the *absolute* uncertainty is 0.05 cm.

- To indicate the magnitude of uncertainty, R_1=10.40 cm in the above illustration is written as $R_1 = (10.40 \pm 0.05)$ cm. This means that R_1 can take values in the range between 10.35 cm to 10.45 cm.
- The absolute uncertainty 0.05 cm is also known as the *maximum uncertainty* in the reading.

- The absolute uncertainty should always be **rounded off to 1 significant figure only.**

- In general, all readings can be recorded in the form $R \pm \Delta R$ in which ΔR is the absolute uncertainty. R should be rounded to the **same number of decimal places as the uncertainty ΔR.**

  For example (10.0 ± 0.1) cm should *not* be written as (10 ± 0.1) cm

Relative Uncertainties

The uncertainty of a measured value can also be presented as a percent or a simple fraction.

(a) The fractional uncertainty of $R = \dfrac{\Delta R}{R}$

(b) The percentage uncertainty of $R = \dfrac{\Delta R}{R} \times 100\%$

For example, the measurement (208 ± 1) mm,

Absolute uncertainty = 1 mm (1 s.f.)

Fractional uncertainty = $\dfrac{1}{208}$ = 0.0048 (2 s.f.)

Percentage uncertainty = 0.0048 x 100% = 0.48% (2 s.f.)

Note that the absolute uncertainty has units and always to 1 s.f., whereas fractional and percentage uncertainties are ratios and are dimensionless and always to 1 or 2 s.f.

While the absolute uncertainty is an indication of the scale sensitivity of the measuring instrument used, the percentage uncertainty is useful to compare whether the error is negligible or significant.

Practice 9

A metre rule has a precision to 1 mm, vernier callipers to 0.1 mm and a micrometer screw gauge to 0.01 mm. If these instruments were to be used to measure the diameter of a wire which is 2.5 mm, what would be the respective percentage uncertainties?

Practice 10

A student makes measurements from which he calculates the speed of sound as 327.66 m s^{-1}. He estimates that his result contains a percentage error of ± 3%. Give his result reduced to the appropriate number of significant figures.

3.4.2 Propagation of Uncertainties

LO (l) assess the uncertainty in a derived quantity by addition of actual, fractional or percentage uncertainties, or numerical substitution (a rigorous statistical treatment is not required).

The result of an experiment is seldom obtained by a single measurement; very often it is obtained by measuring a few related quantities. The overall estimate of uncertainty is called the **consequential uncertainty**.

There are established statistical rules for calculation of consequential uncertainty from individual pieces of information. The A-level course only requires a simplified version of the statistical treatment.

The guiding principle in all cases is to consider the maximum uncertainty i.e. the worst case scenario.

Basic Rules of Consequential Uncertainties at a Glance

Suppose A and B are measured independent quantities, ΔA and ΔB are the corresponding (actual) uncertainties.

Given that n and m are numerical constants, e.g. ½, 3, π.

Addition
If $R_1 = A + B$, then $\Delta R_1 = \Delta A + \Delta B$

Subtraction
If $R_2 = A - B$, then $\Delta R_2 = \Delta A + \Delta B$

Adding absolute uncertainties

Product
If $R_4 = A \times B$

$$\frac{\Delta R_4}{R_4} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Quotient
If $R_5 = \dfrac{A}{B}$

$$\frac{\Delta R_5}{R_5} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Adding fractional uncertainties

Some common situations:

1. If $Z = mA + nB$, then $\Delta Z = |m|\Delta A + |n|\Delta B$ Note: m, n are numerical constants

2. If $Z = kA^m B^n$, where k is a numerical constant, then $\dfrac{\Delta Z}{Z} = |m|\dfrac{\Delta A}{A} + |n|\dfrac{\Delta B}{B}$

3. Other functions: e.g. $Z = \sin A$, $Z = \ln A$

   Use the general approach: $\Delta Z = ½ (Z_{max} - Z_{min})$

Propagation Rules of Consequential Uncertainties – Explained.

<u>Additional and subtraction:</u>

Suppose we need to add the measurements A: (20 ± 2) cm and B: (10 ± 1) cm

The best estimate for the sum of these two measurements is 20 cm + 10 cm = 30 cm
The smallest possible sum in adding these two measurements is 18 cm + 9 cm = 27 cm.
The largest possible sum in adding these two measurements is 22 cm + 11 cm = 33 cm.

Therefore, the range of possible sum ranges from 27 cm to 33 cm.

The sum can be expressed as (30 ± 3) cm.

Instead of going through the tedious steps like the above, apply the short-cut rule! Verify the rule for subtraction!

<u>Multiplication and division:</u>

Suppose you need to multiply A: (20 ± 2) cm by B: (10 ± 1) cm.

The best estimate of the product is 20 cm x 10 cm = 2.0×10^2 cm^2
The smallest possible value would be 18 cm x 9 cm = 1.6×10^2 cm^2,
The largest reasonable value would be 22 cm x 11 cm = 2.4×10^2 cm^2.
The product must be $(2.0 \pm 0.4) \times 10^2$ cm^2.

Check this rule for division for yourself.

Practice 11:

A rectangle has a length l = (34.3 ± 0.6) cm and breadth b = (21.8 ± 0.5) cm.
Calculate the perimeter P and area A of the rectangle and express their values with their absolute uncertainties.

Practice 12:

The formula for the period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Such a pendulum is used to determine g. The fractional error in the measurement of the period T is $\pm x$ and in the measurement of the length l is $\pm y$. Assume there is no other sources error, what is the fractional error in the calculated value of g?

For complicated functions of Z, we can use the first principle method of numerical substitution to find the max and min values.

Absolute uncertainty = ½ (maximum possible value of Z – minimum possible value of Z)

$$\Delta Z = \frac{1}{2}(Z_{max} - Z_{min})$$

Or simply,

Absolute uncertainty = maximum possible value of Z – Z

$$\Delta Z = Z_{max} - Z$$

The difference in methods may result in slightly different uncertainty results. All answers are acceptable as long as the method is logical.

Practice 13:

Consider $S = x \cos\theta$ for x = (2.0 ± 0.2) cm, θ = (53 ± 2) °. Find S with its uncertainty.

Definition List

Physical Quantity	A term that is used to include numerically measurable features of many different items. It has a numerical magnitude and a unit.
Base quantities	The seven physical quantities of the SI system by which all other physical quantities are defined.
Base Units	The seven units of the SI system, related to the base quantities, whose magnitude is defined without referring to other units.
Derived quantities	They are physical quantities that are derived from base quantities and can be expressed in terms of products and quotients of base quantities.
Dimensionless quantity	A quantity without an associated physical dimension, it is a pure number.
Homogeneous Equation	Equation where base units of all the terms are the same.
Experimental errors	Uncertainties in measured quantities that arise from different sources due to: (a) limitations of observer, (b) measuring instrument used, (c) method used.
Random errors	Random errors are present when the measured values produced errors of different magnitudes and signs. These readings are scattered about the mean value with no fixed pattern.
Systematic errors	Systematic errors are present when the measured values produced errors of the same magnitude and sign.
Precision	The precision of a measurement is how close the experimental values are to each other.
Accuracy	The closeness of a reading on an instrument to the true value of the quantity being measured.
Uncertainty	The range of values on both sides of a measurement in which the actual value of the measurement is expected to lie.
Absolute uncertainty	Absolute uncertainty is the actual numerical uncertainty. Eg, measurement of length ($L \pm \Delta L$), ΔL is the absolute uncertainty.
Fractional uncertainty	Fractional uncertainty is the ratio of absolute uncertainty to the measured value of a quantity.
Vector	Vector is a quantity having both magnitude and direction.
Scalar	Scalar is a quantity that has magnitude, not direction.

APPENDIX A

National Measurement System

National Metrology Centre (NMC) is the custodian of the national physical measurement standards in Singapore and is responsible for establishing and maintaining the nation's highest level of physical measurement standards. This can be traced to the International System of Units (SI) established under the Metre Convention, a worldwide diplomatic treaty on metrology.

The way the standards are defined can be changed and Singapore will want to keep track of this development to ensure we meet the national standards. An example is the change in the way 1 kg is being defined.

Singapore has one of the hundred or so pieces that nations have purchased from BIPM[2] to date. And this precious piece is housed inside the NMC. Singapore's prototype kilogram is actually a replica of the world's official 1 kg, also known as Le Grand K (the grand kilogram).

Singapore bought the piece in 2003, and it was hand-carried here from Paris. Scientists at NMC handle it more carefully than they would a baby. They use only tongs to pick it up. They wear gloves because the slightest trace of oil from their fingers would add weight to this 1 kg. Scales at the NMC lab go down to the microgram, or a millionth of a gram, so every trace matters.

Singapore's prototype kilogram outweighs Le Grand K by about 340 micrograms. So it's almost impossible to cast metals down to one microgram. Apparently, even Le Grand K is no longer a "perfect" kilogram. Over the course of a century, it has actually shed a few micrograms.

This development worries scientists who require precise definitions of the kilogram for other measurements such as voltage. The kilogram is the only base unit defined by an artefact. To eliminate the uncertainty raised by the current definitions, scientists have been studying how to redefine 1 kg using a natural constant called Planck constant.

Such precision might seem to be relevant only to scientists, but it has its place in everyday tasks too. For instance, Singapore's definitive 1 kg and stainless steel replicas are used to calibrate weighing scales in the pharmaceutical and petrochemical industries.

Precision is important for business, trade and life in general. After all, we would want to know that our medicines and chemicals are safe, and we should get no less than what we pay for.

Redefining the SI units

In November 2018, at the General Conference on Weights and Measures, the global metrology community agreed a revision to the SI. The decision means that, for the first time, all seven of the base units will be defined in terms of constants of nature – such as the speed of light, the Planck constant and the Avogadro constant. Using seven defining constants as the basis for the SI will mean that the definitions of all the base units will stay stable into the future. The revision will bring in new definitions of the ampere, kilogram, kelvin (and, consequently, degree Celsius) and mole.

From May 2019, all the base units of the SI will be defined in terms of constants of nature – the most stable quantities we have ever encountered.

The kilogram is defined by taking the fixed numerical value of the Planck constant h to be $6.626\ 070\ 15 \times 10^{-34}$ when expressed in the unit J s, which is equal to kg $m^2\ s^{-1}$, where the metre and the second are defined in terms of c and Δv. [c: speed of light in vacuum; Δv: caesium frequency.]

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[2] Bureau International des Poids et Mesures (BIPM), the Paris-based agency that acts as the international custodian for weights and measures.

APPENDIX B

SUMMARY OF KEY QUANTITIES, SYMBOLS AND UNITS

The following list illustrates the symbols and units that will be used in question papers.

Base Quantities

Quantity	*Usual symbols*	*Usual unit*	*Quantity*	*Usual symbols*	*Usual unit*
mass	m	kg	electric current	I	A
length	l	m	thermodynamic temperature	T	K
time	t	s	amount of substance	n	mol

Other Quantities

Quantity	*Usual symbols*	*Usual unit*	*Quantity*	*Usual symbols*	*Usual unit*
distance	d	m	elementary charge	e	C
displacement	s, x	m	electric potential	V	V
area	A	m^2	electric potential difference	V	V
volume	V, v	m^3	electromotive force	E	V
density	ρ	kg m^{-3}	resistance	R	Ω
speed	u, v, w, c	m s^{-1}	resistivity	ρ	Ω m
velocity	u, v, w, c	m s^{-1}	electric field strength	E	N C^{-1}, V m^{-1}
acceleration	a	m s^{-2}	permittivity of free space	ε_0	F m^{-1}
acceleration of free fall	g	m s^{-2}	magnetic flux	Φ	Wb
force	F	N	magnetic flux density	B	T
weight	W	N	permeability of free space	μ_0	H m^{-1}
momentum	p	N s	force constant	k	N m^{-1}
work	w, W	J	Celsius temperature	θ	°C
energy	E, U, W	J	specific heat capacity	c	J K^{-1} kg^{-1}
potential energy	E_p	J	molar gas constant	R	J K^{-1} mol^{-1}
kinetic energy	E_k	J	Boltzmann constant	k	J K^{-1}
heating	Q	J	Avogadro constant	N_A	mol^{-1}
change of internal energy	ΔU	J	number	N, n, m	
power	P	W	number density (number per unit volume)	n	m^{-3}
pressure	p	Pa	Planck constant	h	J s
torque	T	N m	work function energy	Φ	J
gravitational constant	G	N kg^{-2} m^2	activity of radioactive source	A	Bq
gravitational field strength	g	N kg^{-1}	decay constant	λ	s^{-1}
gravitational potential	ϕ	J kg^{-1}	half-life	$t_{1/2}$	s
angle	θ	°, rad	relative atomic mass	A_r	
angular displacement	θ	°, rad	relative molecular mass	M_r	
angular speed	ω	rad s^{-1}	atomic mass	m_a	kg, u
angular velocity	ω	rad s^{-1}	electron mass	m_e	kg, u
period	T	s	neutron mass	m_n	kg, u
frequency	f	Hz	proton mass	m_p	kg, u
angular frequency	ω	rad s^{-1}	molar mass	M	kg
wavelength	λ	m	proton number	Z	
speed of electromagnetic waves	c	m s^{-1}	nucleon number	A	
electric charge	Q	C	neutron number	N	

Tutorial Questions

Part 1 Physical Quantities and SI Units

1(a) Express the following derived quantities in terms of base units. Give the alternative units, if applicable.

Derived quantities	Defining Equation	Derived units	Alternative unit(s)
Area			
Moment			
Pressure			
Work done			
Power			
Resistance			

1(b) The energy of a photon of light of frequency *f* is given by *hf*, where *h* is the Planck constant. What are the base units of *h*? [kg m^2 s^{-1}]

2 The drag coefficient C_D of a car moving with speed *v* through air of density ρ is given by $C_D = \dfrac{F}{\frac{1}{2}\rho A v^2}$ where *F* is the drag force exerted on the car and *A* is the maximum cross-sectional area of the car perpendicular to the direction of travel. Show that C_D is dimensionless (i.e. unitless in this case).

3 By considering the base units of the quantities, determine whether the following expressions for *v* the velocity of ocean waves is possible. ρ is the density of seawater, *g* the acceleration of free fall, *h* the depth of the ocean and λ the wavelength of the wave.

(A) $v = \sqrt{g\lambda}$ (B) $v = \sqrt{\dfrac{g}{h}}$ (C) $v = \sqrt{\rho g h}$ (D) $v = \sqrt{\dfrac{g}{\rho}}$

4 The density ρ and the pressure *P* of a gas are related by the expression

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

where *c* and γ are constants.

(i) Determine the base unit of pressure *P*. [kg m^{-1} s^{-2}]
(ii) Given that the constant γ has no unit, determine the unit of *c*. [m s^{-1}]
(iii) Using your answer to (ii), suggest what quantity may be represented by the symbol *c*.

5 Bernoulli's equation, which applies to fluid flow, states that

$$p + h\rho g + \frac{1}{2}\rho v^2 = k$$

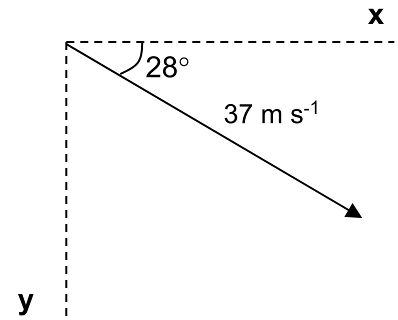
where *p* is the pressure of the fluid, *h* is the height of the fluid, ρ is the density of the fluid, *g* is the acceleration due to gravity, *v* is the velocity of the fluid flow and *k* is a constant.
(i) Show that the terms on the left hand side of the equation have the same SI base units.
(ii) What are the SI base units for *k*? [kg m^{-1} s^{-2}]

Part 2 Scalars and Vectors

6 Draw the two perpendicular components x and y associated with the given vector in the directions shown. Calculate the magnitude of the two components.
Note: The diagrams are not draw to scale.

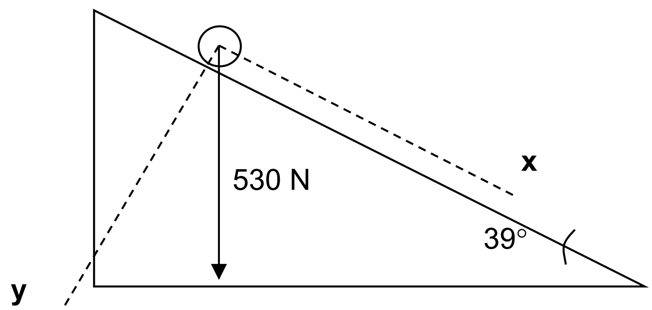
(a) component x: …………………….

component y: ……………………..



(b) component x: …………………….

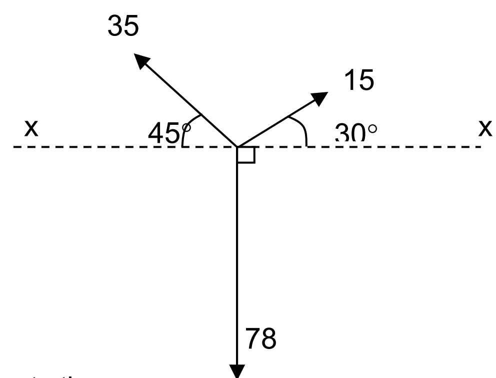
component y: ……………………..



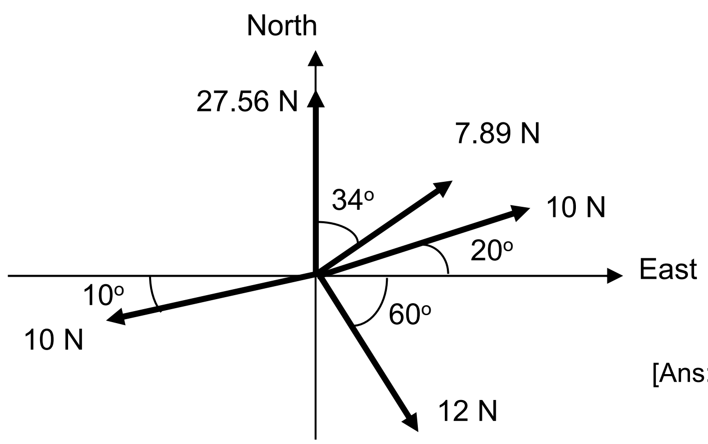
7 Three coplanar forces act on a point as shown.

(i) Determine the resultant vector (magnitude and direction with reference to x – x direction.
[Ans: 47.3 N, 76° acw from –ve x-axis]

(ii) State the smallest force that must be applied such that the resultant is along x – x only.
[Ans: 45.8 N, 90° acw from +ve x-axis.]



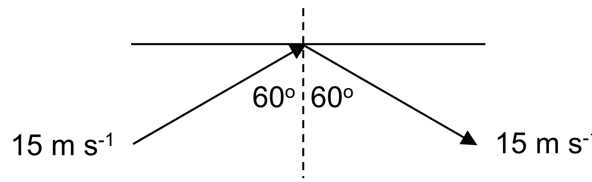
8 The 5 forces shown act on an object. Find the resultant force due to them.



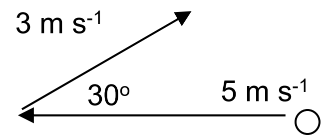
[Ans: 27.3 N, 21.4° E of N]

9a A tennis player was able to return a serve straight back. The ball travelled at 35 m s^{-1} just before it was caught in the racket, and 30 m s^{-1} after it was hit. Show the velocity vectors and hence determine the change in velocity. [Ans: 65 m s^{-1} away from the racket]

9b A ball bounced off a wall with as follows:
What is the change of velocity?
[Ans: 15 m s^{-1}, vertically downwards]



9c Find the change in velocity of a ball if it changes its velocity from 5 m s^{-1} horizontal to the ground to 3 m s^{-1} at 30° upwards after hit by the racket.
[Ans: 7.7 m s^{-1}, 11 ° ccw from horizontal]



Part 3 Errors & Uncertainties

10 A micrometer, reading to ±0.01 mm, gives the following results when used to measure the diameter *d* of a uniform wire: (J2000/I/2)

1.02 mm, 1.02 mm,1.01 mm, 1.02 mm, 1.02 mm

When the wire is removed and the jaws are closed, a reading of – 0.02 mm is obtained.
Which of the following gives the value of *d* with a precision appropriate to the micrometer?

A 1.0 mm **B** 1.00 mm **C** 1.038 mm **D** 1.04 mm

11 When comparing systematic and random errors, the following pairs of properties of errors in an experimental measurement may be contrasted:

P1: error can possibly be eliminated
P2: error cannot possibly be eliminated
Q1: error is of constant sign and magnitude
Q2: error is of varying sign and magnitude
R1: error will be reduced by averaging repeated measurements
R2: error will not be reduced by averaging repeated measurements

Which properties apply to random errors?

A P1, Q1, R2 **B** P1, Q2, R1 **C** P2, Q2, R1 **D** P2, Q1, R1

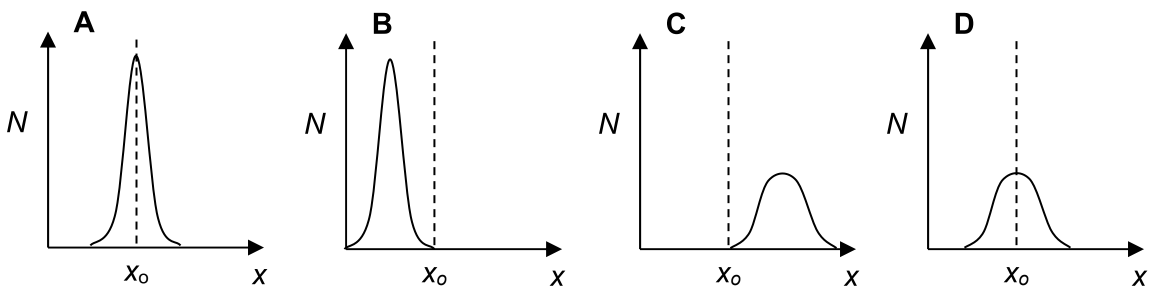
12 Which of the following experimental techniques reduces the systematic error of the quantity being investigated?

A	Timing a large number of oscillations to find a period
B	Measuring the diameter of a wire repeatedly and calculating the average
C	Adjusting an ammeter to remove its zero error before measuring a current
D	Plotting a series of voltage and current readings for an ohmic device on a graph and using its gradient to find resistance

13 An object of mass 1.000kg is placed on four different balances. For each balance the reading is taken five times. The table shows the values obtained together with the means. Which balance has the smallest systematic error but is not very precise? (N2002/I/2)

balance	Reading/ kg					mean/kg
	1	2	3	4	5	
A	1.000	1.000	1.002	1.001	1.002	1.001
B	1.011	0.999	1.001	0.989	0.995	0.999
C	1.012	1.013	1.012	1.014	1.014	1.013
D	0.993	0.987	1.002	1.000	0.983	0.993

14 A quantity x is measured many times and the number N of measurements giving a value x is plotted against x. The true value of the quantity is x_o.
Which graph best represents precise measurements with poor accuracy?



15 A student uses a micrometer screw gauge to measure the diameter of a wire. He fails to notice that with the gauge fully closed, the reading is not zero. (J2002/2/2)

(a) State and explain whether the omission introduces a random error or a systematic error into the readings of the diameter.
(b) Explain why the readings are precise but not accurate.

16 The quantities p and q are measured with estimated errors Δp and Δq. The fractional uncertainty in p/q is at most

A $\Delta p + \Delta q$ **B** $\Delta p - \Delta q$ **C** $\dfrac{\Delta p}{p} + \dfrac{\Delta q}{q}$ **D** $\dfrac{\Delta p}{p} - \dfrac{\Delta q}{q}$

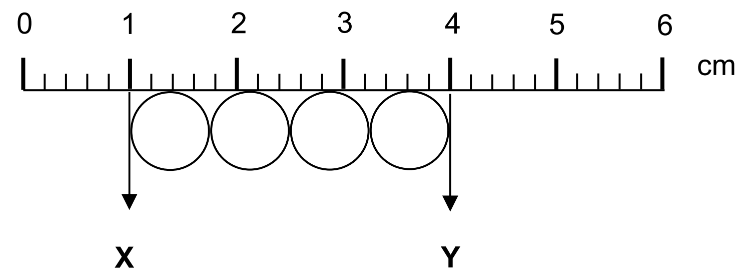
17 The power loss P in a resistor is calculated using the formula $P = V^2/R$.
The uncertainty in the potential difference V is 3% and the uncertainty in the resistance R is 2%. What is the percentage uncertainty in P? (modified from J2002/I/5)

18 A student attempts to measure the diameter a steel ball by using a metre rule to measure four similar balls in a row.

The student estimates the positions on the scale to be as follows.

X (1.0 ± 0.1) cm
Y (4.0 ± 0.1) cm

What is the diameter of a steel ball together with its associated uncertainty? (modified J99/P1/2)



[Ans: (0.75 ± 0.05) cm]

19 A student takes the following readings of the diameter of a wire: 1.52 mm, 1.48 mm, 1.49 mm, 1.51 mm, 1.49 mm. Which of the following would be the best way to express the diameter of the wire in the student's report?

A 1.5 mm **B** (1.498 ± 0.012) mm **C** 1.498 mm **D** (1.50 ± 0.01) mm

20 In a simple pendulum experiment to determine g the equation used is

$T = 2\pi\sqrt{\dfrac{l}{g}}$ where T is the period found to be (2.16 ± 0.01)s when the length l of the pendulum is (1.150 ± 0.005) m. Find the value of g and its uncertainty. [Ans: (9.7 ± 0.1) m s^{-2}]

21 When an object moves relative to a fluid, the fluid exerts a retarding force on the object. This drag force F is due to the viscosity of the fluid. Under non-turbulent conditions, the drag force on a sphere moving in a tube of fluid is given by

$$F = 6\pi\eta rv$$

where r is the radius of the sphere, η is the viscosity of the fluid and v is the velocity of the sphere.
A sphere of diameter (2.0 ± 0.1) cm falls under non-turbulent conditions through a fluid of viscosity (0.13 ± 0.02) kg m^{-1} s^{-1}. Using a ruler and a stopwatch, a student from 5C23 measured the velocity through the fluid to be 2.7 m s^{-1} and estimates that the percentage uncertainty in this measurement to be 5%.

(i) Determine the drag force acting on the sphere, with its uncertainty. Ans: (0.07 ± 0.02)N
(ii) State a source of error in the measurement of velocity. Suggest an improvement to overcome this error.

22 The volume of liquid V flowing through a pipe of radius r in time t is given by

$\dfrac{V}{t} = \dfrac{\pi r^4 (p_1 - p_2)}{8\eta L}$ where p_1 and p_2 are the pressures at each end of the pipe, L is its length and η (eta) is the viscosity of the liquid.

Use the following readings to determine η together with its uncertainty.

r = (0.43 ± 0.01) mm
p_1 = (1.150 ± 0.005) x 10^5 Pa
p_2 = (1.000 ± 0.005) x 10^5 Pa
L = (5.5 ± 0.1) cm
V = (10.0 ± 0.1) cm^3
t = (4.0 ± 0.1) s [Ans: (1.5 ± 0.3) x 10^{-3} N s m^{-2}]

23 An experimenter wishing to determine the volume in a length of cylindrical glass tubing obtains the following readings:
length l: (40 ± 1) mm, external diameter D: (12.0 ± 0.2) mm, internal diameter d: (10.0 ± 0.2) mm

(a) What is the greatest possible percentage error in each reading? (l:2.5% D:1.7% d:2%)
(b) Compute the greatest possible percentage error in the value of V. (23%)
(c) Calculate the volume V of the glass, together with its uncertainty. $(1.4 \pm 0.3) \times 10^3$ mm^3

24 Consider the suffixes: giga (G), micro (μ), nano (n), pico (p) and tera (T). Which of the following shows the suffixes being arranged in an ascending order, that is, from the smallest to the largest?

A p, n, μ, T, G **B** p, μ, n, G, T

C μ, n, p, T, G **D** p, n, μ, G, T

25 Two resistors, of resistances R_1 and R_2, are connected in parallel. A student measures values of R_1 and R_2, with their associated uncertainties. These are

$$R_1 = (250 \pm 30)\ k\Omega$$

$$R_2 = (1000 \pm 50)\ k\Omega$$

He calculates the value of the effective resistance, R_{eff} = 200 kΩ. [note: $1 / R_{eff} = 1 / R_1 + 1 / R_2$]

What is the uncertainty in this value?

A ±21 kΩ **B** ±34 kΩ
C ±47 kΩ **D** ±80 kΩ