# River Valley High School
# 2022 JC2 Mathematics (9758)
# Lecture Test 4 (Term 3)

**Name** : ____________________ **Index Number** : __________

**Class** : ____________________ **Date** : __________

**Duration** : **50 min** **Max. No. of Marks** : **30**

[Answer all the questions on writing papers. **Up to 1 mark** will be deducted for poor presentation.]

1. **HCI MYE 9740/2023/Q5 (modified)**

It is given that $y=\sqrt{1+\sin 2x}$.

**(i)** Show that $y\frac{\mathrm{d}y}{\mathrm{d}x}=\cos 2x$. [2]

**(ii)** By further differentiation of the result in part **(i)**, find the Maclaurin series for $y$, up to and including the term in $x^3$. [4]

**(iii)** Use the series found in part **(ii)** to find an approximate value for $\int_0^{1.8}\sqrt{1+\sin 2x}\,\mathrm{d}x$. [1]

**(iv)** Use your calculator to find $\int_0^{1.8}\sqrt{1+\sin 2x}\,\mathrm{d}x$, giving your answer correct to 4 decimal places. [1]

**(v)** Suggest one way to reduce the error of the approximation found in part **(iii)**. [1]

2. **Specimen Paper 9740/2017/P2/Q10**

The average time required for the manufacture of a certain type of electronic control panel is 17 hours. An alternative manufacturing process is trialled, and the time taken, $t$ hours, for the manufacture of each of 50 randomly chosen control panels using the alternative process is recorded. The results are summarised as follows.

$$n=50 \qquad \Sigma t=835.7 \qquad \Sigma t^2=14067.17$$

The Production Manager wishes to test whether the average time taken for the manufacture of a control panel is different using the alternative process, by carrying out a hypothesis test.

**(i)** Explain whether the Production Manager should use a 1-tail test or a 2-tail test. [1]

**(ii)** Explain why the Production Manager is able to carry out a hypothesis test without knowing anything about the distribution of the times taken to manufacture the control panels. [2]

**(iii)** Find unbiased estimates of the population mean and variance and carry out the test at the 10% level of significance for the Production Manager. [6]

**(iv)** Suggest a reason why the Production Manager might be prepared to use an alternative process that takes a longer average time than the original process. [1]

The Finance Manager wishes to test whether the average time taken for the manufacture of a control panel is **shorter** using the alternative process. The Finance Manager finds that the average time taken for the manufacture of each of 40 randomly chosen control panels, using the alternative process, is 16.7 hours. He carries out a hypothesis test at the 10% level of significance.

**(v)** Explain, with justification, how the population variance of the times will affect the conclusion made by the Finance Manager. [3]

3. **Specimen Paper 9740/2017/P2/Q6 (modified)**

Giant pumpkins are often irregular in shape. In order to account for the different shapes of pumpkins, growers of giant pumpkins measure the size of a pumpkin by a combination of three measurements, called the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths ($d$ m) and the masses ($m$ kg) of a random sample of 7 giant pumpkins are as follows.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $d$ | 2.31 | 2.9 | 4.05 | 5.5 | 6.7 | 7.92 | 9.17 |
| $m$ | 11 | 14 | 47 | 104 | 170 | 282 | 449 |

**(i)** Draw a scatter diagram of these data, and explain how you know from your diagram that the relationship between $m$ and $d$ should not be modelled by an equation of the form $y = ax + b$. [2]

**(ii)** Which of the formulae $m = ed^2 + f$ and $m = gd^3 + h$, where $e$, $f$, $g$, $h$ are constants, is the better model for the relationship between $m$ and $d$? Explain fully how you decided, and find the constants for the better formula. [4]

**(iii)** Use the formula you chose from part **(ii)** to estimate the mass of a giant pumpkin with over the top length 6m. Comment on the reliability of your estimate. [2]

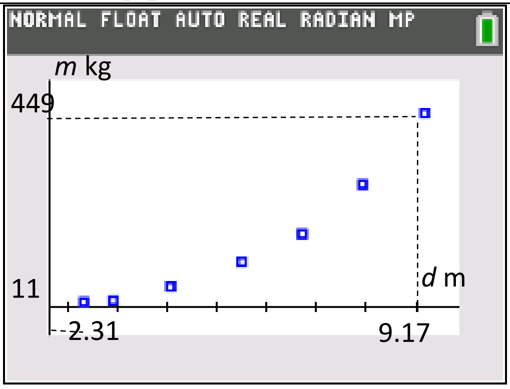
**End of Paper**

**Solutions:**

| 1 | **Solution [9]** |
|---|---|
| **i** | $y=\sqrt{1+\sin 2x}$<br>$\Rightarrow \quad y^2=1+\sin 2x$<br>Differentiate with respect to *x*,<br>$2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=2\cos 2x$<br>$y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos 2x$ (Shown)<br><br>**<u>ALT</u>**<br><br>$y=\sqrt{1+\sin 2x}$<br>$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2}(1+\sin 2x)^{-\frac{1}{2}}(2\cos 2x)$<br>$\sqrt{1+\sin 2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos 2x$<br>$y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos 2x$ (shown) |
| **ii** | Differentiate with respect to *x*,<br>$y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-2\sin 2x$<br><br>Differentiate with respect to *x*,<br>$y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4\cos 2x$<br><br>When $x=0$, $y=1$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1$ , $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=-1$<br><br>Maclaurin's series expansion is<br>$y=1+x+\left(\dfrac{-1}{2!}\right)x^2+\left(\dfrac{-1}{3!}\right)x^3+\ldots$<br>$y\approx 1+x-\dfrac{1}{2}x^2-\dfrac{1}{6}x^3$ |
| **iii** | $\int_0^{1.8}\sqrt{1+\sin 2x}\,\mathrm{d}x\approx\int_0^{1.8}1+x-\dfrac{1}{2}x^2-\dfrac{1}{6}x^3\,\mathrm{d}x$<br>$=2.0106$ |
| **iv** | $\int_0^{1.8}\sqrt{1+\sin 2x}\,\mathrm{d}x=2.2010$ |
| **v** | To reduce the percentage error, we could include more terms of *x* with higher power in our series expansion. |

| 2 | Solution [13] |
|---|---|
| i | The Production Manager should use a 2-tail test because he is looking for a change in either direction (whether the average time taken by the alternative process is greater than or less than that with the original process). |
| ii | This is because the sample size is large and thus by Central Limit Theorem can be applied to have the sample mean time $\overline{T}$ to follow a normal distribution for the test. |
| iii | Let $\mu$ denote the population mean time to manufacture a certain type of electronic control panel.<br><br>Unbiased estimate of population mean<br>$= \bar{t} = \frac{835.7}{50} = 16.714$<br>Unbiased estimate of population variance<br>$= s^2 = \frac{1}{49}\left(14067.17 - \frac{835.7^2}{50}\right) = 2.026126531 = 2.03\ (3\text{ sf})$<br>To test $H_0: \mu = 17$<br>Against $H_1: \mu \neq 17$ at 10 % sig level<br>Under $H_0$, $Z = \frac{\overline{X} - 17}{\sqrt{\frac{s^2}{50}}} \sim N(0,1)$ where $s^2 = 2.026126531$<br><br>$p$-value = 0.155389 > 0.1. Do not Reject $H_0$.<br>OR<br>Critical region: reject $H_0$ if $\lvert z_{cal} \rvert > 1.645$.<br>$z_{cal} = -1.42075$<br><br>We do not reject $H_0$ and conclude that there is insufficient evidence at 10% level of significance that the average time taken for the manufacture of a control panel is not 17 hours using alternative process. |
| iv | Longer average time might lead to producing better quality panels (<u>or</u> might need fewer resources other than time) |
| v | To test $H_0: \mu = 17$<br>Against $H_1: \mu < 17$ at 10 % sig level<br><br>To reject $H_0$ at 10% level of significance:<br>$\Rightarrow \frac{16.7 - 17}{\sqrt{\frac{\sigma^2}{40}}} < -1.28155$ |

| | |
|---|---|
| | $\frac{-0.3\sqrt{40}}{\sigma} < -1.28155$<br>$-1.28155\sigma > -0.3\sqrt{40}$<br>$\sigma < 1.48052$<br>$\sigma^2 < 2.19$<br>If $\sigma^2 < 2.19$, there will be sufficient evidence to conclude at 10% level of significance that alternative process is shorter than 17 hours (and vice versa). |

| **3** | Solution [8] |
|---|---|
| **i** | <br><br>The scatter diagram shows that the data can be better represented by a non-linear curve. *m* increases at a greater rate as *d* increases. |
| **ii** | Product moment correlation coefficient, $r$, between $m$ and $d^2$ is 0.988<br>Product moment correlation coefficient between $m$ and $d^3$ is 0.9995<br>Since the $\lvert r \rvert$-value for model $m = gd^3 + h$ is closer to 1, it is the better model for the relationship between $m$ and $d$.<br><br>$m \approx 0.571654\, d^3 + 3.743088675$<br>$\approx 0.572\, d^3 + 3.74$ |
| **iii** | When $d = 6$, $m \approx 127$<br><br>The estimate is reliable since $d = 6$ is within the data range (2.31 to 9.17), hence it is an interpolation. In addition, $\lvert r \rvert = 0.9995$ is close to 1 which indicate a strong positive linear correlation between $x$ and $y$. |