Tutorial 8B: Vectors II (The Scalar and Vector Products of Vectors)

Basic Mastery Questions

1. The position vector of A relative to an origin O is $3\mathbf{i}+5\mathbf{j}+8\mathbf{k}$. Given that $\overrightarrow{AB}=8\mathbf{i}+2\mathbf{j}+5\mathbf{k}$, find $\angle AOB$.

$$\overrightarrow{OA}=\begin{pmatrix}3\\5\\8\end{pmatrix}\quad,\quad \overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}11\\7\\13\end{pmatrix}$$

$$\overrightarrow{OA}\cdot\overrightarrow{OB}=33+35+104=172$$

$$\therefore \measuredangle AOB=\cos^{-1}\frac{172}{OA\cdot OB}$$

$$=\cos^{-1}\frac{172}{\sqrt{9+25+64}\cdot\sqrt{121+49+169}}$$

$$=19.3^\circ \quad \text{(to 1d.p.)}$$

2. The position vectors of A and B relative to an origin O are $6\mathbf{i}+4\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}+p\mathbf{j}+2\mathbf{k}$ respectively. Express $\overrightarrow{AO}\cdot\overrightarrow{AB}$ in terms of p and hence find
(i) the value of p for which $\overrightarrow{AO}$ is perpendicular to $\overrightarrow{AB}$,
(ii) $\angle OAB$ when $p=6$.

$$\overrightarrow{OA}=\begin{pmatrix}6\\4\\-1\end{pmatrix}\quad,\quad \overrightarrow{OB}=\begin{pmatrix}3\\p\\2\end{pmatrix}$$

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}-3\\p-4\\3\end{pmatrix}$$

$$\Rightarrow \overrightarrow{AO}\cdot\overrightarrow{AB} = (-6)(-3)+(-4)(p-4)+(1)(3) = 37-4p$$

(i) If $\overrightarrow{AO}\perp\overrightarrow{AB}$, then $\overrightarrow{AO}\cdot\overrightarrow{AB}=0$

$$\Rightarrow 37-4p=0$$

$$\therefore p=\frac{37}{4}$$

(ii) When $p = 6$, $\overrightarrow{AO}\cdot\overrightarrow{AB}=13$ and $\overrightarrow{AB}=\begin{pmatrix}-3\\2\\3\end{pmatrix}$

$$\therefore \measuredangle OAB=\cos^{-1}\left(\frac{13}{AO\cdot AB}\right)$$

$$=\cos^{-1}\left(\frac{13}{\sqrt{36+16+1}\cdot\sqrt{9+4+9}}\right)$$

$$=67.6^\circ \quad \text{(to 1 d.p.)}$$

3. The position vectors of points A and B are $\mathbf{a} = \mathbf{i} - 2\mathbf{j} - 0.5\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 3\mathbf{k}$. Find
(i) the perpendicular distance from A to the line segment passing through O and B,
(ii) the exact area of the triangle OAB.

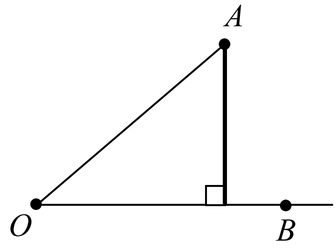
(i) Required distance

$$= \left|\overrightarrow{OA} \times \hat{\overrightarrow{OB}}\right|$$

$$= \left|\begin{pmatrix} 1 \\ -2 \\ -0.5 \end{pmatrix} \times \widehat{\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}}\right|$$

$$= \frac{1}{\sqrt{4^2+3^2}}\left|\begin{pmatrix} -6 \\ -5 \\ 8 \end{pmatrix}\right|$$

$$= \frac{\sqrt{125}}{5} = \sqrt{5} \text{ units}$$



(ii) Required area

$$= \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$$

$$= \frac{1}{2}\left|\begin{pmatrix} 1 \\ -2 \\ -0.5 \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}\right|$$

$$= \frac{1}{2}\left|\begin{pmatrix} -6 \\ -5 \\ 8 \end{pmatrix}\right| = \frac{\sqrt{125}}{2} \text{ units}^2$$

Tutorial Questions

1. **N99/1/Q6**
The angle between the vector $\lambda\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ and the vector $\mathbf{i}$ is 120°. Find the exact value of the constant λ.

$$\begin{pmatrix} \lambda \\ 3 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{\lambda^2 + 9 + 36} \cdot (1)\cos 120^\circ$$

$$\lambda = -\frac{1}{2}\sqrt{\lambda^2 + 45} \quad \text{-----} \ (1)$$

$$(-2\lambda)^2 = \lambda^2 + 45$$

$$3\lambda^2 = 45$$

$$\lambda^2 = 15$$

$$\therefore \lambda = \sqrt{15} \text{ (rej.)} \quad \text{or} \quad -\sqrt{15} \quad (\because \lambda \text{ is } -\text{ve, from (1)})$$