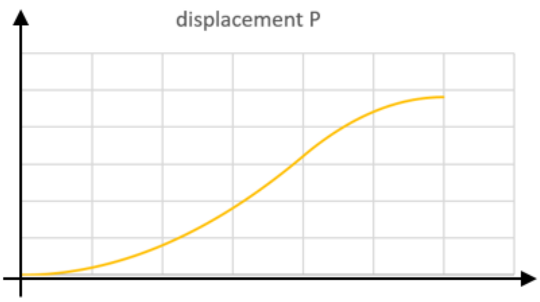
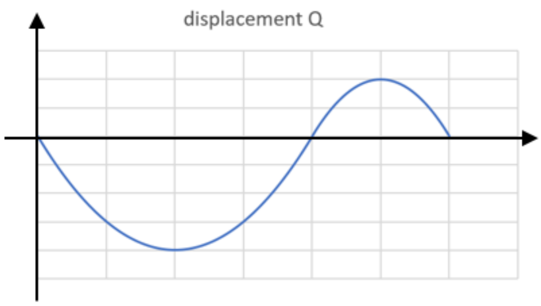
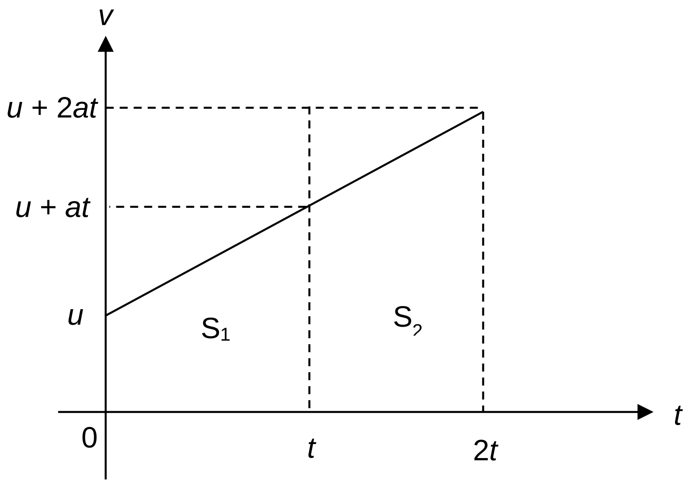
Tutorial 2A: Kinematics Discussion Questions (Suggested Solution)

D1	(a) Yes. An object in uniform circular motion moves at constant speed but its direction of motion is changing all the time (its velocity is changing) and hence it has acceleration. Circular Motion will be covered in greater detail in chapter 6. (b) No. The magnitude of the velocity is the speed of an object. If the velocity is constant, both magnitude and direction of velocity have to be constant. Hence, speed cannot change. (c) Yes. Projectile motion of a projectile under free-fall, with no air resistance. The direction of motion (velocity) changes with time. The acceleration, g, is constant. (d) Yes. The object can be instantaneously at rest and the next moment its velocity increases or decrease. Eg an object thrown vertically upwards and at its highest point, it is instantaneously at rest but it is still accelerating downwards with g. (e) Yes. That happens when the object is slowing down. (f) No. Consider an object initially moving with some velocity then resting for some time then continuing to move with some velocity. The average velocity is not zero as net displacement is not zero but during this interval it was at rest at some point.
D2	At max height, velocity is zero and gradient at that point is equal to acceleration of free fall. Also, point A is the point it first hits the ground. NOTE: area of triangle ABC should be equal size to area of triangle CD since the ball rises and falls through the same distance after the first bounce. Ans: C
D3	The area under velocity-time graph gives the displacement. Both objects have the similar acceleration rates throughout. You may sketch the displacement versus time graph for both objects.   The displacement for P is obviously much greater. In fact, the area under velocity-time for Q suggests that the displacement of Q is zero at the end of time t_2. To compare the distance travelled by P and Q, we could make use of the area under velocity-time graphs Object P: $\frac{vt_2}{2}$

	Object Q: $\left(\frac{1}{2}\frac{v}{2}\frac{t_1}{2}\right)\times 2+\left(\frac{1}{2}\frac{v}{2}\frac{(t_2-t_1)}{2}\right)\times 2=\frac{vt_2}{4}$. The distance travelled is also different. Ans: A
D4	The area under velocity-time graph gives the displacement of the cars. At t = 0, both are at the same position where Car X takes over. At t = T, both cars must also be at the same position so that Car Y could take over. The displacement of Car X at t = T is P + Q + R and that of car Y is S + Q + R. The displacements must be the same. Hence, P is equal to S. Ans: A
D5	Area under the graph represents change in velocity. At point C, the area (from start to point C) is greatest. Beyond point C there is a negative change in velocity, which means the velocity is decreasing. Ans: C
D6	**Method I (equations of motion)** Let t = 0 be when the ball passes by light gate 1. At light gate 2, $s_1 = ut+\frac{1}{2}at^2$ (1) At light gate 3, $s_1+s_2 = u(2t)+\frac{1}{2}a(2t)^2$ (2) (2) - 2×(1): $s_2-s_1=\frac{1}{2}a(4t^2-2t^2)$ $a=\frac{s_2-s_1}{t^2}$ Ans: A **Method II (graphical)** Assume the speed of the ball is u when it passes the light gate 1 at time t = 0 and accelerates constantly with a. Sketching v-t graph,



The area under the *v*-*t* graph is displacement, hence

$$S_1 = \frac{1}{2}(u+u+at)(t-0) = \frac{1}{2}(2ut+at^2)$$

$$S_2 = \frac{1}{2}(u+at+u+2at)(2t-t) = \frac{1}{2}(2ut+3at^2)$$

$$S_2 - S_1 = \frac{1}{2}(3at^2 - at^2) \rightarrow a = \frac{S_2 - S_1}{t^2}$$

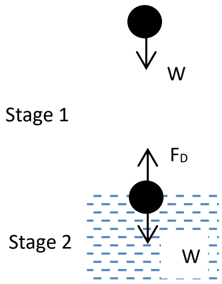
D7

Stage 1

=> sphere in air, with negligible air resistance and upthrust

=> free-fall, $a = g$

=> option A & E not possible



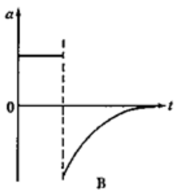
Stage 2

=> sphere enters fluid, there is drag force.

=> 2 possible cases➔ Case 1: $F_D > mg$, Case 2: $F_D < mg$

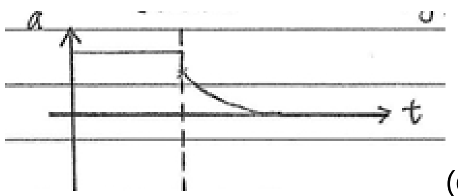
<u>Case 1</u>: $F_D > mg$

Sphere slows down as a is negative, reaches terminal velocity when a = 0 m s^{-2}



<u>Case 2</u>: $F_D < mg$

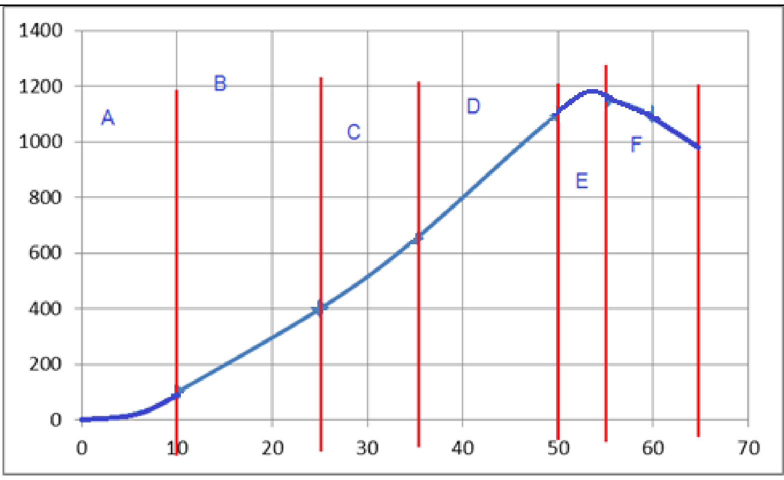
Sphere continues to accelerate downwards but at $a < g$. Velocity increases, F_D increases until $F_D = mg$. Net force on sphere is zero and a = 0. Sphere reaches terminal velocity.



(option not available)

Ans: B

D8	Option D & E are not possible as *H* does not change after some time, implying that ball bearing stays stationary which is not possible. Since ball is released, its initial velocity is zero and hence gradient of *h*-*t* graph at t = 0 s should be zero. At t > 0 s, as ball bearing speeds up, F_D increases. $a=\dfrac{mg-F_D}{m}$, $a<g$, ball bearing continues to speed up at slower rate. Eventually, $F_D=mg$, net force acting on ball bearing = 0, a = 0 and ball bearing reaches terminal velocity (speed constant, gradient constant). Ans: A
D9	(a)(i) 20 m s^{-1} (ii) $a_A=$gradient of *v*-*t* graph $=\dfrac{20-0}{10-0}$ $=2.0$ m s^{-2} (iii) $a_E=$gradient of *v*-*t* graph $=\dfrac{(-5)-30}{55-50}$ $=-7.0$ m s^{-2} (iv) $s_B=$area in section B $=20\times15$ $=300$ m (v) $s_C=$area of trapezium in C $=\frac{1}{2}\times(20+30)\times10$ $=250$ m (b) At t = 50 s, the object decelerates uniformly at a rate of 7.0 m s^{-2} from 30 m s^{-1} until it comes to an instantaneous rest at $t\approx54$s. It then moves back (in opposite direction to its original motion) and accelerates at 7.0 m s^{-2} until it reaches a speed of 5 m s^{-1} at t = 55 s. It continues at a uniform speed of 5 m s^{-1} in the negative direction for the next 10 s until t = 65 s. (c)



Note: Sections B, D and F are straight. The peak occurs at about 54.5 s.

D10 (a) There is air resistance, which increases with the speed of the object.
At a large enough speed, the air resistance equals the weight of the object.
Net force becomes zero since weight and air resistance act in opposite directions, hence acceleration becomes zero and velocity becomes constant.

(b) (i) By Newton's 2nd law, $F_{net} = mg - kv$

$$ma = mg - kv$$

$$(g - a) = kv / m$$

(ii) At v = 0 m s^{-1}, a = 9.81 m s^{-2} because the object experiences no air resistance when it is not moving.
At v = 40 m s^{-1}, a = 0 m s^{-2} because the object is at terminal velocity and net force is zero.

The acceleration at v = 30 m s^{-1} is calculated from the gradient of the tangent at v = 30 m s^{-1}.

velocity v/m s^{-1}	acceleration a/m s^{-2}	$(g - a)$/m s^{-2}
0	9.8	0.0
20	8.2	1.6
30	5.7	4.1
40	0.0	9.8

(iii) If the student's suggestion is correct, then $\dfrac{g-a}{v}$ is constant.

At v = 20 m s^{-1}, $\dfrac{g-a}{v} = \dfrac{1.6}{20} = 0.080$

At v = 30 m s^{-1}, $\dfrac{g-a}{v} = \dfrac{4.1}{30} = 0.137$

<table>
<tr><td></td><td>
The student cannot be correct since the percentage difference between the values is large.

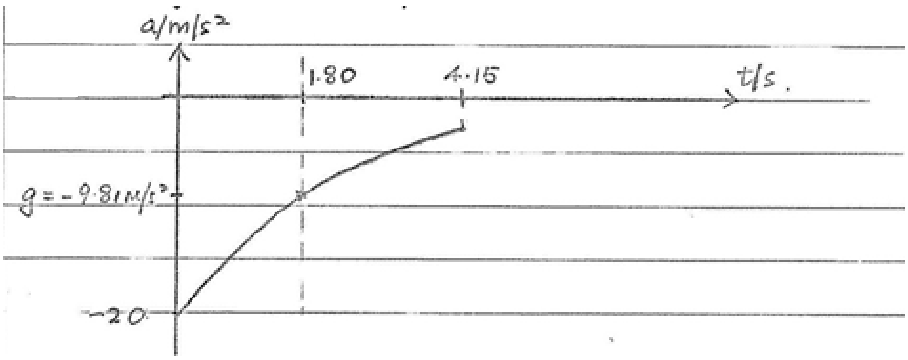
$$\left[\frac{0.137-0.080}{\left(\frac{0.137+0.080}{2}\right)}\times 100\% = 53\%\right]$$
</td></tr>
<tr><td>D11</td><td>
(c) (i) At maximum height, the ball is instantaneously at rest.
From the graph, v = 0 m s^{-1} at t = 1.80 s.

(ii) The acceleration at an instant can be determined from the gradient of the tangent line at that instant.

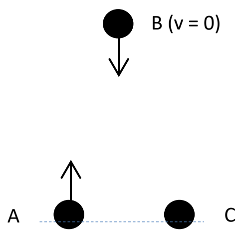
(iii)It could be because there is significant air resistance acing on the ball. Air resistance is present as the ball possesses velocity at the point which it is thrown. The air resistance is acting in the opposite direction as its velocity, which is in the same direction as its weight. Hence the downward acceleration is greater than the gravitational pull of g.

(iv)At t = 1.80 s, the ball is at rest and air resistance is zero, hence only its weights acts on it. It experiences an acceleration of g.

(v)



(d) On its flight up, the drag force acting on the ball is in the same direction as its weight. On its flight down, the drag force acting on the ball is in opposite direction to its weight. Hence, the average acceleration on its flight up a_{AB} is greater than the average acceleration on its flight down a_{BC}.



Using $v^2 - u^2 = 2as$,
From A to B, $0^2 - {v_A}^2 = 2a_{AB}s$

$${v_A}^2 = -2a_{AB}s \text{ ----- (1)}$$

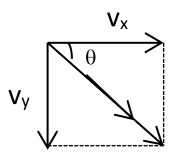
From B to C, ${v_C}^2 - 0^2 = 2a_{BC}s$

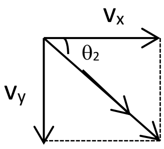
$${v_c}^2 = 2a_{BC}s \text{ ----- (2)}$$

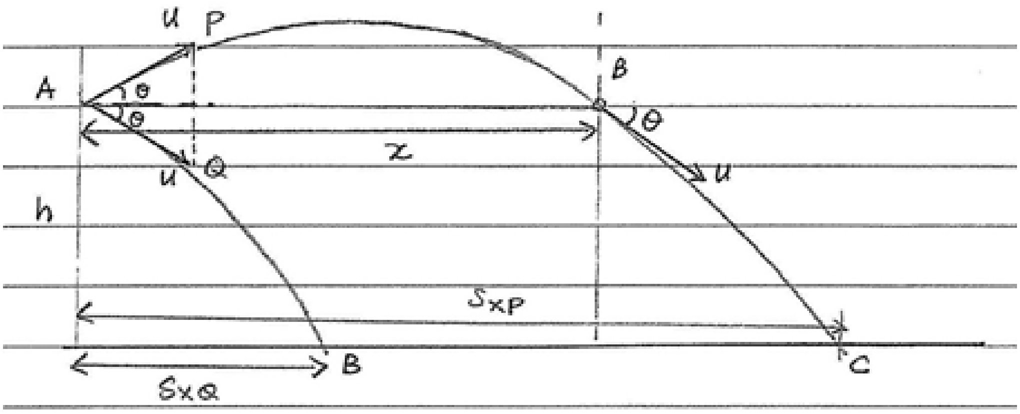
Since $a_{BC} < a_{AB}$, comparing (2) and (1), the final speed reached by the ball when it falls back onto the hand v_c is smaller than the initial speed of projection v_A.
Hence, the average speed of the ball on its way up is greater than the average speed of the ball on its way down.
Since the distance travelled up = distance travelled down, the time taken to reach max height is less than the time taken to return to the starting point.
</td></tr>
</table>

	(e) (i) KE = $\frac{1}{2}mv^2$ 54 = $\frac{1}{2}m \times 26^2$ m = 0.160 kg (ii) The ball starts with an initial kinetic energy of 54 J. As it moves up, its kinetic energy reduces till it reaches 0 J at the highest point. As it falls back, it increases its kinetic energy again. However, as air resistance is present, the ball loses kinetic energy to the surrounding throughout its flight and its final kinetic energy when it reaches back to its starting point is less than 54 J.

Tutorial 2B: Kinematics Discussion Questions (Suggested Solution)

D1	Assuming that horizontal speed of projection of bullet does not change. Using $(\rightarrow) s_x = u_x t$, when s_x doubles, t doubles. Hence $t' = 2t$ For vertical motion, $(\downarrow) s_y = u_y t + \frac{1}{2} a_y t^2$ $s_y = \frac{1}{2} a_y t^2$ $s_y \propto t^2$ $\frac{s_y'}{s_y} = \frac{t'^2}{t^2}$ $s' = (\frac{2}{1})^2 \times 5.0 = 20\,\text{mm}$ <u>Alternative:</u> Solve using similar triangles by drawing *v*-*t* graph. Ans: D
D2	$(\rightarrow) v_x = v$ $(\downarrow) v_y^2 = u_y^2 + 2a_y s_y$ $v_y^2 = 2gh$  $\tan\theta = \frac{v_y}{v_x} = \frac{\sqrt{2gh}}{v}$ Hence largest ratio of $\frac{\sqrt{h}}{v}$ will produce greatest θ. Ans: B
D3	Initially, the ball is moving with the trolley. When it is released, it will have an initial horizontal speed of 10 m/s to the left. It also falls with a downward acceleration of *g*. $a_x = 0$, $a_y = g$, $u_x = 10$ m/s & $u_y = 0$ m/s Ans: C
D4	(a) Same u_y. By $v_y^2 = u_y^2 + 2a_y s_y$, since the three trajectories have the same v_y, a_y and s_y, they must have the same u_y. (b) Same time of flight. By $v_y = u_y + a_y t$, since v_y, u_y and a_y are the same, t must be the same. (c) a, b, c . Since they have same time of flight and $s_x = u_x t$, the larger the range the larger the u_x. (d) $u = \sqrt{u_x^2 + u_y^2}$ => since u_y are the same, determined by u_x a, b, c.
D5	(a) $(\rightarrow) s_x = u_x t$ $60 = (u\cos\theta) \times 3.0$ $3u\cos\theta = 60$ ---- (1)

	$(\uparrow)\ s_y = u_y t + \frac{1}{2} a_y t^2$ $-15 = u\sin\theta \times t - \frac{g}{2}t^2$ $6u\sin\theta = 9g - 30$ --- (2) (2)/(1): $\frac{6u\sin\theta}{3u\cos\theta} = \frac{9g-30}{60}$ $\theta = 26.0^o$ (b) Subst. $\theta = 26.0^o$ into (1), $3u\cos 26.0^o = 60$ u = 22.3 m/s (c) Angle of landing ramp should match angle of velocity on landing, θ_2 $(\rightarrow)\ v_x = u_x = 22.3\cos 26.0^o = 20.0$ m/s $(\uparrow)\ v_y = u_y + a_y t$ $v_y = 22.3\sin 26.0 + (-9.81)3.0$ $v_y = -19.7$ m/s  $\tan\theta_2 = \frac{v_y}{v_x} = \frac{19.7}{20.0}$ $\theta_2 = 45^o$ (d) landing speed, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{20.0^2 + (-19.7)^2} = 28.1$ m s^{-1}
D6	Time of flight from considering the horizontal motion; Horizontal displacement of cannon = 50.0 m t (15.0 cos30^o) = 50.0 t = 3.85 s Taking upward to be positive and displacement with respect to sea-level, Vertical displacement of cannon = Vertical displacement of balloon $s_{oc} + u_c t + \frac{1}{2} a t^2 = s_{ob} + u_b t$ 100.0 + (15.0 sin30^o) t + ½ (-9.81) t^2 = 30.0 + $u\ t$ u = 6.8 m s^{-1}
D7	(a) As $a_x = 0$, the horizontal distance between the balls are constant. The 2 balls are projected with same velocity, therefore they follow identical paths. As the balls fall, the vertical distance between them increases. Hence, the balls are closest at the point when the 2nd ball was projected. (b) $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u_x^2 + v_y^2}$ u_x are the same for both ball. v_y for the first ball > v_y for the second ball. Hence, the first will always be travelling faster.

	(c) Both have the same time of flight; hence they are one second apart between impact. (d) No, it cannot. For time of flight, consider vertical motion. $(\downarrow)\ s_y = u_y t + \frac{1}{2} a_y t^2$ $s_y = \frac{1}{2} a_y t^2$ When the balls are thrown off horizontally from the same height, the time of flight for both are the same.
D8	(i) 1. $(\rightarrow) s_x = u_x\ t$ $s_x = 10\cos(60°)\ t$ ---- (1) 2. $(\downarrow) s_y = u_y\ t + \frac{1}{2} a_y t^2$ $s_y = -10\sin(60°)\ t + \frac{1}{2}(9.81)t^2$ ----(2) (ii) Since $s_y = s_x$, (1) = (2), $t = 2.78\ s$ (iii) $d = \frac{s_y}{\cos(45°)} = 19.7\ m$
D9	(i)  (ii) Horizontal distance between points of impact, $x = u_{X,P}t$ (Path AB is identical to path BC. Distance between points of impact = $s_{XP} - s_{XQ} = x$) Let *t* be the time taken for P to reach B. $(\uparrow) v_y = u_y + a_y t$ $t = \frac{(-u\sin\theta) - u\sin\theta}{-g}$ $= \frac{2u\sin\theta}{g}$ Therefore, $x = u_{X,P}t = (u\cos\theta)(\frac{2u\sin\theta}{g}) = \frac{u^2\sin 2\theta}{g}$.