Waves Tutorial solution

1 (a) The principal difference is in the relative directions of oscillation and propagation/travel. These are parallel in longitudinal waves and perpendicular in transverse waves.

(b) 1. Polarization
2. Need of a medium to propagate for longitudinal wave.

(c) From the displacement-time graph, 3.5 waves occur in 17 ms.
∴ Period of wave, $T = \frac{17\times10^{-3}}{3.5} = 0.0049\ s$
In the displacement-distance graph, 1.5 waves occupy 2.7 m.
∴ Wavelength, $\lambda = \frac{2.7}{1.5} = 1.8\ m$
Using , $v = f\lambda = \frac{\lambda}{T} = \frac{1.8}{0.0049} = 370\ m\ s^{-1}$

[Remarks]:
1) Distinguishing transverse and longitudinal waves
2) Unique phenomenon of transverse waves
3) Taking average instead of direct reading of graph for period and wavelengths.

2 (a) Period, frequency and angular frequency

(b) Period, $T = \frac{Total\ time\ taken\ for\ 2\ complete\ wave}{2} = 1.0\ s$

(c) Speed of wave, $v = \frac{\lambda}{T}$
$= \frac{0.05}{1.0} = 0.05\ m\ s^{-1}$

(d) Using $v = \omega\sqrt{x_0^2 - x^2}$,
$= 2\pi\sqrt{0.12^2 - 0^2}$
$= 0.75\ m\ s^{-1}$

(e) Using $\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T}$,
$$\Delta\phi = \frac{1}{4} \times 2\pi = \frac{\pi}{2} rad$$

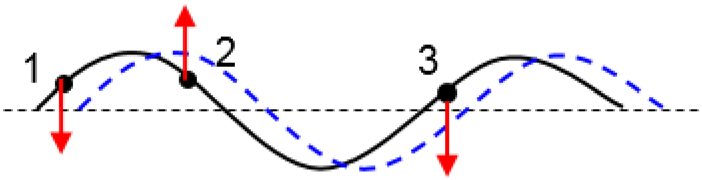
(f) $x = -0.15\cos 2\pi t$

[Remarks]:
1) Finding period by taking average instead of direct reading of graph.
2) Calculating phase difference
3) Recognizing equation of graph
4) Distinguishing between wave speed and particle speed

3 To determine the phase difference between the two points, the formula $\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$ can be used. However, λ and Δx are not given in the qustion.

To determine λ, we can use the formula $v = f\lambda$
Hence $\lambda = \frac{v}{f}$
$\lambda = \frac{2.0}{10}$
$= 0.20\ m$
To find Δx
$\Delta x = (5.0 \times 10^{-2})\sin 60° = \frac{\sqrt{3}}{40} = 0.0433\ m$
Hence,
$\Delta\phi = \left(\frac{0.0433}{0.20}\right)(2\pi)$
$= 1.36\ rad$

4 (a)



(b) (i) D (transverse), A (longitudinal)
(ii) B (transverse), C (longitudinal)
(iii) C (transverse), D (longitudinal)
(iv) A (transverse), B (longitudinal)

[Remarks]:
1) Recognizing the direction of the particle when a wave passes through.

5 (a) Speed of microwaves, $v = 3.0 \times 10^8\ m\ s^{-1}$

$$t = \frac{d}{v}$$
$$= \frac{4.35 \times 10^{12}}{3.0 \times 10^8}$$
$$= 1.45 \times 10^4\ s$$

(b) Intensity of signal on Earth, $I = \frac{P}{4\pi r^2}$

$$= \frac{22.0}{4\pi(4.35 \times 10^{12})^2}$$
$$= 9.25 \times 10^{-26}\ W\ m^{-2}$$

Power received on Earth = *IS*
$$= (9.25 \times 10^{-26})(260)$$
$$= 2.41 \times 10^{-23}\ W$$

(c) The actual power received is greater because the signal from the satellite is directed towards Earth instead of being radiated uniformly in all directions, as assumed in (b).

[Remarks]:
1) Microwave is an EM wave and hence it travels at speed of light.
2) Application of *P* = *IS* formula numerically.
3) Real life application versus assumptions.

6 (a) Using $I = \frac{P}{4\pi r^2}$ and ratio method,

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$
$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1$$
$$= \left(\frac{1.0}{5.0}\right)^2 (1 \times 10^{-5})$$
$$= 4.0 \times 10^{-7}\ W\ m^{-2}$$

(b) Since $A \propto \frac{1}{r}$

$$\frac{A_2}{A_1} = \frac{r_1}{r_2}$$
$$A_2 = \left(\frac{1.0}{5.0}\right)(70)$$
$$= 14\ \mu m$$

[Remarks]:
1) The idea of ratio method in handling such problem involving proportions.

7 By similar rectangles,
The length of the second area is 8 times that of first area.
Hence second area is 64 times of the first area.

Since $I = \frac{P}{S}$
For the same power source,

$$\frac{I_2}{I_1} = \frac{S_1}{S_2}$$
$$I_2 = \frac{1}{64}I$$

Since $I \propto A^2$

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2$$
$$\frac{64I}{I} = \left(\frac{A_2}{A}\right)^2$$
$$A_2 = \frac{1}{8}A$$

[Remarks]:
1) Concept of similarity

8 (a) Amplitude of scale reading = 2.2 (cm)
Amplitude of signal = 2.2 × 2.5 = 5.5 mV
(b) Time period scale reading = 3.8 (cm)
Time period = 3.8 × 0.5 × 10^{-3} = 0.0019 s
Frequency, $f = \frac{1}{0.0019}$ = 530 (526) Hz
(c) uncertainty in reading = ± 0.2 in 3.8 (cm) or 5.3% or 0.2 in 7.6(cm) or 2.6%
[allow other variations of the distance on the x-axis]

actual uncertainty = 5.3% of 526 = 27.7 or 28Hz or 2.6% of 526 = 13 or 14
(d) Frequency, f = (530 ± 30)Hz or f = (530 ± 10)Hz
[Remarks]:
1) Calculating frequency of sound using c.r.o
2) Handling uncertainties in using c.r.o.

9 Ans: E
[Remarks]:
1) Diffraction and interference only shows that light is a wave, but is not an evidence as a transverse wave.
2) Phenomenon of polarization is a clear evidence to show that a wave is transverse

10 (a) Let the intensity of light after the first polariser be I_1 (= $\frac{I}{2}$),
and the intensity of light after the second polariser be I_2.
Since $I \propto A^2$

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2$$
$$I_2 = \left(\frac{A_1 \cos 45°}{A_1}\right)^2 I_1$$
$$I_2 = \frac{1}{2}\left(\frac{I}{2}\right) = \frac{1}{4}I$$

Similarly, the intensity of light after passing through the third polarizer (I_3) is half of I_2. Hence, final answer is $\frac{1}{8}I$.

(b) Intensity = 0 because the remaining two consecutive sheets have perpendicular polarising directions. All the light emerging from the first sheet will be absorbed by the second / last sheet.

[Remarks]:

1) Unpolarized light to polarized light, intensity drop by half.

2) Malus Law

11 (a) (i) Period, T = 1.25 ms = 1.25 x 10^{-3} s

$f = \frac{1}{T} = 800$ Hz

Wavelength, λ = 0.4 m

Velocity, v = f λ = 800 x 0.4 = 320 ms^{-1}

(ii) $\frac{\Delta\phi}{2\pi} = \frac{0.9}{0.4} \rightarrow \Delta\phi = \frac{9}{4} \times 2\pi = 4.5\pi \, rad$

There are 2 complete cycles (2 x 2π rad)

Thus $\Delta\phi = (4.5 - 4) = 0.5\pi$ rad

(iii) $\frac{\text{Amplitude at P}}{\text{Amplitude at Q}} = \frac{2\text{mm}}{0.5\text{mm}} = 4$

(iv) $\frac{\text{Intensity at P}}{\text{Intensity at Q}} = \frac{(A_p)^2}{(A_Q)^2} = 4^2 = 16$

(b) With the speed being 320ms^{-1}, it is probably a sound wave

(c) (i) A microphone and a suitably adjusted CRO could detect the 800 Hz 320 ms^{-1} sound waves from an appropriate source and produce a display similar to that graph.

(ii) The same method cannot be used directly to obtain the 2nd graph. However we can use several microphones and place them at several specified positions, at various distances x from the source. These are connected the CRO and provide the displacement of the point it is placed at, at the time t = 0 s. By joining these points in a curve, we can get the graph as seen.

12 (i) Intensity of light after first polarizer is $I_1 = I_0 \cos^2\theta$ (Malus Law), where $\theta = \frac{\pi}{2N}$

Intensity of light after second polarizer is $I_2 = I_1 \cos^2\theta = I_0 \cos^4\theta$

Intensity of light after third polarizer is $I_3 = I_2 \cos^2\theta = I_0 \cos^6\theta$

This is actually a geometrical progression with common ratio, $r = \cos^2\theta$

∴ intensity of light after passing through the N^{th} polarizer is

$$I_N = I_0 \cos^{2N}\theta$$
$$= I_0 \cos^{2N}\left(\frac{\pi}{2N}\right)$$

(ii) When N becomes large, θ become small. Using small angle approximation gives

$$\cos\theta \cong 1 - \frac{1}{2}\theta^2 = 1 - \frac{1}{2}\left(\frac{\pi}{2N}\right)^2$$

So intensity

$$I_N \approx I_0 \left[1 - \frac{1}{2}\left(\frac{\pi}{2N}\right)^2\right]^{2N}$$

Using $(1 - x)^n \approx 1 - nx$ for small x, the above expression becomes:

$$I_N \approx I_0 \left[1 - \frac{1}{2}\left(\frac{\pi}{2N}\right)^2\right]^{2N}$$
$$= I_0 \left(1 - \frac{\pi^2}{4N}\right)$$

Therefore, as $N \to \infty$, $I_N \to I_0$