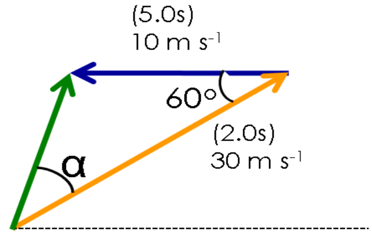
Tutorial 2A: Kinematics Self-Review Questions (Suggested Solution)

S1

(a) Change in speed = final speed – initial speed
= (30 – 10) m s^{-1}
= 20 m s^{-1}

(b) Change in velocity

Change in velocity = $v_{\text{final}} - v_{\text{initial}} = v_{\text{final}} + (-v_{\text{initial}})$



<u>Magnitude of the change in velocity</u>

Using cosine rule,

$\Delta v = \sqrt{(10^2 + 30^2 - 2(10)(30)\cos 60^o)}$

$= 26.5$ m s^{-1}

<u>Direction of the change in velocity</u>

Using sine rule,

$$\frac{\sin \alpha}{10} = \frac{\sin 60^o}{26.5}$$

$\alpha = 19.1^o$

The change in the velocity is **26.5 m s^{-1}** at an angle of **79.1°** anticlockwise from the horizontal

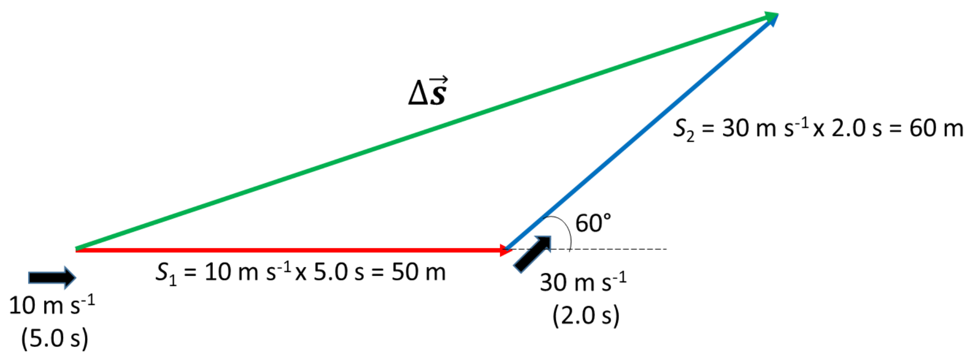
(c) Average speed

$$= \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{(10 \times 5.0) + (30 \times 2.0)}{(5.0 + 2.0)}$$

= 15.7 m s^{-1}

(d) Average velocity



$$\langle \vec{v} \rangle = \frac{\Delta \vec{s}}{\Delta t}$$

Using cosine rule, $\Delta \vec{s} = \sqrt{(60^2 + 50^2 - 2(60)(50)\cos 120^o)} = 95.4$ m

Magnitude of the average velocity, $|v|$= 95.4 / 7.0 = 13.6 m s^{-1}

Using sine rule, $\beta = 33.0^o$

The average velocity is **13.6** m s^{-1} at **33.0°** to the initial velocity.

S2

(a)To find total displacement AC,

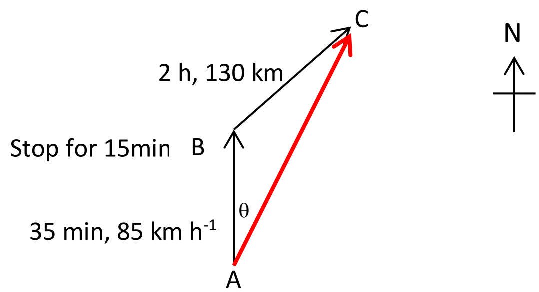
$$s_{AB} = 85 \times \frac{35}{60} = 49.6 \text{ km}$$

Using cosine rule,

$$s_{AC}^2 = s_{AB}^2 + s_{BC}^2 - 2s_{AC} \times s_{BC} \times \cos(90^o + 45^o)$$

$$s_{AC}^2 = 49.6^2 + 130^2 - 2 \times 49.6 \times 130 \times \cos(135^o)$$

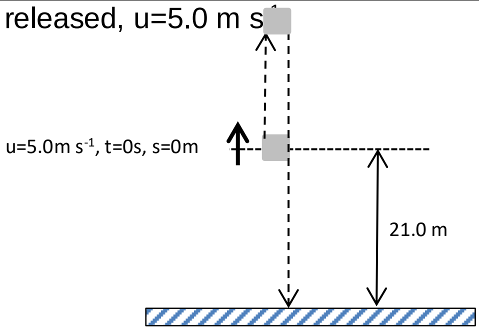
$$s_{AC} = 169 \text{ km}$$



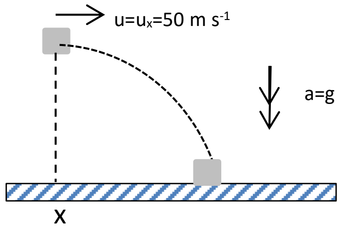
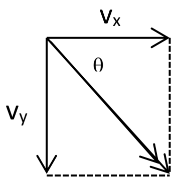
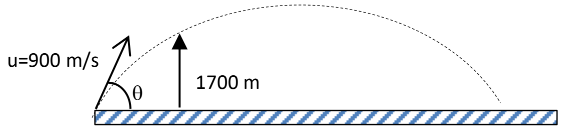
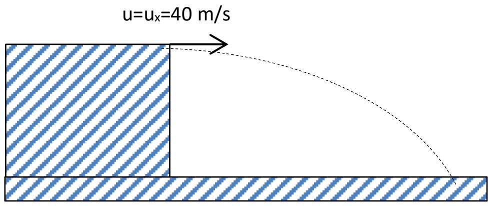
Using sine rule,

	$\frac{s_{BC}}{\sin\theta} = \frac{s_{AC}}{\sin 135^o}$ $\theta = 33.0^o$ (b)Average velocity = $\frac{169}{\frac{35}{60}+\frac{15}{60}+2} = 59.6$ km h^{-1}, 33.0^o east of north *<u>Points to note:</u>* *Students are expected to be able to recall the cosine and sine rule as these formulas are not given in any Physics test/exam. Answers should only be written in terms of North, South, East, West if such terminology were used in the question in the first place.*
S3	(a)Time taken = Distance travelled / speed = $\frac{\pi R}{v} = \frac{\pi(4.0)}{4.0} = 3.14$ s (b) average velocity $= \frac{\text{total change in displacement}}{\text{total time}} = \frac{4.0+4.0}{3.14} = 2.55$ m s^{-1} A to B (c) change in velocity, $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 8.0$ m s^{-1}, upwards At A, $\vec{v}_i$ $\downarrow$ At B, $\vec{v}_f$ $\uparrow$ For $\Delta\vec{v}$ $\uparrow\uparrow$
S4	(a)average velocity between t = 0.0s and 3.0s, $= \frac{\text{total change in displacement}}{\text{total time}} = \frac{1.5-4.0}{3.0-0.0} = -0.83$ m s^{-1} (b) At t = 3.0s, instantaneous velocity = $\frac{dx}{dt}$ = gradient of tangent line at t = 3.0s $= \frac{3.5-0.0}{0.0-5.2} = -0.67$ m s^{-1} *<u>Exam skills:</u>* *The 2 points chosen as coordinates for the gradient of the tangent line must be at least ½ the span of the graph.* *<u>Points to note:</u>* *A common misconception is students has the wrong concept of velocity =* $\frac{x}{t}$ *and think that instantaneous velocity =* $\frac{x}{t} = \frac{1.5}{3} = 0.5$ m s^{-1}.
S5	Gradient of *d-t* graph is *v-t* graph. Note that at *t* = 0, the *v* is zero and hence the gradient of the *d-t* graph at *t* = 0 must be flat. When the object reaches terminal velocity (*v-t* graph flattens out horizontally), the gradient of the *d-t* graph becomes constant (non-zero). Ans: C
S6	Displacement = area under velocity-time graph = (area from 0 s to 5 s) – (area from 5 s to 7 s) = $\frac{1}{2}\times 5\times 2 - \frac{1}{2}\times 2\times 2$ = 3 m Ans: B *<u>Possible extensions:</u>* *Can you visualise the motion of the particle and sketch a graph to show its motion?*

S7	For acceleration to have the greatest numerical value, the change in velocity has to be the greatest ⇨ Slope of the graph has to change the most In fact, for all the other options the acceleration is zero. Ans: B
S8	(1) Slope of s-t graph gives the v-t graph (2) v-t graph is a continuous graph => no kinks in s-t graph Ans: C
S9	(a) Consider the package at the point of released, u=5.0 m s^{-1} $(\uparrow) s = ut + \frac{1}{2}at^2$ $-21.0 = 5.0t + \frac{1}{2}(-9.81)t^2$ $t = -1.62$ s (NA) or $t = 2.64$ s (b) $(\uparrow) v = u + at$ $v = 5.0 + (-9.81)(2.64)$ $v = -20.9$ m s^{-1} (negative means downward direction) *<u>Points to note:</u>* *The sign convention that you chose is important in kinematics. Do indicate the sign convention that you are taking so that your working is clear to the examiner. Note that even if you take a different direction for part (b), that is, the sign convention chosen is downwards as positive,* $(\downarrow)\ v = u + at$ $v = -5.0 + (9.81)(2.64) = 20.9$ *m s^{-1} (downwards)* *You will still obtain the same answer!*



Tutorial 2B: Kinematics Self-Review Questions (Suggested Solution)

S1	Parabolic trajectory => projectile motion in the absence of air resistance (a) speed: not constant (b) acceleration: constant (c) horizontal component velocity: constant (d) vertical component velocity: not constant *(due to gravitational pull)*
S2	(a) Using $(\downarrow)\ s_y = u_y t + \tfrac{1}{2} a_y t^2$ $300 = 0 + \tfrac{1}{2}(9.81)t^2$ $t = 7.82$ s  (b) To find *v*, we will need v_x and v_y. $(\rightarrow) v_x = u_x = u = 50$ m s^{-1} $(\downarrow)\ v_y{}^2 = u_y{}^2 + 2a_y s_y$ $v_y{}^2 = 0 + 2(9.81)(300)$ $v_y = 76.7$ m s^{-1} $v^2 = v_x{}^2 + v_y{}^2$ $v = \sqrt{50^2 + 76.7^2} = 91.6$ m s^{-1} $\tan\theta = \dfrac{76.7}{50}$ $\theta = 57.0^o$ below horizontal  *<u>Points to note:</u>* *Velocity is a vector quantity, hence the direction must be given, if not marks will be deducted. A good practice is to draw the vector diagram, indicate the angle θ in the diagram, and write a short description of the direction.* (c) $(\rightarrow)\ s_x = u_x t + \tfrac{1}{2} a_x t^2$ $s_x = 50 \times 7.82 = 391$m
S3	Using $(\uparrow)\ s_y = u_y t + \tfrac{1}{2} a_y t^2$ $1700 = (900 \sin 70^o)t + \tfrac{1}{2}(-9.81)t^2$ $t = 170$s or $t = 2.03$s Ans: A 
S4	To find *v*, we will need v_x and v_y. $(\rightarrow) v_x = u_x = u = 40$ m s^{-1} $(\downarrow) v_y = u_y + a_y t$ $v_y = 0 + 9.81(3) = 29.4$ m s^{-1} $v = \sqrt{40^2 + 29.4^2} = 50$ m s^{-1} Ans: C 

S5	$(\rightarrow)\ s_x = u_x t + \tfrac{1}{2} a_x t^2$ $s_x = 24 \times 4.0 = 96\,\text{m}$ $(\downarrow)\ s_y = u_y t + \tfrac{1}{2} a_y t^2$ $s_y = 0 + \tfrac{1}{2} \times 9.81 \times 4.0^2 = 78m$ Ans: A
S6	$(\rightarrow)\ s_x = u_x t + \tfrac{1}{2} a_x t^2$ $1.00 = u_x t ----(1)$ $(\downarrow)\ s_y = u_y t + \tfrac{1}{2} a_y t^2$ $0.52 = 0 + \tfrac{1}{2} \times 9.81 t^2$ $t = 0.326\,\text{s}$ -------(2) Subs. (2) into (1), $u_x = \dfrac{1.00}{0.326} = 3.07$ m s^{-1}
S7	$(\rightarrow)\ s_x = u_x t$ $150 = (40\cos\theta)t ----(1)$ Let t be the time taken for the particle to reach 150 m horizontal range. $(\uparrow)\ s_y = u_y t + \tfrac{1}{2} a_y t^2$ $0 = u\sin\theta - g(t/2)$ $t = \dfrac{2(40)\sin\theta}{g} ----(2)$ Subst. (2) into (1), $150 = (40\cos\theta) \times \left(\dfrac{2 \times 40 \times \sin\theta}{g}\right)$ $150 = \dfrac{40^2 \sin 2\theta}{9.81}$ $\theta = 33.4^o$ or $\theta = 56.6^o$

S8	Let the initial speed be *u.* $(\rightarrow)$ $s_x = u_x t$ $100 = (u \cos \theta)(2.0)$ $u \cos \theta = 50$ -------------- (1) $(\uparrow)$ $s_y = u_y t + \frac{1}{2} a_y t^2$ $0 = (u \sin \theta)(2.0) + (0.5)(-9.81)(2.0)^2$ $u \sin \theta = 9.81$ ------------- (2) Taking (2) / (1), $\tan \theta = 9.81/50$ $\theta = 11^{\circ}$