NAME		INDEX NO.		CLASS	

NORTHLAND SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal Academic

ADDITIONAL MATHEMATICS — 4049/01

Paper 1

27 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

Setter: Mr Chen Weizhong

Vetter: Ms Joanne Yap

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Express each of $2x^2+4x-1$ and $-x^2+x-8$ in the form $a(x+b)^2+c$, where a, b and c are constants. [4]

(b) Use your answers from part **(a)** to explain why the curves with equations $y=2x^2+4x-1$ and $y=-x^2+x-8$ will not intersect. [2]

2 Use the substitution $u = e^{2x}$ to show that the solution to the equation $9e^{2x} + 14 = 8e^{-2x}$ can be expressed in the form $e^{x} = k$, where k is a constant to be found. [3]

3 A right circular cylinder tank has a radius of $(\sqrt{5}-\sqrt{3})$ m and a volume of $(26\sqrt{3}-20\sqrt{5})\pi$ m^3. **Without using a calculator**, express the height of the cylinder in the form $(\sqrt{a}-\sqrt{b})$ m, where a and b are integers. [6]

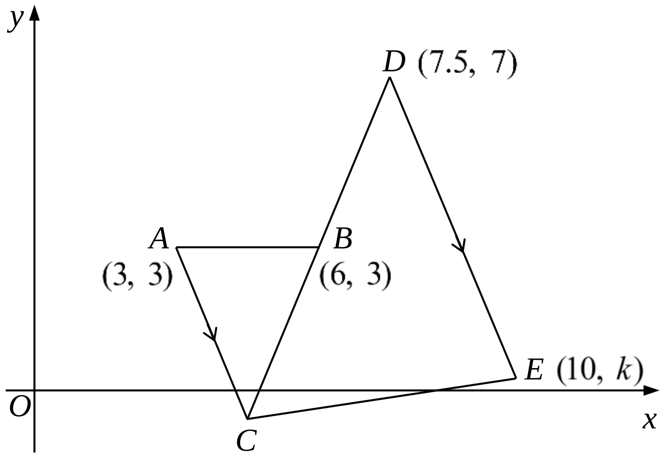
4 $\mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 3\sin x - 4\cos 2x$. Given that $\mathrm{f}\left(\frac{\pi}{6}\right) = 8$ and $\mathrm{f}'(\pi) = 9$, find $\mathrm{f}(x)$. [6]

5 **(a)** **(i)** State, in terms of π, the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$. [1]

(ii) Explain why the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ cannot be $-\frac{\pi}{4}$. [1]

(b) The acute angles A and B are such that $2\sin(A+B) = 1 - 2\sin(A-B)$ and $\cos B = \frac{1}{3}$. Without using a calculator, find the exact value of $\tan A$. [4]

6



The diagram shows an isosceles triangle *ABC* in which $AC = BC$. Point *A* is $(3,\ 3)$, *B* is $(6,\ 3)$ and *C* lies below the *x*-axis. The line *CB* is extended to the point *D* with coordinates $(7.5,\ 7)$. A line is drawn from *D*, parallel to *AC*, to the point *E* $(10,\ k)$ and *C* is joined to *E*.

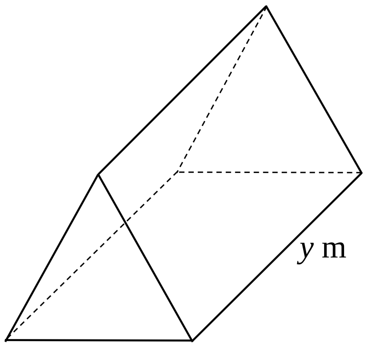
(a) Given that the area of the triangle *ABC* is 6 square units, find the value of *k*. [5]

(b) Using this value of k, find the area of triangle CDE. [2]

7 **(a)** A particle moves along the curve $y = \tan x$ in such a way that the y-coordinate of the particle is decreasing at a rate of 0.12 units per second. Find the rate of change of the x-coordinate of the particle at the instant when $y = 1$. [4]

(b) When an object is subject to different pressures, the pressure, P pascals, and the area, A square centimetres, of the object in contact with the surface are related by the formula $PA = k$, where k is a constant. Given that when $P = 120$, $A = 2$, find the rate at which A is changing with respect to P when $A = 2$. [4]

8



The diagram shows a triangular prism with an equilateral triangle as its cross-section. The total length of the 9 edges is 48 cm.

(a) Show that the total volume, V cm^3, of the prism is given by

$$V = \frac{\sqrt{3}x^2(8-x)}{2}.$$

[4]

(b) Given that x can vary, find the stationary value of V. [4]

9 **(a)** **(i)** Factorise x^3+1 and x^3-1. [2]

(ii) Hence write 6^6-1 as a product of four integers. [2]

(b) A polynomial, P, is $2x^3-3x^2-11x+6$, where k is a constant.

(i) Find the remainder when P is divided by $2x+1$. [2]

The quadratic expression $2x^2 + ax + 3$ is a factor of *P*.

(ii) Find the value of *a* and hence factorise *P* completely. [4]

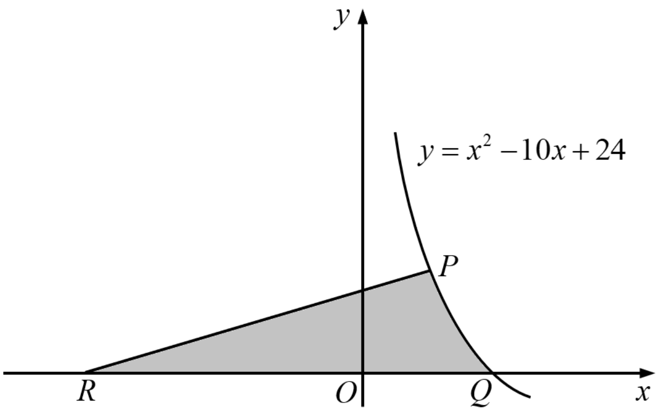
10 **(a)** **(i)** Write down the general term in the binomial expansion of $\left(x+\frac{1}{2x^3}\right)^{12}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, or otherwise, determine the term independent of x in the binomial expansion of $\left(x+\frac{1}{2x^3}\right)^{12}$. [2]

(b) The first three terms in the expansion of $(1+ax)\left(1+\frac{x}{2}\right)^n$, in ascending powers of x, are $1+x-5x^2$. Find the value of each of the constants a and n. [6]

11



The diagram shows part of the curve $y = x^2 - 10x + 24$ cutting the x-axis at Q. The point P lies on the curve and the gradient of the tangent to the curve at P is -4. The normal to the curve at P meets the x-axis at R. Find the area of the shaded region bounded by the curve, the line PR and the x-axis. [10]

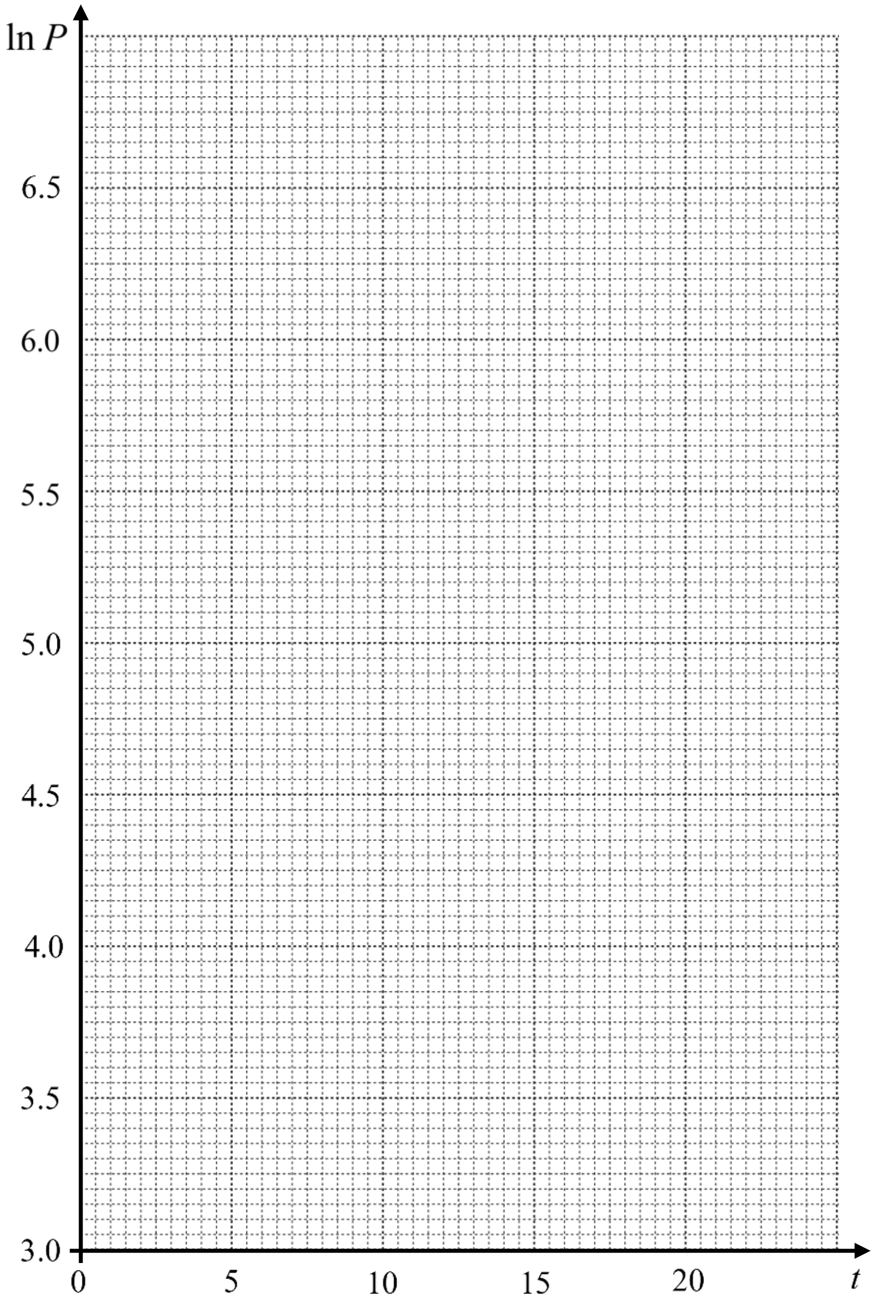
Continuation of working space for question **11**.

12 **(a)** Since 1995, the population of a small town has been steadily decreasing due to a widespread disease. The table shows the estimated population, P, in thousands, in 2000, 2005, 2010 and 2015.

Year	2000	2005	2010	2015
t (years)	5	10	15	20
P (thousands)	260	168	109	70

A demographer believed that these figures can be modelled by the formula $P = ab^t$, where a and b are constants.

(i) On the grid below, plot $\ln P$ against t and draw a straight line graph. [2]



(ii) Estimate the population of the small town in 1995. [2]

(iii) Find the year in which the population first drops below 200 000. [2]

(b) A formula for working out the displacement, s, for a moving vehicle travelling with an acceleration for a certain amount of time, t, is $s = ut + \frac{1}{2}at^2$, where a and u are constants. Values of s for different values of t have been collected. Explain clearly how a straight line graph can be drawn using the recorded data, and state how the values of a and u could be obtained from the line. [4]