

# INNOVA JUNIOR COLLEGE JC 2 PRELIMINARY EXAMINATION 2

in preparation for General Certificate of Education Advanced Level **Higher 2** 

CANDIDATE NAME	
CIVICS GROUP	
Mathematics	9740/02
Paper 2	16 September 2015
	3 hours
Additional materials:	Answer Paper Cover Page List of Formulae (MF 15)

## READ THESE INSTRUCTIONS FIRST

### Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.Write in dark blue or black pen on both sides of the paper.You may use a soft pencil for any diagrams or graphs.Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.



This document consists of 6 printed pages.

#### Section A: Pure Mathematics [40 marks]

**1** The parametric equations of a curve are

$$x = 1 + e^t$$
,  $y = e^{2t}$ .

- (i) Find the equation of the normal to the curve at the point  $P(1+e^p, e^{2p})$ . [3]
- (ii) This normal meets the x-axis at the point Q. Find the cartesian equation of the locus of the mid-point of PQ as p varies. [4]

2 (i) Find the general solution of the differential equation  $x^3 \frac{d^2 y}{dx^2} = 2 - x$ . [3]

- (ii) It is given that y = 1 when x = 1. On a single diagram, sketch three members of the family of solution curves for x > 0. [5]
- 3 (i) Given that  $f(x) = \frac{\sin x + 2}{\cos 2x + 3}$ , where x is sufficiently small, find the series expansion of f(x) in ascending powers of x, up to and including the term in  $x^2$ . [4]
  - (ii) Use your answer to part (i) to give an approximation for  $\int_0^n f(x) dx$  in terms of *n*. Evaluate this approximation in the case where n = 0.5, leaving your answer in 6 decimal places. [3]
  - (iii) Use your calculator to find an accurate value for  $\int_0^{0.5} f(x) dx$ , correct to 6 decimal places. Explain why this value is more than the approximation obtained in part (ii). [3]

4 (a) The complex number z satisfies the relations  $|z-3| \le 3$  and |z-3-3i| = |z|.

- (i) Illustrate both of these relations on a single Argand diagram. [3]
- (ii) Find exactly the maximum and minimum possible values of  $|z|^2$ . [4]
- (b) The complex number w is given by  $\left(\frac{-\sqrt{3}+i}{\sqrt{2}-i\sqrt{2}}\right)^2$ . Without using a calculator, find
  - (i) |w| and the exact value of  $\arg w$ , [4]
  - (ii) the set of values of n, where n is a positive integer, for which  $w^n w^*$  is a real number. [4]

### Section B: Statistics [60 marks]

5 A manufacturing company has three factories that produce packets of instant noodles. The manager wants to test whether the lead content in its latest batch of instant noodles produced exceeds the legally permitted levels. The number of packets of instant noodles produced from each factory for the latest batch is shown in the table below.

Factory	Number of packets produced
A	4000
В	2000
С	1000

To carry out the test, a sample of 100 packets will be chosen from this batch of instant noodles produced.

- (i) Describe how the sample could be chosen using stratified sampling. [2]
- (ii) State one advantage of using stratified sampling in this context. [1]

- 6 A jackpot game machine at an arcade contains 4 slots where each of the first 2 slots displays any of the twelve zodiac signs and each of the next 2 slots display any of the twenty-six letters of the alphabets A–Z. The jackpot is won if the 4 slots display two identical zodiac signs and two identical letters. Find the probability that a random game played at the machine results in
  - (i) two different zodiac signs and two different vowels, [2]
  - (ii) winning the jackpot,
  - (iii) exactly two identical zodiac signs or exactly two identical letters or both. [3]
- 7 Bernard is carrying out an experiment with a fair tetrahedral die, which has its four triangular faces numbered from '6' to '9', and a biased 10-sided die numbered from '1' to '10'.
  - (i) Bernard rolls the fair die 9 times. Find the probability that the die shows a '8' between 3 and 7 times, inclusive. [2]
  - (ii) Bernard now rolls the fair die 65 times. Use a suitable approximate distribution, which should be stated, to find the probability that the die shows a '9' more than 12 times.

The probability that the biased die shows a '9' is  $\frac{1}{25}$ .

- (iii) Bernard rolls the biased die 65 times. Use a suitable approximate distribution, which should be stated, to find the probability that the biased die shows a '9' more than 5 times.
  [3]
- 8 A large office building is busy during the five weekdays, Monday to Friday, and less busy during the two weekend days, Saturday and Sunday. The block is illuminated by fluorescent light tubes which frequently fail and must be replaced with new tubes. It is assumed that the number of fluorescent tubes that fail on a particular weekday has the distribution Po(1.2). The number of fluorescent tubes that fail on a particular weekend day is also assumed to be an independent random variable with distribution Po(0.5).
  - (i) Find the probability that at least 8 fluorescent light tubes fail in a period of five consecutive weekdays.

A week refers to a complete seven-day week.

(ii) Given that a total of 10 fluorescent light tubes fail during a week, find the probability that at most 2 fluorescent light tubes fail during Saturday and Sunday.

[3]

[2]

(iii) Using a suitable approximation, find the probability that at most 30 fluorescent light tubes fail during a period of 4 weeks. [3]

- 9 The continuous random variable X has the distribution  $N(\mu, \sigma^2)$ . It is known that P(X > 2a) = 0.10 and P(X < a) = 0.30.
  - (i) Find E(X) and Var(X) in terms of *a*. [5]
  - (ii) Given that  $X_1$ ,  $X_2$  and  $X_3$  are three independent observations of X, find P $(X_1 + X_2 - 2X_3 > a)$ . [4]
- 10 A firm of solicitors claims that the average duration of the interviews with their clients is 45 minutes.
  - (a) A random sample of 12 interviews is chosen, and the time taken for each interview, x minutes, is noted. The results are shown in the following data.
    - 53 40 61 48 51 43 50 35 42 55 65 60
    - (i) Calculate unbiased estimates of the population mean and variance. [2]
    - (ii) Stating a necessary assumption, carry out a test to determine, at the 5% significance level, whether the firm is understating the average interview time. You should define any symbols that you use. [5]
  - (b) Another sample of 60 interviews is chosen and the time taken for each interview, in minutes, is noted. The sample mean is found to be *m* minutes and the sample standard deviation is 9 minutes. A test is carried out at the 10% level of significance to determine the validity of the firm's claim. Find the set of values of *m* for which the firm's claim is not valid. [5]
- (a) A random sample of six students is taken from those who sat for Mathematics and General Paper examinations, and their marks, x and y, each out of 100, are given in the table.

Mathematics ( <i>x</i> )	34	41	70	86	62	56
General Paper (y)	32	34	63	55	45	58

(i)	Sketch a scatter diagram for the data.	[1]
(ii)	Find the equations of the regression line of	

- (a) y on x, (b) x on y. [2]
- (iii) Sketch the above two regression lines on the scatter diagram in part (i) and mark the point  $(\overline{x}, \overline{y})$  on the diagram. [3]

(b) An experiment is being conducted to measure how the volume of a substance, v (in cm<sup>3</sup>), varies with time t (in minutes) in a particular chemical reaction. The results are given in the table.

t (minutes)	10	15	20	25	30	35	40
$v (cm^3)$	1.75	1.47	1.22	1.05	0.94	0.88	0.85

(i) Calculate, correct to 4 decimal places, the value of the product moment correlation coefficient betweeen v and t. Explain whether your answer suggests that a linear model is appropriate. [2]

The scatter diagram for the data is shown below.



It is proposed that the volume of the substance v can be modelled by the formula  $v = a + \frac{b}{t}$ , where a and b are constants, and b > 0.

- (ii) Explain why the scatter diagram for the data is consistent with this proposed model. [1]
- (iii) Calculate, correct to 4 decimal places, the value of the product moment correlation coefficient betweeen v and  $\frac{1}{t}$ . Comment on the value obtained.

[2]

(iv) Use a suitable regression line to give the best estimate that you can of the volume of the substance at the 28th minute of the chemical reaction. [2]