CANBERRA SECONDARY SCHOOL

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2022 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/01

26 August 2022
2 hours 15 minutes
0800h – 1015h

Name: ______________________________ (   ) **Class:** ________

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
		90

This question paper consists of 19 printed pages including the cover page.

Setter: Mr Muhamad Lathif Yunus

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0, a \neq 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

Answer all the questions

1 (a) Find the range of values of x for which $(x+1)^2-8(x+1)+12>0$. [2]

(b) Using part (a), explain if $(x+1)^2-8(x+1)+12$ will be positive or negative when $x=4$. [1]

2 Given that $2^{2x-5}\times5^{x+3}=8^x\times5^{2x}$, evaluate 10^x. [3]

3 It is given that A and B are acute angles, $\sin(A-B)=\frac{1}{2}$ and $\sin A\cos B=\frac{3}{4}$.

(a) Find the value of $\sin(A+B)$. [3]

(b) Explain why $\sin(180+A)\cos(180+B)=\sin A\cos B$. [2]

4 (a) Show that the derivative of $\frac{\cos x}{\sin x} = -\text{cosec}^2 x$. [2]

(b) Hence, evaluate $2\int \sin^2 x - \text{cosec}^2 x \, dx$. [3]

5 **(a)** Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants. [2]

(b) Hence, state the maximum value of y and the corresponding value of x. [1]

(c) Sketch the graph of $y = 4x - 4x^2 - 3$. [2]

(d) Using the sketch from **(c)**, explain why $4x - 4x^2 - 3 = 0$ has no solutions. [1]

6 (a) Solve $\log_9 y^3 - 2 = \log_3 y$. [3]

(b) Solve $e^x - 1 = 6e^{-x}$. [3]

7 (a) Prove the identity $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$. [3]

(b) Hence, solve $\frac{1+\sin(x+70°)}{\cos(x+70°)}+\frac{\cos(x+70°)}{1+\sin(x+70°)}=4$ for $0° \le x \le 360°$. [3]

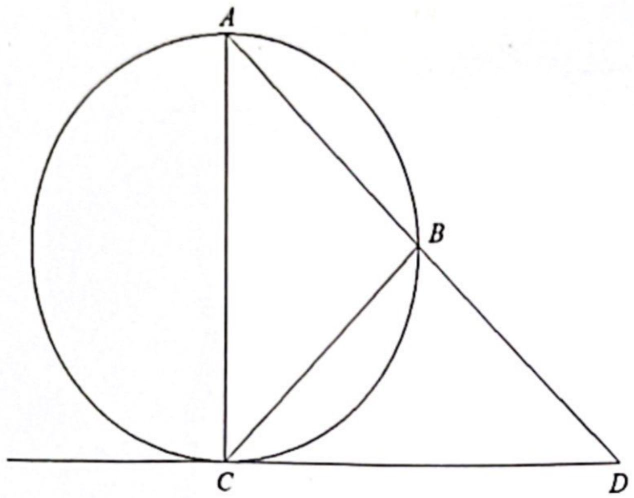
8 A function is defined by $y = \ln(3x-2)+2x$.

(a) Find an expression for $\frac{dy}{dx}$. [2]

(b) Explain why the function $y = \ln(3x-2)+2x$ does not exist for $x \le \frac{2}{3}$. [2]

(c) Determine whether $y = \ln(3x-2)+2x$ is an increasing or decreasing function for all values of x except $x \le \frac{2}{3}$. [2]

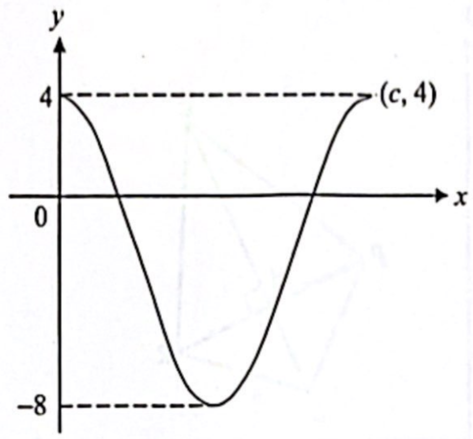
9 The figure below shows point A, B and C on a circle. The line CD is tangent to the circle and AC is the diameter of the circle. The line AD intersects the line CB at B.



(a) Show that ΔABC is similar to ΔCBD. [3]

(b) Find another triangle similar to ΔABC. Explain why this triangle is similar to ΔABC. [3]

10 The diagram shows the graph of $y = a\cos\frac{1}{2}x + b$ where x is in radians.

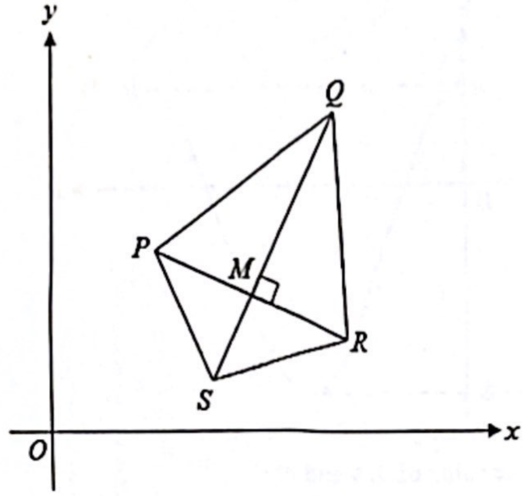


(a) Write down the value of a, b and c. [3]

(b) Explain what happens to a, b and c if the graph $y = a\cos\frac{1}{2}x + b$ is reflected about the x-axis. [1]

(c) By adding a suitable line to the diagram, find the number of solutions for $0 < x < c$, that will satisfy the equation $a\cos\frac{1}{2}x + b + 6 = \frac{2x}{\pi}$. [3]

11 Solutions to this question by accurate drawing will not be accepted.



In the quadrilateral $PQRS$ shown in the diagram, QS is the perpendicular bisector of PR. The coordinates of P and R are (3, 6) and (11, 2) respectively.

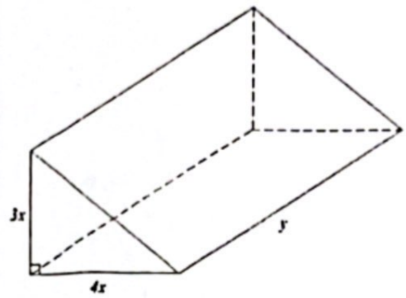
(a) Find the equation of the line QS. [3]

Given that $S(5.5, 1)$ and $3MS = SQ$,

(b) Find the coordinates of Q. [2]

(c) Hence, find the area of $PQRS$. [2]

12 A packaging box (as shown in the figure below) has a uniform triangular cross section consisting of a right-angled triangle with sides $4x$ cm and $3x$ cm. The length of the box is y cm and its volume is 1500 cm^3.



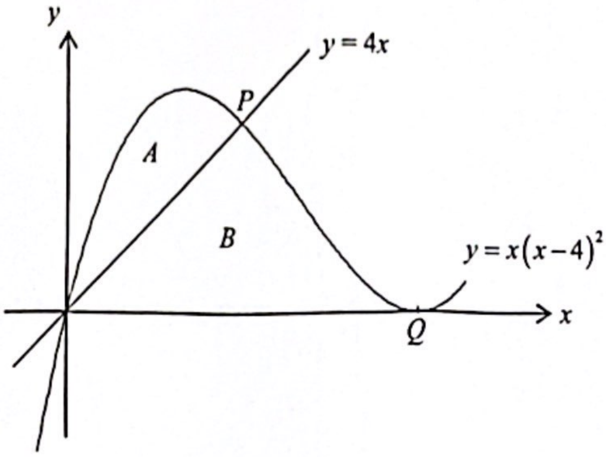
(a) Express y in terms of x. [2]

(b) If the total surface area of the box is $A\text{ cm}^2$, show that $A=\dfrac{3000}{x}+12x^2$. [2]

(c) Find the value of x for which A has a stationary value. [3]

(d) Calculate this stationary value of A and determine whether it is a maximum or a minimum. [3]

13 The diagram below shows line $y = 4x$ that intersects the curve $y = x(x-4)^2$ at point P. The curve touches the x-axis at point Q.
Area A is defined as the area bounded by the curve and the line. (See diagram below).
Area B is defined as the area bounded by the line, curve and the x-axis. (See diagram below).



(a) Find the coordinates of P and Q. [4]

(b) Hence, find the ratio of the area of A to the area of B. [5]

14 A particle P starts at a point 2 m away from the origin O and travels in a straight line so that its velocity v ms $^{-1}$ is given by $v = 8t - 4t^2$ where t is the time in seconds measured from the start of the motion.

(a) Find the initial acceleration of particle P. [1]

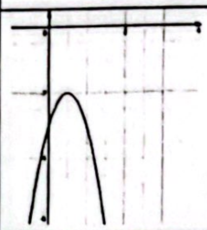
(b) Is the acceleration always increasing or decreasing? Explain your answer. [1]

(c) Calculate the acceleration when the particle is instantaneously at rest. [2]

(d) Find the total distance travelled by P in the first 4 seconds. [4]

(e) Explain why the velocity of the particle can never be more than 4 m/s. [3]

End of Paper

1a	$x<1$ or $x>5$	1b	Since $x=4$ is not in the range, it will give a negative value for $(x+1)^2-8(x+1)+12$
2	$10^x=\frac{125}{32}=3\frac{29}{32}$	3a	$\sin(A+B)=\frac{3}{4}+\frac{1}{4}=1$
4b	$=x-\frac{1}{2}\sin 2x+\frac{2\cos x}{\sin x}+c$	5a	$-4\left(x-\frac{1}{2}\right)^2-2$
5b	$\left(\frac{1}{2},-2\right)$	5c	
5d	Since the graph never intersects the x – axis, $4x-4x^2-3=0$ has no solutions	6a	$y=3^4=81$
6b	$x=\ln 3=1.10$	7b	$x=230°,\ 350°$
8a	$\frac{dy}{dx}=\frac{3}{3x-2}+2$	8b	$y=\ln(3x-2)+2x$ does not exist for $3x-2\leq 0$ $x\leq\frac{2}{3}$ will result in having to evaluate the natural logarithm of 0 or a negative number which is undefined.
8c	Therefore $\frac{dy}{dx}=\frac{3}{3x-2}+2>0$, increasing function.	10a	$a=\frac{4-(-8)}{2}=6$, $b=-2$, $c=\frac{2\pi}{\left(\frac{1}{2}\right)}=4\pi$
10b	Values of a and b will change sign, while c remains unchanged	10c	2 solutions
11a	$y=2x-10$	11b	Therefore $Q(10, 10)$
11c	45 square units	12a	$y=\frac{1500}{6x^2}=\frac{250}{x^2}$
12c	$x=5$	12d	When $x=5$, $\frac{d^2A}{dx^2}>0$ (Minimum point)
13a	$P(2, 8)$ $Q(4, 0)$	13b	5 : 11
14a	8 m/s^2	14b	$\frac{da}{dt}=-8<0$ (decreasing)
14c	–8 m/s^2	14d	32 m
14e	$v=-4(t-1)^2+4$ Max velocity = 4 when t = 1		