

**2024 JPJC J2 H2 Prelims Paper 1 Solutions**

**1**

Let  $v_n = an^3 + bn^2 + cn + d$

$$v_1 = -9 : a + b + c + d = -9 \dots \dots (1)$$

$$v_2 = 7 : 8a + 4b + 2c + d = 7 \dots \dots (2)$$

$$v_3 = 47 : 27a + 9b + 3c + d = 47 \dots \dots (3)$$

$$v_4 = 141 : 64a + 16b + 4c + d = 141 \dots \dots (4)$$

Use GC:  $a = 5, b = -18, c = 35, d = -31$

$$v_n = 5n^3 - 18n^2 + 35n - 31$$

**2**

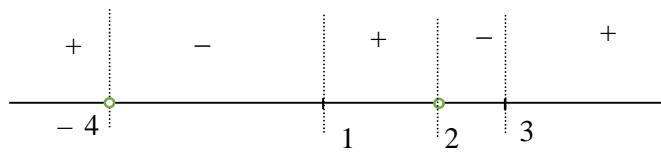
$$\frac{11x - 19}{x^2 + 2x - 8} \leq 2$$

$$\frac{11x - 19}{x^2 + 2x - 8} - 2 \leq 0$$

$$\frac{11x - 19 - 2x^2 - 4x + 16}{x^2 + 2x - 8} \leq 0$$

$$\frac{-2x^2 + 7x - 3}{x^2 + 2x - 8} \leq 0$$

$$\frac{(2x-1)(x-3)}{(x+4)(x-2)} \geq 0$$



$$x < -4 \text{ or } \frac{1}{2} \leq x < 2 \text{ or } x \geq 3$$

Replace  $x$  by  $e^{-x}$

$$e^{-x} < -4 \text{ or } \frac{1}{2} \leq e^{-x} < 2 \text{ or } e^{-x} \geq 3$$

$$(\text{no solution}), \ln \frac{1}{2} \leq -x < \ln 2 \text{ or } -x \geq \ln 3$$

$$-\ln 2 < x \leq \ln 2 \text{ or } x \leq -\ln 3$$

**3**

$$y = 5 \sin \theta \Rightarrow \frac{dy}{d\theta} = 5 \cos \theta$$

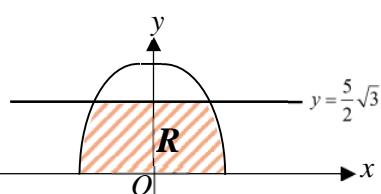
$$y = 0, 0 = 5 \sin \theta \Rightarrow \theta = 0$$

$$y = \frac{5}{2}\sqrt{3}, \frac{5}{2}\sqrt{3} = 5 \sin \theta \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

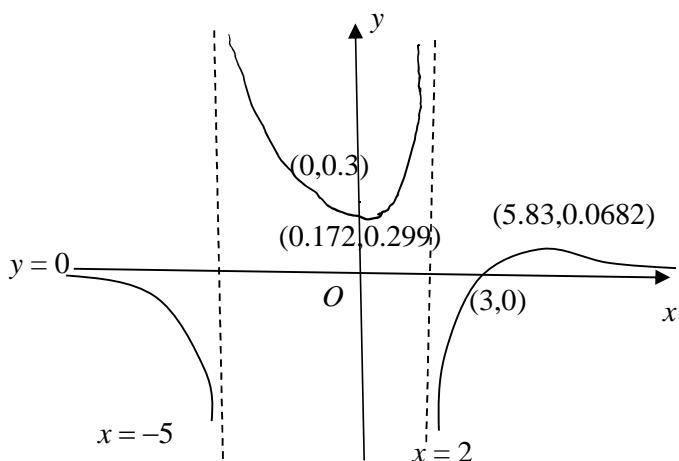
Volume

$$= \pi \int_0^{\frac{5\sqrt{3}}{2}} x^2 dy$$

$$= \pi \int_0^{\frac{5\sqrt{3}}{2}} \sqrt{25 - y^2} dy$$



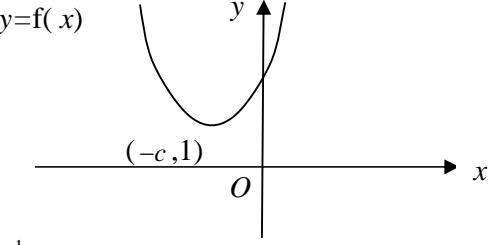
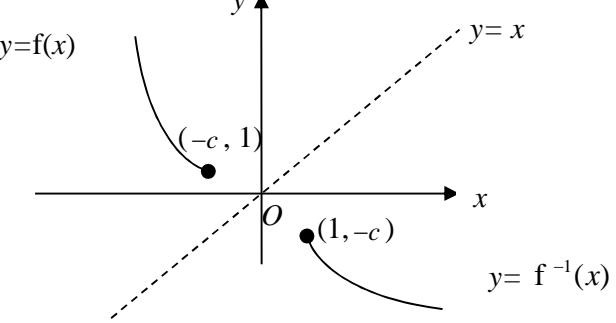
$$\begin{aligned}
&= \pi \int_0^{\frac{\pi}{3}} \sqrt{25 - 25 \sin^2 \theta} (5 \cos \theta) d\theta \\
&= 5\pi \int_0^{\frac{\pi}{3}} \sqrt{25(1 - \sin^2 \theta)} \cos \theta d\theta \\
&= 25\pi \int_0^{\frac{\pi}{3}} \sqrt{\cos^2 \theta} \cos \theta d\theta \\
&= 25\pi \int_0^{\frac{\pi}{3}} \cos^2 \theta d\theta \\
&= \frac{25}{2}\pi \int_0^{\frac{\pi}{3}} 1 + \cos 2\theta d\theta \\
&= \frac{25}{2}\pi \left[ \theta + \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{3}} \\
&= \frac{25}{2}\pi \left[ \frac{\pi}{3} + \frac{1}{2} \sin \frac{2\pi}{3} \right] \\
&= \frac{25}{2}\pi \left[ \frac{\pi}{3} + \frac{\sqrt{3}}{4} \right] \\
&= \frac{25}{6}\pi^2 + \frac{25}{8}\sqrt{3}\pi \quad (\text{exact})
\end{aligned}$$

**4(i)****(ii)**

$$\begin{aligned}
y = f(x) &= \frac{x-3}{(x-2)(x+5)} \\
\rightarrow y = f(2x) &= \frac{2x-3}{(2x-2)(2x+5)} \\
\rightarrow y = (2)f(2x) &= (2) \frac{2x-3}{(2x-2)(2x+5)} \quad y = \frac{2x-3}{(x-1)(2x+5)}
\end{aligned}$$

Scaling parallel to the  $x$ -axis by scale factor  $\frac{1}{2}$ .

Scaling parallel to the  $y$ -axis by scale factor 2.

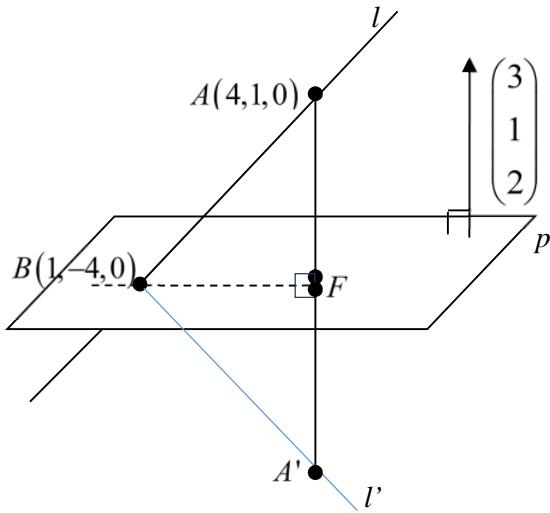
<b>5(i)</b>	<p><math>y=f(x)</math></p>  <p><math>f^{-1}</math> exists for <math>x \leq -c</math>  <math>k = -c</math></p>
<b>(ii)</b>	$y = e^{(x+c)^2}$ $(x+c)^2 = \ln(y)$ $x+c = \pm\sqrt{\ln(y)}$ $x = -c \pm \sqrt{\ln(y)}$ <p>Since <math>x \leq -c</math>, <math>x = -c - \sqrt{\ln(y)}</math></p> $f^{-1}(x) = -c - \sqrt{\ln x}, x \geq 1$
<b>(iii)</b>	 <p>Graphs of <math>f</math> and <math>f^{-1}</math>  are reflections of each other about the line <math>y = x</math></p>
<b>(iv)</b>	<p>Range of <math>f = [1, \infty)</math> and Domain of <math>g = (0, \infty)</math>  Range of <math>f \subseteq</math> Domain of <math>g</math>  Hence <math>gf</math> exists  <math display="block">gf(x) = \ln e^{(x+c)^2}</math> <math display="block">= (x+c)^2, x \leq -c</math> </p>

<b>6(i)</b>	$\overrightarrow{AB} = \underline{b} - \underline{a}$ $\overrightarrow{AC} = (m\underline{a} + n\underline{b}) - \underline{a} = (m-1)\underline{a} + n\underline{b}$ <p>Area of triangle <math>ABC</math></p> $= \frac{1}{2}  \overrightarrow{AB} \times \overrightarrow{AC} $ $= \frac{1}{2}  (\underline{b} - \underline{a}) \times ((m-1)\underline{a} + n\underline{b}) $
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	$  \begin{aligned}  &= \frac{1}{2}  (m-1)(\underline{b} \times \underline{a}) + n(\underline{b} \times \underline{b}) - (m-1)(\underline{a} \times \underline{a}) - n(\underline{a} \times \underline{b})  \\  &= \frac{1}{2}  (m-1)(\underline{b} \times \underline{a}) + \underline{0} + \underline{0} + n(\underline{b} \times \underline{a})  \\  &= \frac{1}{2}(m+n-1) \underline{b} \times \underline{a}  \\  &= \frac{1}{2}(m+n-1) \underline{b}  \underline{a} \sin 30^\circ \\  &= \frac{1}{2}(m+n-1) \underline{b} (4 \underline{b} )\left(\frac{1}{2}\right) \\  &= (m+n-1) \underline{b} ^2  \end{aligned}  $
(ii)	$\overrightarrow{OD} = \frac{1}{2}\underline{a} \quad \overrightarrow{OE} = \frac{3}{5}\underline{b}$
(iii)	<p> <math>\underline{l}_{BD} : \underline{r} = \underline{b} + \lambda_1 \overrightarrow{BD}</math>  <math>= \underline{b} + \lambda_1 (\underline{d} - \underline{b})</math>  <math>= \underline{b} + \lambda_1 \left( \frac{1}{2}\underline{a} - \underline{b} \right)</math>  <math>= \underline{b} + \lambda (\underline{a} - 2\underline{b}), \text{ where } \lambda = \frac{\lambda_1}{2}</math>  <math>= \lambda \underline{a} + (1 - 2\lambda) \underline{b} \text{ (shown)}</math> </p> <p> <math>\underline{l}_{AE} : \underline{r} = \underline{a} + \mu_1 \overrightarrow{AE}</math>  <math>= \underline{a} + \mu_1 (\underline{e} - \underline{a})</math>  <math>= \underline{a} + \mu_1 \left( \frac{3}{5}\underline{b} - \underline{a} \right)</math>  <math>= \underline{a} + \mu (3\underline{b} - 5\underline{a}), \text{ where } \mu = \frac{\mu_1}{5}</math>  <math>= (1 - 5\mu) \underline{a} + 3\mu \underline{b}</math> </p> <p>Since lines <math>BD</math> and <math>AE</math> meet,</p> $\lambda \underline{a} + (1 - 2\lambda) \underline{b} = (1 - 5\mu) \underline{a} + 3\mu \underline{b}$ <p>Comparing coefficients of <math>\underline{a}</math>,</p> $\lambda = 1 - 5\mu \Rightarrow \lambda + 5\mu = 1 \quad \dots (1)$ <p>Comparing coefficients of <math>\underline{b}</math>,</p> $1 - 2\lambda = 3\mu \Rightarrow 2\lambda + 3\mu = 1 \quad \dots (2)$ <p>Using GC, <math>\lambda = \frac{2}{7}, \mu = \frac{1}{7}</math></p> $  \begin{aligned}  \overrightarrow{OF} &= \frac{2}{7}\underline{a} + \left[ 1 - 2\left(\frac{2}{7}\right) \right] \underline{b} \\  &= \frac{2}{7}\underline{a} + \frac{3}{7}\underline{b}  \end{aligned}  $

<b>7(a)</b>	$l: \underline{r} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}, t \in \mathbb{R} \quad \text{---(1)}$ <p>For plane <math>p</math>, <math>\underline{n} = \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix} \times \begin{pmatrix} -3 \\ a \\ 1 \end{pmatrix} = \begin{pmatrix} -5-a \\ -4 \\ a-15 \end{pmatrix}</math></p> <p>Since <math>l</math> and <math>p</math> do not meet in a unique point,</p> $\begin{pmatrix} -5-a \\ -4 \\ a-15 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = 0$ $3(-5-a) - 20 = 0$ $-3a = 35$ $a = -\frac{35}{3}$
<b>(b)(i)</b>	<p>Given <math>a = 7</math>,</p> $\underline{n} = \begin{pmatrix} -5-7 \\ -4 \\ 7-15 \end{pmatrix} = -4 \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ <p><math>p: \underline{r} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = -1 \quad \text{--- (2)}</math></p> <p>Subst. (1) into (2):</p> $\left[ \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \right] \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = -1$ $13 + 14t = -1$ $t = -1$ <p>Position vector of the point of intersection,</p> $\underline{B} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$ $\therefore B(1, -4, 0)$

(ii)



To find  $F$ , the foot of perpendicular from  $A$  to  $p$ :

$$l_{AF}: \vec{r} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, s \in \mathbb{R}$$

$$\left[ \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right] \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = -1$$

$$13 + 14s = -1$$

$$s = -1$$

$$\overrightarrow{OF} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

Since  $F$  is the mid-point of  $AA'$ ,

$$\overrightarrow{OF} = \frac{\overrightarrow{OA} + \overrightarrow{OA'}}{2}$$

$$\overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$$

$$= 2 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$$

$$\overrightarrow{BA'} = \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -4 \end{pmatrix}$$

$$l': \vec{r} = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -3 \\ 3 \\ -4 \end{pmatrix}, \alpha \in \mathbb{R}$$

$$\frac{1-x}{3} = \frac{y+4}{3} = -\frac{z}{4}$$

8 (i)	<table border="1"> <thead> <tr> <th>Month</th><th>Amount owed at beginning of the month</th><th>Amount owed at the end of the month</th></tr> </thead> <tbody> <tr> <td>1</td><td><math>200000(1.005)</math></td><td><math>200000(1.005) - x</math></td></tr> <tr> <td>2</td><td><math>200000(1.005)^2 - (1.005)x</math></td><td><math>200000(1.005)^2 - (1.005)x - x</math></td></tr> <tr> <td>3</td><td><math>200000(1.005)^3 - (1.005)^2x - (1.005)x</math></td><td><math>200000(1.005)^3 - (1.005)^2x - (1.005)x - x</math></td></tr> </tbody> </table> <p>Amount owed at the end of <math>n</math> months</p> $= 200000(1.005)^n - (1.005)^{n-1}x - (1.005)^{n-2}x - \dots - 1.005x - x$ $= 200000(1.005)^n - x[1 + 1.005 + \dots + (1.005)^{n-2} + (1.005)^{n-1}]$ $= 200000(1.005)^n - \frac{x[1 - 1.005^n]}{1 - 1.005}$ $= 200000(1.005)^n - 200x[1.005^n - 1]$	Month	Amount owed at beginning of the month	Amount owed at the end of the month	1	$200000(1.005)$	$200000(1.005) - x$	2	$200000(1.005)^2 - (1.005)x$	$200000(1.005)^2 - (1.005)x - x$	3	$200000(1.005)^3 - (1.005)^2x - (1.005)x$	$200000(1.005)^3 - (1.005)^2x - (1.005)x - x$
Month	Amount owed at beginning of the month	Amount owed at the end of the month											
1	$200000(1.005)$	$200000(1.005) - x$											
2	$200000(1.005)^2 - (1.005)x$	$200000(1.005)^2 - (1.005)x - x$											
3	$200000(1.005)^3 - (1.005)^2x - (1.005)x$	$200000(1.005)^3 - (1.005)^2x - (1.005)x - x$											
(ii)	$200000(1.005)^n - 200x[1.005^n - 1] \leq 0$ $200000(1.005)^n - 200(1500)[1.005^n - 1] \leq 0$ $300000 - 100000(1.005)^n \leq 0$ $(1.005)^n \geq 3$ $n \geq \frac{\ln 3}{\ln 1.005}$ $n \geq 220.27$ <p>Alternatively, Use GC table</p> <table border="1"> <thead> <tr> <th><math>n</math></th><th><math>200000(1.005)^n - 200(1500)[1.005^n - 1]</math></th></tr> </thead> <tbody> <tr> <td>219</td><td>1896.19</td></tr> <tr> <td>220</td><td>405.67 &gt; 0</td></tr> <tr> <td>221</td><td>-1092.30 &lt; 0</td></tr> </tbody> </table> <p><math>n = 221</math></p>	$n$	$200000(1.005)^n - 200(1500)[1.005^n - 1]$	219	1896.19	220	405.67 > 0	221	-1092.30 < 0				
$n$	$200000(1.005)^n - 200(1500)[1.005^n - 1]$												
219	1896.19												
220	405.67 > 0												
221	-1092.30 < 0												
	<p>At the end of 220 months, Selena owed</p> $200000(1.005)^{220} - 200(1500)[1.005^{220} - 1] = \$405.67$ <p>Last repayment amount to be repaid on the 221<sup>st</sup> month  <math>= \\$405.67 \times 1.005 = \\$407.70</math> (2d.p.)</p> <p>221 months = 18 years 5 months</p> <p>Full repayment on: <u>31 May 2043</u></p>												
(iii)	$200000(1.005)^{120} - 200x[1.005^{120} - 1] \leq 0$ $363879.3468 - 163.8793468x \leq 0$ $x \geq 2220.410039$ <p><math>\\$2220.42</math> (2d.p.) [<math>\\$2220.41</math> not accepted]</p>												

<b>9(a)</b>	<p>Sub. <math>z = 1 + 2i</math> into <math>z^4 - z^3 - 9z^2 + sz + t = 0</math></p> $(1+2i)^4 - (1+2i)^3 - 9(1+2i)^2 + s(1+2i) + t = 0$ $(-7-24i) - (-11-2i) - 9(-3+4i) + s(1+2i) + t = 0$ $(31+s+t) + (2s-58)i = 0$ <p>Comparing imaginary parts,  <math>2s - 58 = 0</math></p> $\underline{\underline{s = 29}}$ <p>Comparing real parts,  <math>31 + s + t = 0</math></p> $t = -31 - s$ $= \underline{\underline{-60}}$ <p>Now <math>z^4 - z^3 - 9z^2 + 29z - 60 = 0</math>  Using GC the other roots are <math>\underline{1-2i}, \underline{3}, \underline{-4}</math>.</p> <p><b>Alternative solution</b>  Since <math>z^4 - z^3 - 9z^2 + sz + t = 0</math> is a polynomial equation with real coefficients and <math>1 + 2i</math> is a root, <math>1 - 2i</math> is another root.</p> <p>Quadratic factor = <math>[z - (1+2i)][z - (1-2i)]</math>  <math>= [(z-1)-2i][(z-1)+2i]</math>  <math>= (z-1)^2 - (2i)^2</math>  <math>= z^2 - 2z + 5</math>  Let <math>z^4 - z^3 - 9z^2 + sz + t = (z^2 - 2z + 5)(z^2 + az + b)</math>.</p> <p>By comparing coefficients,  <math>z^3: -1 = a - 2 \Rightarrow a = 1</math>  <math>z^2: -9 = b - 2a + 5 \Rightarrow b = -12</math>  <math>z: s = -2b + 5a = 29</math>  constant term: <math>t = 5b = -60</math>  Now <math>z^4 - z^3 - 9z^2 + 29z - 60 = 0</math>  Using GC (polyroot finder), the other roots are <math>\underline{1-2i}, \underline{3}, \underline{-4}</math>.</p>
<b>(b)</b>	$\arg\left(\frac{w^3}{iw^*}\right) = \arg w^3 - \arg(iw^*)$ $= 3\arg w - (\arg i + \arg w^*)$ $= 3\arg w - \left(\frac{\pi}{2} - \arg w\right)$ $= 4\arg w - \frac{\pi}{2}$

	<p>For <math>\frac{w^3}{iw^*}</math> to be purely imaginary,</p> $\arg\left(\frac{w^3}{iw^*}\right) = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$ $4\arg(w) - \frac{\pi}{2} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$ $4\arg(w) = 0, \pi, 2\pi, -\pi, 3\pi, -2\pi$ $\arg(w) = \frac{\pi}{4}, 0, \frac{\pi}{2}, \frac{-\pi}{4}, \frac{3\pi}{4}, -\frac{\pi}{2}$ <p>Since <math>w = a + ib</math> and <math>a</math> and <math>b</math> are positive real numbers, <math>0 &lt; \arg(w) &lt; \frac{\pi}{2}</math>.</p> $\therefore \arg(w) = \frac{\pi}{4}$ $\tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{4}$ $\frac{b}{a} = 1$ $b = a$ $w = a + ia$
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10(i)	$\frac{dv}{dt} = 2e^{-0.1t}$ $v = \int 2e^{-0.1t} dt$ $v = 2\left(\frac{e^{-0.1t}}{-0.1}\right) + c$ $v = -20e^{-0.1t} + c$ <p>When <math>t = 0, v = 0</math></p> $0 = -20e^0 + c$ $c = 20$ $v = 20 - 20e^{-0.1t}$ <p>Subt <math>v = 10</math></p> $10 = 20 - 20e^{-0.1t}$ $20e^{-0.1t} = 10$ $e^{-0.1t} = \frac{1}{2}$ $-0.1t = \ln\frac{1}{2}$ $t = -10\ln\frac{1}{2} = 10\ln 2 \quad (\text{exact})$
(ii)	<p>As <math>t \rightarrow \infty, e^{-0.1t} \rightarrow 0, v \rightarrow 20</math></p> <p>Eventually, the speed <u>increases</u> and <u>tend to 20 ms<sup>-1</sup></u></p>

<b>(iii)</b>	$-2 \frac{dw}{dt} = (w-3)(w+2)$ $\frac{1}{(w-3)(w+2)} \frac{dw}{dt} = -\frac{1}{2}$ $\int \frac{1}{(w-3)(w+2)} dw = \int -\frac{1}{2} dt$
	<p><u>Method 1: Partial fractions</u></p> $\frac{1}{(w-3)(w+2)} = \frac{A}{(w-3)} + \frac{B}{(w+2)}$ $1 = A(w+2) + B(w-3)$ $1 = -5B \Rightarrow B = -\frac{1}{5}$ $1 = 5A \Rightarrow A = \frac{1}{5}$ $\frac{1}{5} \int \frac{1}{(w-3)} - \frac{1}{(w+2)} dw = \int -\frac{1}{2} dt$ <p><u>Method 2: Completing the square</u></p> $\int \frac{1}{w^2 - w - 6} dw = \int -\frac{1}{2} dt$ $\int \frac{1}{\left(w - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2} dw = \int -\frac{1}{2} dt$ $\frac{1}{2(\frac{5}{2})} \ln \left  \frac{w - \frac{1}{2} - \frac{5}{2}}{w - \frac{1}{2} + \frac{5}{2}} \right  = \int -\frac{1}{2} dt$
	$\frac{1}{5} [\ln  w-3  - \ln  w+2 ] = -\frac{1}{2} t + c$ $\frac{1}{5} \ln \left  \frac{w-3}{w+2} \right  = -\frac{1}{2} t + c$ $\ln \left  \frac{w-3}{w+2} \right  = -\frac{5}{2} t + 5c$ $\left  \frac{w-3}{w+2} \right  = e^{-\frac{5}{2}t+5c}$ $\frac{w-3}{w+2} = \pm e^{-\frac{5}{2}t+5c}$ $\frac{w-3}{w+2} = A e^{-\frac{5}{2}t}, \quad A = \pm e^{5c}$ <p>Sub <math>w = 18</math>,</p> $\frac{18-3}{18+2} = A e^0$ $A = \frac{15}{20} = \frac{3}{4}$ $\frac{w-3}{w+2} = \frac{3}{4} e^{-\frac{5}{2}t}$ $w-3 = \frac{3}{4} w e^{-\frac{5}{2}t} + \frac{3}{2} e^{-\frac{5}{2}t}$

	$w - \frac{3}{4}we^{-\frac{5}{2}t} = 3 + \frac{3}{2}e^{-\frac{5}{2}t}$ $w = \frac{3 + \frac{3}{2}e^{-\frac{5}{2}t}}{1 - \frac{3}{4}e^{-\frac{5}{2}t}}$ $w = \frac{12 + 6e^{-\frac{5}{2}t}}{4 - 3e^{-\frac{5}{2}t}}$
(iv)	<p>The speed will not fall below <math>3 \text{ ms}^{-1}</math>.</p>

11(i)	$y = x \tan \theta - \frac{10x^2}{2u^2 \cos^2 \theta}$ $-1.8 = 15 \tan\left(\frac{\pi}{4}\right) - \frac{10(15)^2}{2u^2 \cos^2\left(\frac{\pi}{4}\right)}$ $-1.8 = 15(1) - \frac{10(15)^2}{2u^2\left(\frac{1}{2}\right)}$ $\frac{10(15)^2}{u^2} = 15 + 1.8$ $u^2 = \frac{10(15)^2}{16.8}$ $u = 11.6$
(ii)	$y = x \tan \theta - \frac{10x^2}{2(10)^2 \cos^2 \theta}$ <p>When <math>y = -1.8</math></p> $\therefore -1.8 = x \tan \theta - \frac{x^2}{20 \cos^2 \theta}$ $-36 \cos^2 \theta = 20x \tan \theta \cos^2 \theta - x^2$ $x^2 - 20x \sin \theta \cos \theta - 36 \cos^2 \theta = 0$ $x^2 - 10x \sin 2\theta - 18(1 + \cos 2\theta) = 0$ $x^2 - 10x \sin 2\theta - 18 \cos 2\theta - 18 = 0 \text{ (Shown)}$

(iii)	<p>Differentiate w.r.t. <math>\theta</math>, we have</p> $2x \frac{dx}{d\theta} - 10 \frac{dx}{d\theta} \sin 2\theta - 20x \cos 2\theta + 36 \sin 2\theta = 0$ <p>At stationary value of <math>x</math>,</p> $\frac{dx}{d\theta} = 0$ $-20x \cos 2\theta + 36 \sin 2\theta = 0$ $36 \sin 2\theta = 20x \cos 2\theta$ $x = \frac{36 \sin 2\theta}{20 \cos 2\theta}$ $x = \frac{9}{5} \tan 2\theta$
(iv)	<p>Sub into equation, we have</p> $\left(\frac{9}{5} \tan 2\theta\right)^2 - 10\left(\frac{9}{5} \tan 2\theta\right) \sin 2\theta - 18 \cos 2\theta - 18 = 0$ $\frac{81}{25} \tan^2 2\theta - 18 \tan 2\theta \sin 2\theta - 18 \cos 2\theta - 18 = 0$ $81 \tan^2 2\theta - 450 \sin 2\theta \tan 2\theta - 450 \cos 2\theta - 450 = 0$ <p>Using GC,</p> $\theta = 0.70883 \approx 0.71 \text{ (2 decimal places)}, \frac{\pi}{2} \text{ (reject)}$ <p>Therefore, stationary value of <math>x = \frac{9}{5} \tan 2(0.70883) = 11.7</math></p>