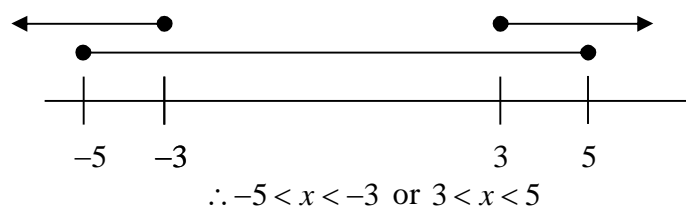# Solutions to Tutorial 1: Inequalities and Equations
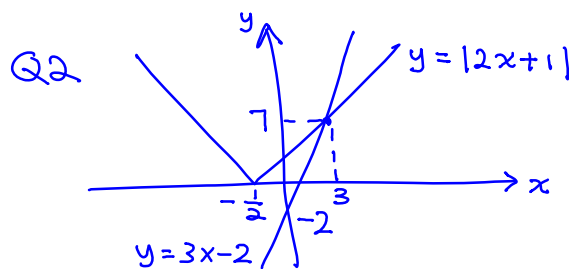
Basic Mastery Questions

1. (a) $x^2 \leq 3 \Rightarrow x^2 - 3 \leq 0 \Rightarrow \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) \leq 0 \Rightarrow -\sqrt{3} \leq x \leq \sqrt{3}$

   (b) $x^4 \leq 9 \Rightarrow x^4 - 9 \leq 0 \Rightarrow \left(x^2 - 3\right)\left(x^2 + 3\right) \leq 0 \Rightarrow x^2 - 3 \leq 0$ (since $x^2 + 3 > 0$)

   $\Rightarrow -\sqrt{3} \leq x \leq \sqrt{3}$ (from 1(a))

   (c) $|x| \leq 3 \Rightarrow -3 \leq x \leq 3$

   (d) $|x+1| > 2 \qquad \Rightarrow x + 1 < -2$ or $x + 1 > 2$

   $\Rightarrow x < -3$ or $x > 1$

   (e) $(x+1)^2 + 3 > 0$

   Since $(x+1)^2 \geq 0$ for all real x, so $(x+1)^2 + 3 > 0$ for all real x $\qquad \therefore x \in \mathbb{R}$

   (f) $x^2 + 2x + 4 < 0 \Rightarrow (x+1)^2 + 3 < 0$

   Since $(x+1)^2 \geq 0$ for all real x, so there is no real solution for x for $(x+1)^2 + 3 < 0$

   (g) $-5 < |x+1| < 2 \Rightarrow 0 \leq |x+1| < 2 \Rightarrow -2 < x + 1 < 2 \qquad \Rightarrow -3 < x < 1$

   (h) $\left|e^{2x}\right| > 2 \qquad \Rightarrow e^{2x} > 2$ (since $e^{2x} > 0$ for all real x)

   $\Rightarrow \ln\left(e^{2x}\right) > \ln 2 \Rightarrow 2x > \ln 2 \Rightarrow x > \dfrac{1}{2}\ln 2 \Rightarrow x > \ln\sqrt{2}$

   (i) $\left|0.5^x\right| < 0.5 \Rightarrow 0 \leq 0.5^x < 0.5 \Rightarrow x \ln 0.5 < \ln 0.5$

   $\Rightarrow x > 1$ (since $\ln 0.5 < 0$)

   (j) $3 < |x| < 5 \Rightarrow -5 < x < -3$ or $3 < x < 5$

   (k) $9 < x^2 < 25 \qquad \Rightarrow 0 < x^2 - 9$ and $x^2 - 25 < 0$

   $\Rightarrow 0 < (x-3)(x+3)$ and $(x-5)(x+5) < 0$

   $\Rightarrow x < -3$ or $x > 3$ and $-5 < x < 5$

   Taking intersection:

   $\therefore -5 < x < -3$ or $3 < x < 5$

2.

Q2

$$|2x+1| \le 3x-2$$
$$x \ge 3$$