Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 1
Preliminary Examination 2020

1		$y = x^2 + kx - 8 + 3k$		
		Lies completely above x-axis		
		$\Rightarrow D < 0$ when $x^2 + kx - 8 + 3k = 0$		
		$(k)^2 - 4(1)(-8+3k) < 0$		
		$k^2 - 12k + 32 < 0$		
		$(k-4)(k-8) < 0$		
		$4 < k < 8$		
2	(i)	$2^{2x-3} = 3^{1-x}$		
		$2^{2x} \times 2^{-3} = 3^1 \times 3^{-x}$		
		$\frac{(2^2)^x}{8} = \frac{3}{3^x}$		
		$(4^x)(3^x) = 24$		
		$12^x = 24$		
	(ii)	$x \lg 12 = \lg 24$		
		$x = \frac{\lg 24}{\lg 12}$		
		$x = 1.3$ (3 s.f.)		
3	(i)	$y = e^{3x}$		
		$\frac{dy}{dx} = 3e^{3x}$		
		Coordinates of $P = \left(k,\ e^{3k}\right)$		
		Gradient of tangent at $P = 3e^{3k}$		
		Equation of tangent:		
		$y - e^{3k} = 3e^{3k}(x-k)$		
		At x-axis, $y = 0$		
		$0 - e^{3k} = 3e^{3k}(x-k)$		
		$-e^{3k} = 3e^{3k}(x-k)$		
		$-1 = 3(x-k)$		
		$x = k - \frac{1}{3}$		
		$\therefore T\left(k - \frac{1}{3},\ 0\right)$		

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	(ii)	Gradient of tangent at $P = 3e^{3k}$		
		For the straight line to cut C twice, it must be <u>steeper</u> than the tangent. Therefore its gradient is greater than $3e^{3k}$.		
4	(i)	$\frac{d}{dx}\left(x^3 \ln x\right)$		
		$= 3x^2 \ln x + x^3\left(\frac{1}{x}\right)$		
		$= 3x^2 \ln x + x^2$		
	(ii)	$\int 3x^2 \ln x + x^2 dx = x^3 \ln x + C_1$		
		$\int 3x^2 \ln x \, dx + \frac{x^3}{3} = x^3 \ln x + C_2$		
		$\int 3x^2 \ln x \, dx = x^3 \ln x - \frac{x^3}{3} + C_2$		
		$\int x^2 \ln x \, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$		
5		$y = \sec^2(2x) - 2$		
		When $x = \frac{1}{2}$, $a = 1.4255$ (5 s.f.)		
		Shaded area		
		$= \left\lvert \int_0^{\frac{\pi}{8}} \sec^2(2x) - 2dx \right\rvert + \frac{1}{2}a - \int_{\frac{\pi}{8}}^{\frac{1}{2}} \sec^2(2x) - 2dx$		
		$= \left\lvert \left[\frac{\tan 2x}{2} - 2x\right]_0^{\frac{\pi}{8}} \right\rvert + \frac{1}{2}(1.4255) - \left[\frac{\tan 2x}{2} - 2x\right]_{\frac{\pi}{8}}^{\frac{1}{2}}$		
		$= \left\lvert \left[\frac{\tan 2\left(\frac{\pi}{8}\right)}{2} - 2\left(\frac{\pi}{8}\right)\right] - 0 \right\rvert + \frac{1}{2}(1.4255)$ $- \left\{\left[\frac{\tan 2\left(\frac{1}{2}\right)}{2} - 2\left(\frac{1}{2}\right)\right] - \left[\frac{\tan 2\left(\frac{\pi}{8}\right)}{2} - 2\left(\frac{\pi}{8}\right)\right]\right\}$		
		$= 0.934$ (3 s.f.)		

6	(i)	Mid-point of $(-15,5)$ and $(1,5)$ = $(-7,5)$		
		x-coordinate of centre = -7		
		Radius = $-7-(-17)=10$		
	(ii)	Let the coordinates of centre be (x, y).		
		$(-7-1)^2+(y-5)^2=100$		
		$(y-5)^2=36$		
		$y-5=6$ $y-5=-6$		
		$y=11$ $y=-1$ (rej.)		
		$\therefore C(-7,\ 11)$		
	(iii)	$(x+7)^2+(y-11)^2=100$		
7	(i)	$(2-3x)^5$		
		$=32-240x+720x^2+\ldots$		
	(ii)	$(1+ax)(2-3x)^5=32-bx-240x^2+\ldots$		
		$(1+ax)(32-240x+720x^2+\ldots)$		
		$=32-240x+720x^2+32ax-240ax^2+\ldots$		
		$=32+(-240+32a)x+(720-240a)x^2+\ldots$		
		Comparing coefficients of x^2:		
		$720-240a=-240$		
		$a=4$		
		Comparing coefficients of x:		
		$-b=-240+32a$		
		$b=112$		
8	(i)	$h=-3\cos(k\pi t)+7$		
		Distance between A and B, and distance between B and C is 3 cm.		
		Thus the distance between A and $C=2\times3=6$ cm.		
	(ii)	Period = 0.5 seconds		

		$\therefore b=\frac{2\pi}{0.5}=4\pi$ rad/s		
		$\therefore k=4$ rad/s		
	(iii) (a)	$-3\cos(4\pi t)+7=8$		
		$\cos(4\pi t)=-\frac{1}{3}$		
		$\alpha=1.2310$ (5 s.f.)		
		$4\pi t=\pi-\alpha, \pi+\alpha$		
		$t=0.152, 0.348$ (3 s.f.)		
	(b)	Duration $=\frac{0.3480-0.1520}{2}=0.0980$ s (3 s.f.)		
9	(i)	$f(x)=3x^3+ax^2+bx-2$		
		$3x+1$ is a factor $\Rightarrow f\left(-\frac{1}{3}\right)=0$		
		$f\left(-\frac{1}{3}\right)=3\left(-\frac{1}{3}\right)^3+a\left(-\frac{1}{3}\right)^2+b\left(-\frac{1}{3}\right)-2=0$		
		$\frac{1}{9}a-\frac{1}{3}b-\frac{19}{9}=0$		
		$a-3b=19$		
		remainder of $4\Rightarrow f(1)=4$		
		$f(1)=3(1)^3+a(1)^2+b(1)-2=4$		
		$a+b=3$		
		$a=7$		
		$b=-4$		
	(ii)	$3x^3+ax^2+bx-2=0$		
		$(3x+1)(x^2+2x-2)=0$		
		$x=-\frac{1}{3}$ or $x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$		

		$=\dfrac{-2\pm\sqrt{12}}{2}$		
		$=\dfrac{-2\pm2\sqrt{3}}{2}$		
		$=-1\pm\sqrt{3}$		
10	(i)	By similar triangles,		
		$\dfrac{y-3}{3}=\dfrac{8}{x}$ $\qquad$ $\dfrac{y}{3}=\dfrac{x+8}{x}$		
		$y-3=\dfrac{24}{x}$ $\qquad$ $y=\dfrac{3x+24}{x}$		
		$y=\dfrac{24}{x}+3$ $\qquad$ $y=3+\dfrac{24}{x}$		
	(ii)	$L=PA+AQ$		
		$=8+x+y$		
		$=8+x+\dfrac{24}{x}+3$		
		$=11+x+\dfrac{24}{x}$		
		$\dfrac{dL}{dx}=1-\dfrac{24}{x^2}=0$		
		$\dfrac{24}{x^2}=1$		
		$x^2=24$		
		$x=\sqrt{24}$ or $x=-\sqrt{24}$ (rej.)		
		$L=11+\sqrt{24}+\dfrac{24}{\sqrt{24}}=20.8\text{ cm (3 s.f.)}$		
	(iii)	$\dfrac{d^2L}{dx^2}=\dfrac{48}{x^3}$		
		$\dfrac{d^2L}{dx^2}>0$		
		$L=20.8$ is a minimum value.		
11	(i)	$A=25+68=93$		

	(ii)	$T=25+68e^{-\frac{t}{8}}$		
		$T-25=68e^{-\frac{t}{8}}$		
		$e^{-\frac{t}{8}}=\frac{T-25}{68}$		
		$-\frac{t}{8}=\ln\left(\frac{T-25}{68}\right)$		
		$t=-8\ln\left(\frac{T-25}{68}\right)$		
	(iii)	At $T=70$, $t=-8\ln\left(\frac{70-25}{68}\right)$		
		$t=3.30$		
	(iv)	$\frac{dT}{dt}=(68)\left(-\frac{1}{8}\right)e^{-\frac{t}{8}}$		
		$\frac{dT}{dt}=-\frac{17}{2}e^{-\frac{t}{8}}$		
		When $t=5$, $\frac{dT}{dt}=-\frac{17}{2}e^{-\frac{5}{8}}$		
		$\frac{dT}{dt}=-4.55$ °C/min		
		Rate of decrease = 4.55 °C/min		
	(v)	Because $68e^{-\frac{t}{8}}>0$,		
		$\therefore T=25+68e^{-\frac{t}{8}}>25+0$		
		$\therefore T>25$		
12	(i)	$y=\sin x-\cos 2x$		
		$\sin x-\cos 2x=0$		
		$\sin x-(1-2\sin^2 x)=0$		
		$2\sin^2 x+\sin x-1=0$		
		$(2\sin x-1)(\sin x+1)=0$		
		$\sin x=\frac{1}{2}$ or $\sin x=-1$		

		$\alpha=\frac{\pi}{6}$		$x=\frac{3\pi}{2}$		
		$x=\frac{\pi}{6},\ \frac{5\pi}{6}$				
	(ii)	$\frac{dy}{dx}=\cos x+2\sin 2x$				
		$\cos x+2\sin 2x=0$				
		$\cos x+4\sin x\cos x=0$				
		$\cos x(1+4\sin x)=0$				
		$\cos x=0$	or	$1+4\sin x=0$		
		$x=\frac{\pi}{2}$		$\sin x=-\frac{1}{4}$ (rej.)		
		$y=2$				
		$\therefore P\left(\frac{\pi}{2},\ 2\right)$			A1	

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1		$y=\frac{3x}{2x-1}$		
		$\frac{dy}{dx}=\frac{3(2x-1)-2(3x)}{(2x-1)^2}$		
		$\frac{dy}{dx}=\frac{-3}{(2x-1)^2}$		
		Given $\frac{dy}{dt}=-0.25$ and $\frac{dx}{dt}=3$,		
		$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$		
		$-0.25=\frac{-3}{(2x-1)^2}\times 3$		
		$(2x-1)^2=\frac{-9}{-0.25}$		
		$(2x-1)^2=36$		
		$2x-1=\pm 6$		
		$2x=-5$ or $2x=7$		
		$x=-\frac{5}{2}$ or $x=\frac{7}{2}$ (rej.)		
		$y=\frac{3\left(-\frac{5}{2}\right)}{2\left(-\frac{5}{2}\right)-1}=\frac{5}{4}$		
2	(i)	Least value = 0 Greatest value = 4		

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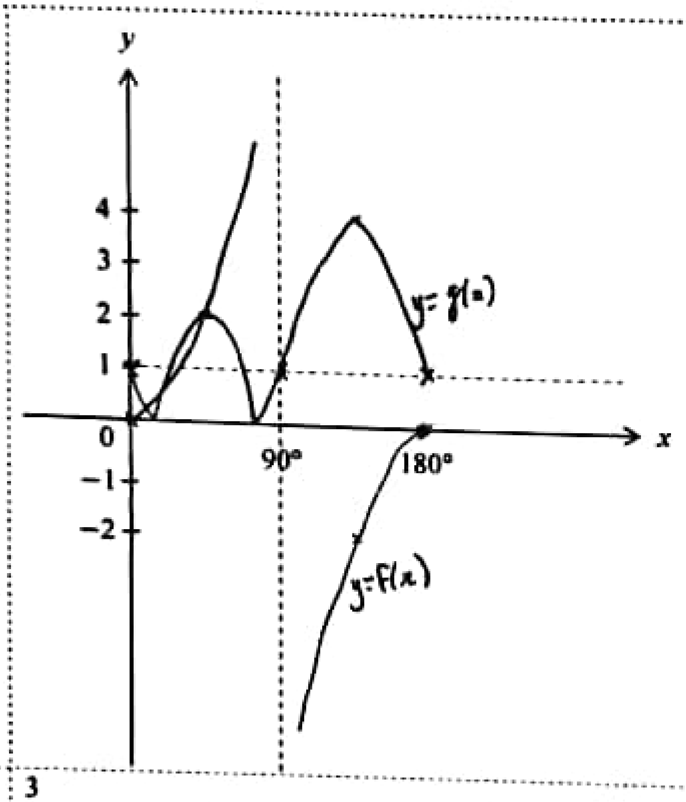
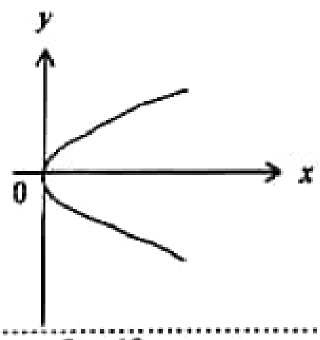
	(ii)		
	(iii)	3	
3	(a)	$\lvert 2x+1 \rvert = 3x-1$	
		$2x+1 = 3x-1$	$2x+1 = -3x+1$
		$x = 2$	$x = 0$ (rejected)
	(b)	$\dfrac{1}{2\log_y 2} + \log_4 2 = \log_4(y+3)$	
		$\left(\dfrac{1}{2}\right)\left(\dfrac{1}{\log_y 2}\right) + \log_4 2 = \log_4(y+3)$	
		$\left(\dfrac{1}{2}\right)(\log_2 y) + \log_4 2 = \log_4(y+3)$	
		$\left(\dfrac{1}{2}\right)\left(\dfrac{\log_4 y}{\log_4 2}\right) + \log_4 2 = \log_4(y+3)$	
		$\log_4 y + \dfrac{1}{2} = \log_4(y+3)$	
		$\log_4 \dfrac{y^2}{(y)(y+3)} = -\dfrac{1}{2}$	
		$\log_4 \dfrac{y}{y+3} = -\dfrac{1}{2}$	

		$\frac{y}{y+3}=4^{-\frac{1}{2}}$		
		$y=3$		
4	(i)	$x^2+3x+6=0$		
		$\alpha+\beta=-3$		
		$\alpha\beta=6$		
		$\alpha^2+\beta^2=(-3)^2-2(6)=-3$		
		$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$		
		$\alpha^3+\beta^3=(-3)(-3-(-6))$		
		$\alpha^3+\beta^3=27$		
	(ii)	Sum $=\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$		
		$=\frac{\alpha^3+\beta^3}{\alpha\beta}$		
		$=\frac{27}{6}$		
		$=\frac{9}{2}$		
		Product $=\left(\frac{\alpha^2}{\beta}\right)\left(\frac{\beta^2}{\alpha}\right)=6$		
		New equation: $x^2-\frac{9}{2}x+6=0$		
5	(i)	 • Shape • Passes through origin		
	(ii)	$y=-2x+12$		
		$(-2x+12)^2=4x$		

		$144-48x+4x^2=4x$		
		$4x^2-52x+144=0$		
		$x^2-13x+36=0$		
		$(x-9)(x-4)=0$		
		$x=4$ or 9		
		$y=4$ or -6		
		Points of intersection are $(4, 4)$ and $(9, -6)$.		
	(iii)	$y=\dfrac{x}{k}+k$		
		$\left(\dfrac{x}{k}+k\right)^2=4x$		
		$\dfrac{x^2}{k^2}+2x+k^2-4x=0$		
		$\dfrac{x^2}{k^2}-2x+k^2=0$		
		Discriminant = $(-2)^2-4\left(\dfrac{1}{k^2}\right)(k^2)$		
		As the discriminant = 0, the line is a tangent for all values of k.		
6	(i)	$L=AB+BC+CD+AD$		
		$L=3+2+3\cos\theta+3\sin\theta+2$		
		$L=7+3\cos\theta+3\sin\theta$		
	(ii)	$3\cos\theta+3\sin\theta=R\cos(\theta-\alpha)$		
		$3\cos\theta+3\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$		
		$3=R\cos\alpha$ and $3=R\sin\alpha$		
		$R^2=3^2+3^2$		
		$R=\sqrt{18}=3\sqrt{2}$		
		$\tan\alpha=\dfrac{3}{3}=1$		
		$\alpha=45°$		
		$\therefore L=7+3\sqrt{2}\cos(\theta-45°)$		
	(iii)	$7+3\sqrt{2}\cos(\theta-45°)=11$		

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		$\cos(\theta-45^\circ)=\frac{4}{3\sqrt{2}}$		
		$\theta-45^\circ=19.47^\circ$		
		$\theta=64.5^\circ$ (1 d.p.)		
7	(i)	Let $\frac{5x^2+4x-3}{x^2(2x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x-1}$		
		$\frac{5x^2+4x-3}{x^2(2x-1)}=\frac{Ax(2x-1)+B(2x-1)+Cx^2}{x^2(2x-1)}$		
		$5x^2+4x-3=Ax(2x-1)+B(2x-1)+Cx^2$		
		When $x=0$: $-3=B(-1)$ $\therefore B=3$		
		When $x=\frac{1}{2}$: $5\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)-3=C\left(\frac{1}{2}\right)^2$ $\frac{5}{4}+2-3=\frac{1}{4}C$ $\frac{1}{4}=\frac{1}{4}C$ $\therefore C=1$		
		When $x=1$: $5+4-3=A(1)+3(1)+1$ $\therefore A=2$		
		<u>**OR**</u> By comparing coefficients:		
		$5x^2+4x-3=2Ax^2-Ax+2Bx-B+Cx^2$		
		$5x^2+4x-3=(2A+C)x^2+(-A+2B)x-B$		
		$\therefore B=3$		
		$-A+2B=4$ $-A+2(3)=4$ $\therefore A=2$		
		$2A+C=5$ $2(2)+C=5$ $\therefore C=1$		

		$\frac{5x^2+4x-3}{x^2(2x-1)}=\frac{2}{x}+\frac{3}{x^2}+\frac{1}{2x-1}$		
	(ii)	$\int_1^5 \frac{5x^2+4x-3}{x^2(2x-1)}\,dx$		
		$=\int_1^5 \frac{2}{x}+\frac{3}{x^2}+\frac{1}{2x-1}\,dx$		
		$=\left[2\ln x+\frac{3}{(-1)x}+\frac{\ln(2x-1)}{2}\right]_1^5$		
		$=\left[2\ln 5+\frac{3}{(-1)(5)}+\frac{\ln(10-1)}{2}\right]-\left[2\ln 1+\frac{3}{(-1)(1)}+\frac{\ln 1}{2}\right]$		
		$=2\ln 5-\frac{3}{5}+\frac{1}{2}\ln 9+3$		
		$=\frac{12}{5}+\ln 25+\ln 3$		
		$=\frac{12}{5}+\ln 75$		
8	(i)	Gradient of $AB=\frac{8-2}{2-4}=-3$		
		Gradient of $BC=\frac{1}{3}$		
		BC: $y-8=\frac{1}{3}(x-2)$		
		$y=\frac{1}{3}x+\frac{22}{3}$		
	(ii)	$\frac{1}{3}x+\frac{22}{3}=x-2$		
		$x=14$		
		$y=12$		
		$\therefore C(14,\ 12)$		
	(iii)	$M(9,\ 7)$		
		Let the coordinates of D be $(x,\ y)$		
		$\left(\frac{2+x}{2},\ \frac{8+y}{2}\right)=(9,7)$		

		$D(16,\ 6)$		
	(iv)	$\text{Area} = \frac{1}{2}\begin{vmatrix} 4 & 16 & 14 & 2 & 4 \\ 2 & 6 & 12 & 8 & 2 \end{vmatrix}$		
		$= \frac{1}{2}\{[(4)(6)+(16)(12)+(14)(8)+(2)(2)]$ $-[(16)(2)+(14)(6)+(2)(12)+(4)(8)]\}$		
		$= 80 \text{ units}^2$		
9	(a) (i)	$\tan 15°$		
		$= \tan(45°-30°)$ or $= \tan(60°-45°)$		
		$= \frac{\tan 45° - \tan 30°}{1+\tan 45° \tan 30°}$		
		$= \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$ or $\frac{\sqrt{3}-1}{1+\sqrt{3}}$		
		$= \frac{(\sqrt{3}-1)(1-\sqrt{3})}{1-3}$		
		$= \frac{\sqrt{3}-1-3+\sqrt{3}}{-2}$		
		$= \frac{2\sqrt{3}-4}{-2}$		
		$= \frac{\sqrt{3}-2}{-1}$		
		$= 2-\sqrt{3}$		
	(ii)	$\sec^2 15° = 1+\tan^2 15°$		
		$= 1+(2-\sqrt{3})^2$		
		$= 1+4-4\sqrt{3}+3$		
		$= 8-4\sqrt{3}$		
		$a = 8,\ b = 4$		
	(b) (i)	$\text{LHS} = \sin A(\cot A + \tan A)$		
		$= \sin A\left(\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)$		

		$=\sin A\left(\dfrac{\cos^2 A+\sin^2 A}{\sin A\cos A}\right)$		
		$=\sin A\left(\dfrac{1}{\sin A\cos A}\right)$		
		$=\dfrac{1}{\cos A}$		
		$=\sec A=\text{RHS}$		
	(ii)	$\sin A(\cot A+\tan A)=-2$		
		$\sec A=-2$		
		$\cos A=-\dfrac{1}{2}$		
		Principal value of $A=120°$		
10	(i)	X: 1, 2, 3, 4, 5; Y: 1.386, 4.290, 5.308, 7.307, 9.307		
		Plot of 5 points accurately. Each error (more than 2 mm away) to deduct one mark.		
		Best fit line passing through all points (except $X=2$) drawn.		

X	1	2	3	4	5
Y	1.386	4.290	5.308	7.307	9.307

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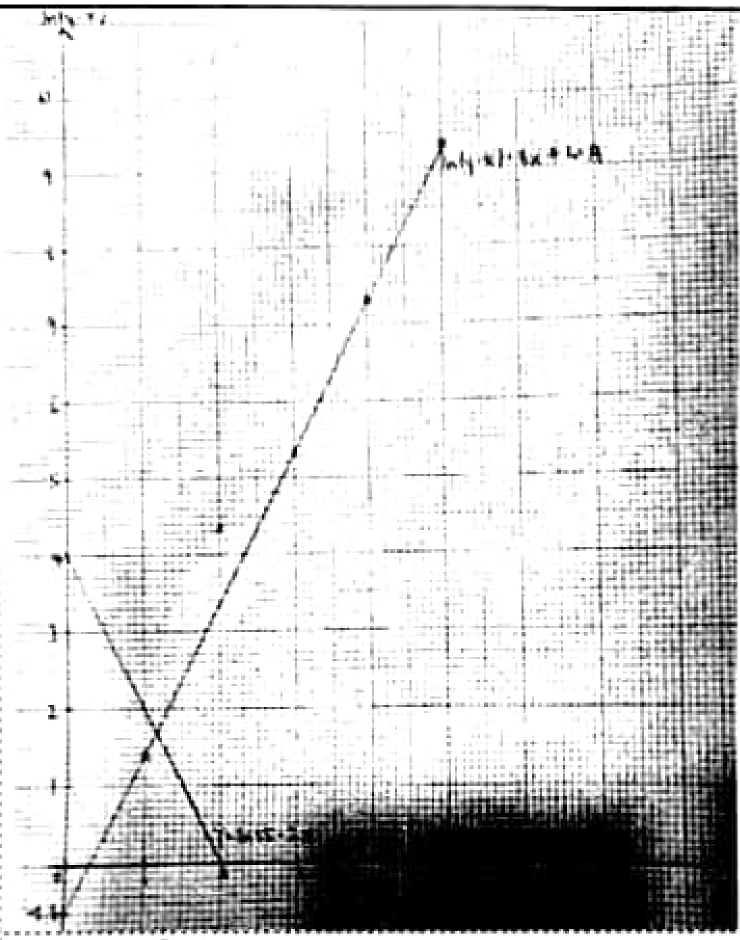

(ii)	$y = x + Ae^{Bx}$		
	$\ln(y - x) = Bx + \ln A$		
	$\ln A = -0.6$		
	$A = e^{-0.6}$		
	$A = 0.55$ (2 d.p.)		
	Gradient $= \dfrac{-0.6 - 9.3}{0 - 5}$		
	$B = 1.98$ (2 d.p.)		
(iii)	Incorrect $x = 2$		
	Corrected $Y = 3.35$		
	$\ln(y - x) = 3.35$		
	$\ln(y - 2) = 3.35$		
	$y - 2 = e^{3.35}$		
	$y = 30.5$ (3 s.f.)		
(iv)	$Ae^{Bx} = 55e^{-2x}$		

Draw $Y = \ln 55 - 2x$

$x = 1.15$

11 (i)

$v = (0.5t-3)^4 - 16 = 0$

$(0.5t-3)^4 = 16$

$0.5t - 3 = \pm 2$

$0.5t - 3 = 2$ and $0.5t - 3 = -2$

$t = 10$; $t = 2$

(ii)

$a = 4(0.5t-3)^3(0.5)$

$a = 2(0.5t-3)^3$

(iii)

When $a = 0$, $2(0.5t-3)^3 = 0$

$0.5t = 3$

The <u>only turning point</u> of v is when $t = 6$.

t	5.9	6	6.1
$\frac{dv}{dt}$	-	0	+
tangent	\	—	/

Because at $t = 6$, v is a minimum value, we can conclude that v is decreasing when $t < 6$.

As $t > 0$, therefore v is decreasing only when $0 < t < 6$.

(iv)

$$s = \frac{(0.5t-3)^5}{(5)(0.5)} - 16t + c$$

$$s = \frac{2}{5}(0.5t-3)^5 - 16t + c$$

When $t = 0$, $s = 0$

$$\frac{2}{5}(-3)^5 + c = 0$$

$$c = \frac{486}{5}$$

$$s = \frac{2}{5}(0.5t-3)^5 - 16t + \frac{486}{5}$$

(v)

When $t = 0$, $s = 0$

When $t = 2$, $s = \frac{262}{5}$

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		When $t=5$, $s=\frac{275}{16}$		
		Distance travelled $=\frac{262}{5}+\left(\frac{262}{5}-\frac{275}{16}\right)=\frac{7009}{80}$		

$2^{2x-3} = 3^{\cancel{x}\,1-x}$

$12^x = (2\times 6)^x = (3$

$2^{2x}.2^{-3} = 3^1.3^{-x}$

$t -$

$\frac{\tau}{1+51-}$