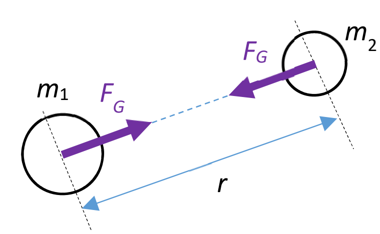
Gravitational Field

Newton's Law of Gravitation – mutual force of attraction, F_G, between two point masses is proportional to the product of their masses (m_1 & m_2) and inversely proportional to the square of their separation, r.

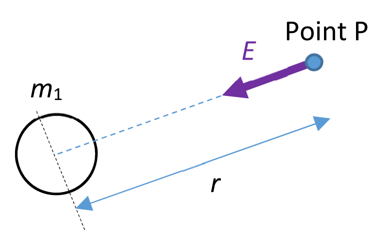
Gravitational Potential Energy – work done, U, by an external agent on a mass in moving it from infinity to that point.



$$F_G = \frac{Gm_1m_2}{r^2} \quad \xrightarrow{g = \frac{F_G}{m_2}} \quad g = \frac{Gm_1}{r^2}, \qquad F_G = m_2 g$$

$$F_G = -\frac{dU}{dr} \qquad U = -\int_\infty^r F_G\,dr \qquad g = -\frac{d\phi}{dr} \qquad \phi = -\int_\infty^r g\,dr$$

$$U = -\frac{Gm_1m_2}{r} \quad \xrightarrow{\phi = \frac{U}{m}} \quad \phi = -\frac{Gm_1}{r}, \qquad U = m\phi$$



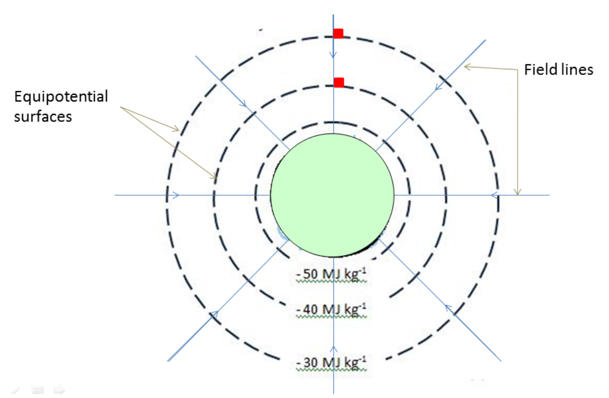
Note: There is NO mass at P.

Gravitational Field Strength, g – at a point in a gravitational field is defined as the gravitational force per unit mass acting on a small mass placed at that point.

Gravitational Potential, ϕ – at a point in a gravitational field is defined as the work done per unit mass by an external agent in bringing a point mass from infinity to that point.

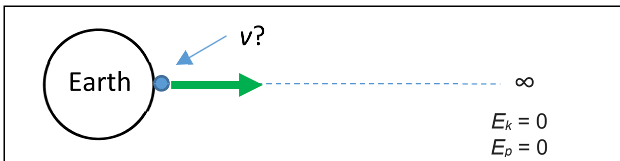
Field Lines
- Tangent of the lines points in the direction of the force on a test mass
- Point in the direction of decreasing gravitational potentials
- Closer lines indicate larger g
- No two lines intersect one another
- Perpendicular to Equipotential Lines

Equipotential Lines
- All points along a line has the same gravitational potential
- No work done when moving a mass along a equipotential line
- Perpendicular to Field Lines



Escape Speed: (Motion of an object that eventually escapes the gravitational pull – contrast with Orbital motion)

The minimum speed for object at the surface of a planet to reach infinity.

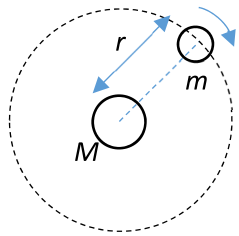
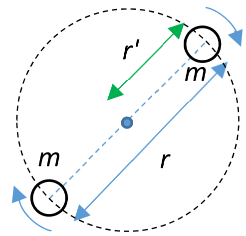


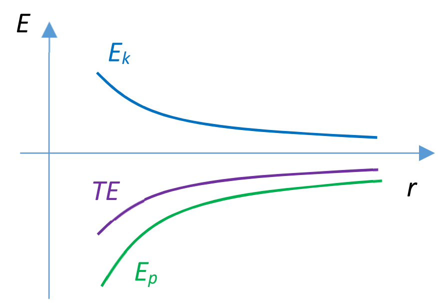
By conservation of energy, total energy = 0,

$$TE = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = 0$$

$$v = \sqrt{\frac{2GM}{r}}$$

Satellites & Planets in Orbit – Circular Motion (Motion of an object ‘bound’ by gravity)

<u>Equation</u>	<u>Examples</u>	
Gravitational Force provides for centripetal force on Satellites, $F_G = F_C$	Satellite / Moon orbiting Earth  $\frac{GMm}{r^2} = \frac{mv^2}{r}$ or $\frac{GMm}{r^2} = mr\omega^2$	Binary Stars  $\frac{Gmm}{r^2} = \frac{mv^2}{r'}$ or $\frac{Gmm}{r^2} = mr'\omega^2$

Period / Frequency of Satellite	Use: $\frac{GMm}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$ $T^2 = \frac{4\pi^2}{GM}r^3$	• Kepler’s Third Law, $T^2 \propto r^3$
Energies of Satellite • KE • GPE • TE	Use: $\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\frac{1}{2}mv^2 = E_k = \frac{GMm}{2r}$ $E_p = -\frac{GMm}{r}$ $TE = E_k + E_p = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2r}$	

Geostationary Satellite – in a geostationary orbit in which the orbiting object remains stationary relative to an observer on the Earth.

Conditions for Geostationary Orbit:

- Centre of the Earth coincides with the centre of orbit of the object/satellite, and, axis of rotation of the Earth coincides with the axis of orbit of the object/satellite. [OR The orbit of the object/satellite lies in the equatorial plane.]
- The object/satellite orbits from West to East.
- The period of one orbit of the object/satellite is 24 hours.