West Spring Secondary School
PRELIMINARY EXAMINATION 2022

Additional Mathematics **4049 / 01**
Paper 1

Secondary 4 Express / 5 Normal (Academic)

Name ______________________________ (   ) **Date** 12 SEP 2022

Class _______________ **Duration** 2 h 15 min

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.

Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINER'S USE
/ 90

This document consists of **23** printed pages and 1 blank page.

Setter(s) Mr Soh Hong Wei

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}bc\sin A$$

1 The line $y-2x=3$ intersects the curve $\frac{y}{x}+y=10$ at points A and B.

Find the coordinates of A and of B. [4]

2 It is known that $\sin\theta = k$, where k is a positive constant, and $\cos\theta$ is negative.

(a) State the quadrant in which θ lies. [1]

(b) Express, in terms of k,

(i) $\text{cosec}\,\theta$, [1]

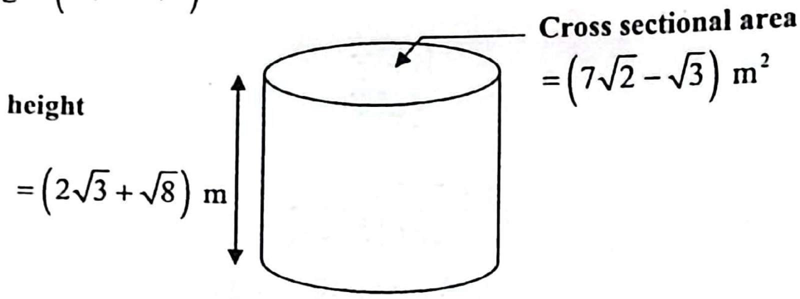
(ii) $\tan\theta$. [3]

3 The equation of a curve is given by $y = e^{x} + e^{-x}$.

Show that y is a decreasing function for $x < 0$. [4]

4 (a) Find the value of integers a and b such that $a - 3\sqrt{5} = \dfrac{b + \sqrt{5}}{2 + \sqrt{5}}$. [4]

(b) The diagram shows a cylinder with cross-sectional area of $\left(7\sqrt{2}-\sqrt{3}\right)$ m^2 and height $\left(2\sqrt{3}+\sqrt{8}\right)$ m.



Find the volume of the cylinder in the form $\left(p+q\sqrt{6}\right)$ m^3, where p and q are integers.

[3]

5 (a) (i) Write down the general term in the binomial expansion of

$\left(x^2 + \frac{1}{x}\right)^{10}$ and state the power of x in this general term. [2]

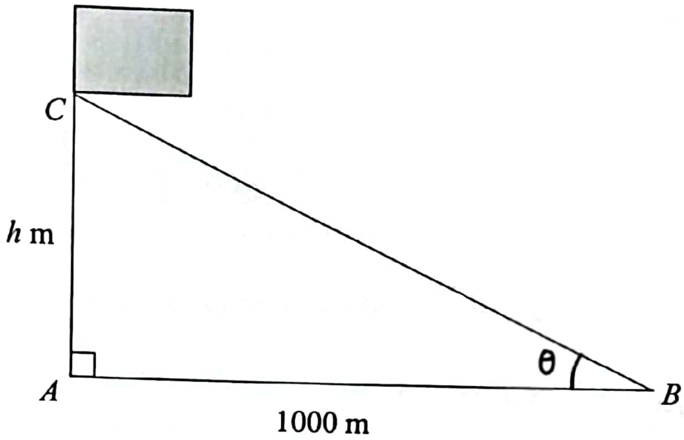
(ii) Explain why there is no term that is independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^{10}$. [2]

(b) Given that the coefficient of x^2 in the expansion of $\left(x^2+\frac{1}{x}\right)^{10}+(a+x)^5$ is -60, find the value of a. [3]

6 (a) Solve the equation $\log_3(x+2)+\log_3(x-2)=\log_3(2x-1)$. [3]

(b) Solve the equation $\log_x 2^2 = (\log_2 x)^2$, giving your answer correct to 3 significant figures. [4]

7 A box is raised vertically from a point A on level ground to a point C. The angle of elevation θ, from an observer at point B, 1000 m horizontally away from A, is increasing at a rate of 0.003 radians per second.



Find the rate of change of the height, h, of the box when θ is $\frac{\pi}{6}$ radians. [5]

8 (a) Express $\frac{4x^2+5}{2x^2-x-1}$ in partial fractions. [5]

(b) Hence, find $\int \frac{4x^2+5}{2x^2-x-1}\,dx$, for $x>1$. [3]

9 The value, $\$V$, of one cryptocurrency, is related to t, the number of years since it was mined in 2012. Investors estimate that the variables V and t are related by the formula

$$V = ae^{kt}, \text{ where } a \text{ and } k \text{ are constants.}$$

The table below gives the value of one cryptocurrency in 2014, 2016, 2018 and 2020.

Year	2014	2016	2018	2020
t (years)	2	4	6	8
V ($)	5000	6800	9200	12600

(a) Plot $\ln V$ against t and draw a suitable straight line graph. [4]

(b) Use your graph to estimate the values of a and k. [3]

(c) Assuming that the model is valid until 2022, estimate the value of one cryptocurrency in 2022, correct to the nearest dollar. [2]

10 The coordinates of three points are $A(1, 9)$, $B(7, -3)$ and $C(4, -6)$.
The perpendicular bisector of AB cuts the x-axis at D.

(a) Find the equation of the perpendicular bisector of AB. [4]

(b) Find the length of *CD*. [2]

(c) Find the area of the quadrilateral *ABCD*. [2]

11 **(a)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x}{(3x+1)^{\frac{1}{2}}}\right) = \dfrac{3x+2}{2(3x+1)^{\frac{3}{2}}}$. [4]

[4]

(b) Hence, evaluate the integral $\int_0^5 \frac{x}{(3x+1)^{\frac{3}{2}}}\,dx$.

12 (a) Using $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$, prove the identity

$$\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 1 + \frac{\sin 2x}{2}.$$

[4]

[4]

(b) Solve the equation $\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = \frac{2}{3}$, for $0° \leq x \leq 180°$.

13 The equation of a curve is $y = x(x-3)^3$.

(a) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(b) By considering the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$, determine the nature of each stationary point. [3]

(c) Ashley says that "If the stationary point on the curve has $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$, the stationary point is a point of inflexion." Do you agree with this statement? Provide an example to justify your answer. [2]

End of Paper 1