CANDIDATE NAME: ……………………………………………………………

CLASS: ………………………   INDEX NUMBER: ……........

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **November 2024**

Secondary 4 Express **2 hours 15 minutes**
Setter: itzpipey :)))

Candidates answer on the question paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

90

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve for a and b if $\frac{\sqrt{24} - a\sqrt{6}}{\sqrt{8} - \sqrt{3}} = \frac{\sqrt{2} + b\sqrt{3}}{2}$. [4]

$$\frac{\sqrt{24} - a\sqrt{6}}{\sqrt{8} - \sqrt{3}} = \frac{\sqrt{2} + b\sqrt{3}}{2}$$

$$(2\sqrt{6} - a\sqrt{6})(2) = (\sqrt{2} + b\sqrt{3})(2\sqrt{2} - \sqrt{3})$$

$$(4-2a)\sqrt{6} = 2(2) + 2b\sqrt{6} - \sqrt{6} - b(3)$$ M1 for expanding

$$(4-2a)\sqrt{6} = (2b-1)\sqrt{6} + 4 - 3b$$

Comparing coeffs of: constant terms,

$$4 - 3b = 0$$ M1 for doing comparisons

$$3b = 4$$

$$b = \frac{4}{3}$$ A1

$\sqrt{6}$ terms,

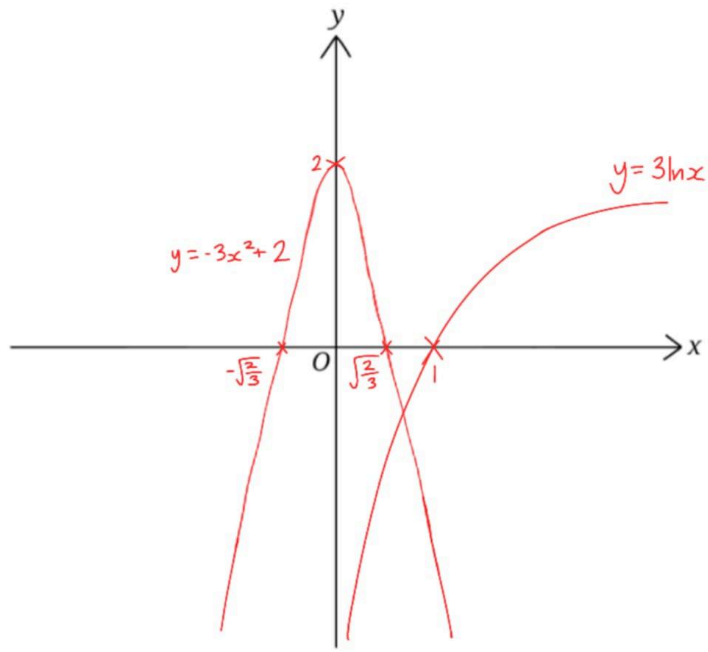
$$4 - 2a = 2b - 1$$

Sub $b = \frac{4}{3}$: $4 - 2a = 2\left(\frac{4}{3}\right) - 1$

$$2a = \frac{7}{3}$$

$$a = \frac{7}{6}$$ A1

2 The curve $y = -x^3 + 5x - 3x\ln x$ has k stationary points. By sketching suitable curves on the axes below, find the value of k. You are to label all axial intercepts in their **exact** form. Please show your workings. [5]



correct axial intercepts — M1
correct curve shapes — M1

$$\frac{dy}{dx} = -3x^2 + 5 - 3\left(1\cdot\ln x + x\cdot\frac{1}{x}\right)$$
$$= -3x^2 + 5 - 3\ln x - 3$$
$$= -3x^2 + 2 - 3\ln x \quad \text{M1 for diff}^{\underline{n}}$$

at stationary points, $-3x^2 + 2 - 3\ln x = 0$

$-3x^2 + 2 = 3\ln x$ (find intersections btwn $y = -3x^2+2$ and $y = 3\ln x$)

↳ M1 for identifying any appropriate curves to sketch

for $y = -3x^2 + 2$,
axial intercepts: $(0, 2)$
↳ $-3x^2 + 2 = 0$
$x^2 = \frac{2}{3}$
$x = \pm\sqrt{\frac{2}{3}}$
$\left(\sqrt{\frac{2}{3}}, 0\right), \left(-\sqrt{\frac{2}{3}}, 0\right)$

for $y = 3\ln x$
axial intercept: $(1, 0)$

one intersection, $\therefore k = 1$ // A1

3 **(a)** State the range of principal values for

(i) $sin^{-1}x$, [1]

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ A1

(ii) $cos^{-1}x$. [1]

$0 \le \cos^{-1}x \le \pi$ A1

(b) You are given that $\frac{sin^4 x - cos^4 x}{T(x)} = sin\ 4x$, where $T(x)$ is a function of x. Show that $T(x) = -\frac{1}{2}cosec2x$. [3]

$$T(x) = \frac{(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{2\sin 2x \cos 2x}$$ M1 for denominator

$$= -\frac{\cos^2 x - \sin^2 x}{2\sin 2x \cos 2x}$$ M1 for simplifying numerator

$$= -\frac{\cos 2x}{2\sin 2x \cos 2x}$$

$$= -\frac{1}{2\sin 2x}$$

M1 for converting and cancelling

$$= -\frac{1}{2}\operatorname{cosec} 2x \text{ (shown)}$$

4 **(a)** By representing 7999999 in the form $a^3 - b^3$, explain why 7999999 is not a prime number. [3]

$$7\,999\,999 = 8000000 - 1$$
$$= 200^3 - 1^3 \quad \text{M1 for } a^3 - b^3$$
$$= (200-1)(200^2 + 200\cdot 1 + 1^2)$$
$$= (199)(40201) \quad \text{M1 for factorising}$$

7999999 is not prime as it has more than two factors 1, 199, 40201, 7999999 // A1

(b) Represent $\frac{48x^2 + 3x - 2}{14x^2 - 7x^3}$ as a sum of partial fractions. [5]

$$\frac{48x^2+3x-2}{14x^2-7x^3} = \frac{48x^2+3x-2}{x^2(14-7x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{14-7x} \quad \text{M1 for correct decomposition}$$

$$48x^2+3x-2 = Ax(14-7x) + B(14-7x) + Cx^2$$

when $x = 0$: $0 + 0 - 2 = 0 + B(14-0) + 0$
$$14B = -2$$
$$B = -\frac{1}{7} \quad \text{M1}$$

when $x = 2$: $48(2^2) + 3(2) - 2 = 0 + 0 + C(2^2)$
$$4C = 196$$
$$C = 49 \quad \text{M1}$$

when $x = 1$: $48(1^2) + 3(1) - 2 = A(1)(14-7) - \frac{1}{7}(14-7) + 49(1^2)$
$$7A = 1$$
$$A = \frac{1}{7} \quad \text{M1}$$

$$\therefore \frac{48x^2+3x-2}{14x^2-7x^3} = \frac{1}{7x} - \frac{1}{7x^2} + \frac{49}{14-7x}$$
$$= \frac{1}{7x} - \frac{1}{7x^2} + \frac{7}{2-x} \;// \quad \text{A1}$$

*accept method of comparing coefficients too!!

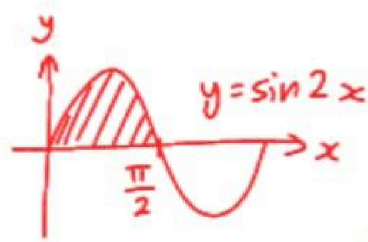
5 Deduce whether the curve $y = e^{-sin^2 x}$ is increasing or decreasing for $0 < x < \frac{\pi}{4}$. [4]

$$\frac{dy}{dx} = -2\sin x\cos x\, e^{-\sin^2 x}$$ M1

$$= -\sin 2x\, e^{-\sin^2 x}$$

for $0 \le x \le \frac{\pi}{4}$, $e^{-\sin^2 x} > 0$

$\sin 2x > 0$ M1



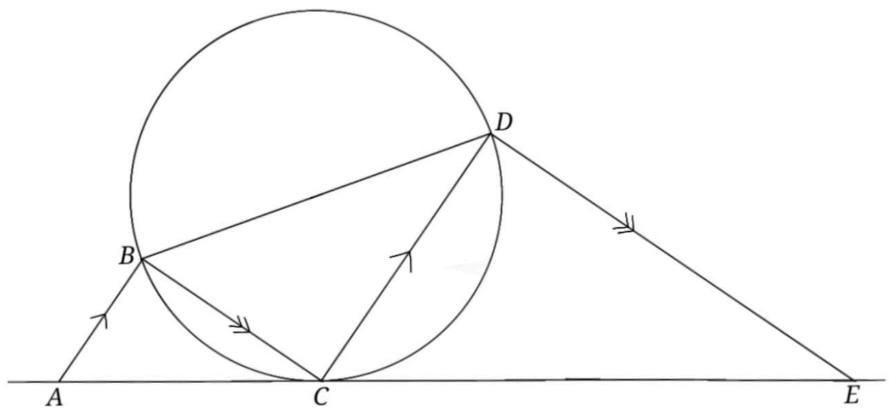
$\sin 2x e^{-\sin^2 x} > 0$

$-\sin 2x e^{-\sin^2 x} < 0$ M1

$\frac{dy}{dx} < 0$

$\therefore$ The curve is decreasing // A1

6



The diagram shows a triangle BCD inscribed in a circle, with BD as the diameter. A and E are points on a tangent of the circle at C such that AB is parallel to CD and BC is parallel to DE.

(a) Show that triangles ABC, BCD and CDE are similar. [3]

$\angle BAC = \angle DCE$ (corr. $\angle$s, $AB // CD$)
$\angle CBD = \angle DCE$ (alt. seg. thm.)
$= \angle BAC$ [A]

$\angle ACB = \angle CED$ (corr. $\angle$s, $BC // DE$)
$\angle BDC = \angle ACB$ (alt. seg. thm.)
$= \angle CED$ [A]

M1 for 1 use of corr. $\angle$s
M1 for 1 use of alt. seg. thm.
M1 for using both again/alt $\angle$s for $\angle ABC$, $\angle BCD$, $\angle CDE$

$\therefore$ by AA similarity test, $\triangle ABC$, $\triangle BCD$, $\triangle CDE$ are similar // (shown)

(b) Explain whether a circle could be drawn intersecting points A, B and C, and state which line would be the diameter. [2]

$\angle BCD = 90°$ (rt. $\angle$ in semicircle)
$\angle ABC = \angle BCD = 90°$ (alt. $\angle$s, $AB // CD$) M1

$\therefore$ Yes, a circle could be drawn intersecting A, B, and C, with AC as the diameter (by converse of rt. $\angle$ in semicircle)

A1

(c) Show that $AB \times CD + BC \times DE = BD^2$. [3]

by pyth thm., $BC^2 + CD^2 = BD^2$ —— ① M1

by similar △s, $\frac{AB}{BC} = \frac{BC}{CD}$

$BC^2 = AB \times CD$ —— ②

$\frac{BC}{CD} = \frac{CD}{DE}$ M1

$CD^2 = BC \times DE$ —— ③

Sub ② & ③ in ① : $AB \times CD + BC \times DE = BD^2$ // (shown)

↳ M1

7 The curve $y = 12x^3 + 56x^2 + 57x - 26$ intersects the line $y = 2x - 1$ at the point $(\frac{1}{3}, \frac{1}{3})$. Find the coordinates of the other intersection point(s) between the and line. [6]

at intersection points

$12x^3 + 56x^2 + 57x - 26 = 2x - 1$

$12x^3 + 56x^2 + 55x - 25 = 0$ M1

$x = \frac{1}{3}$ is a solution to this,

meaning $3x - 1$ is a factor of this M1 for finding factor

$$\begin{array}{r} 4x^2 + 20x + 25 \\ 3x-1 \overline{)12x^3 + 56x^2 + 55x - 25} \\ -(12x^3 - 4x^2) \\ \hline 60x^2 + 55x \\ -(60x^2 - 20x) \\ \hline 75x - 25 \\ -(75x - 25) \\ \hline 0 \end{array}$$

M2 for factorising

$(3x - 1)(4x^2 + 20x + 25) = 0$

$3x - 1 = 0$ or $4x^2 + 20x + 25 = 0$

$x = \frac{1}{3}$ (given)

$(2x + 5)^2 = 0$

$2x + 5 = 0$

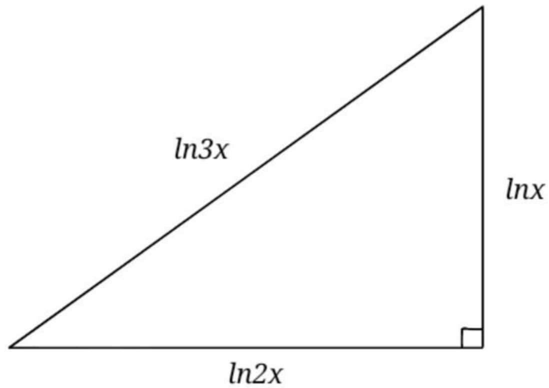
$x = -\frac{5}{2}$ M1

$y = 2(-\frac{5}{2}) - 1$

$= -6$

$\therefore (-\frac{5}{2}, -6)$ A1

8



A right triangle has side lengths of lnx, $ln2x$ and $ln3x$ $units$. Solve for x. [6]

by pyth. thm, $(\ln x)^2 + (\ln 2x)^2 = (\ln 3x)^2$

$(\ln x)^2 + (\ln 2 + \ln x)^2 = (\ln 3 + \ln x)^2$ M1 for expand

Let $u = \ln x$,

$u^2 + (\ln 2 + u)^2 = (\ln 3 + u)^2$

$u^2 + u^2 + 2u\ln 2 + (\ln 2)^2 = u^2 + 2u\ln 3 + (\ln 3)^2$

$u^2 + u(\ln 2^2) - u(\ln 3^2) + (\ln 2)^2 - (\ln 3)^2 = 0$

$u^2 + (\ln \frac{4}{9})u + (\ln 2)^2 - (\ln 3)^2 = 0$ M1 for quadratic format

$$u = \frac{-\ln\frac{4}{9} \pm \sqrt{(\ln\frac{4}{9})^2 - 4(1)[(\ln 2)^2 - (\ln 3)^2]}}{2(1)}$$ M1

$u = 1.34933$ M1 or $u = -0.538408$ (rej, $u > 0$) M1

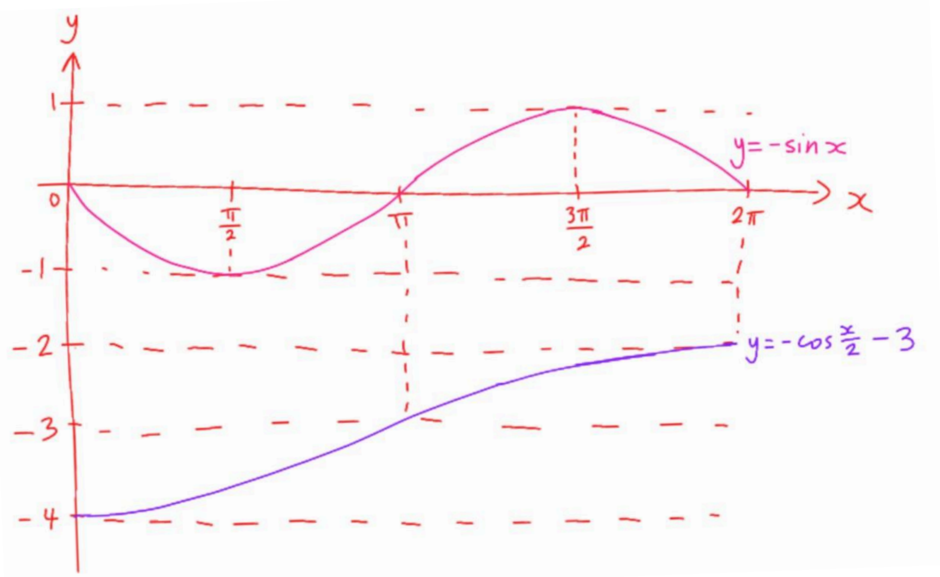
$\ln x = 1.34933$

$x = e^{1.34933}$

$= 3.8548$

$= 3.85$ // (3sf) A1

9 **(a)** Sketch the curves $y =- sinx$ and $y =- cos\frac{x}{2} - 3$ for $0 \le x \le 2\pi$. [4]



(b) Find the area of the region bound by the two curves, the y–axis and the line $x = \pi$. Express your answer in the **exact** form. [4]

$$\text{area} = -\int_0^{\pi}\left(-\cos\frac{x}{2} - 3\right)dx - \left(-\int_0^{\pi} -\sin x \, dx\right)$$ M1 for doing area − area

$$= \int_0^{\pi}\left(\cos\frac{x}{2} + 3\right)dx - \int_0^{\pi}\sin x \, dx$$

$$= \left[2\sin\frac{x}{2} + 3x\right]_0^{\pi} + \left[-\cos x\right]_{\pi}^{0}$$ M1 for each successful integration

$$= 2\sin\frac{\pi}{2} + 3\pi - 2\sin 0 - 0 - \cos 0 + \cos\pi$$

$$= 2(1) + 3\pi + 0 - 1 - 1$$

$$= 3\pi \text{ units}^2$$ A1

10 The number of views a K-pop music video gains, V, after t days can be modelled by $V = Ae^{-k(t-1)} + 50\,000$. The table below shows some information about the video.

Number of days that passed	Number of **total** views of the video (nearest whole number)
1	1050000
2	2098400
3	N

(a) Find the values of A, k and N. [5]

when $t=1$, $Ae^{-k(1-1)} + 50000 = 1050000$

$Ae^{0} = 1000\,000$

$A = 1000\,000$ // A1

when $t=2$, $V = 2098400 - 1050000 = 1048400$

$1000000e^{-k(2-1)} + 50000 = 1048400$

$e^{-k} = 0.9984$

$-k = \ln 0.9984$ M1

$k = -\ln 0.9984$

$= 0.00160128$

$= 0.00160$ // (3sf) A1

when $t=3$, $N = 2098400 + 1000000e^{\ln 0.9984(3-1)} + 50\,000$ M1

$= 3145202.56$

$= 3145203$ (nearest whole number) A1

↳ N must be whole no. as column says

(b) Suggest and explain the value that V gets closer to as time passes. [2]

as t becomes bigger, e^{-kt} gets smaller and closer to 0 M1

$V = 1000000e^{-kt} + 50000$ gets closer to $0 + 50000 = \underline{50\,000}$ // A1

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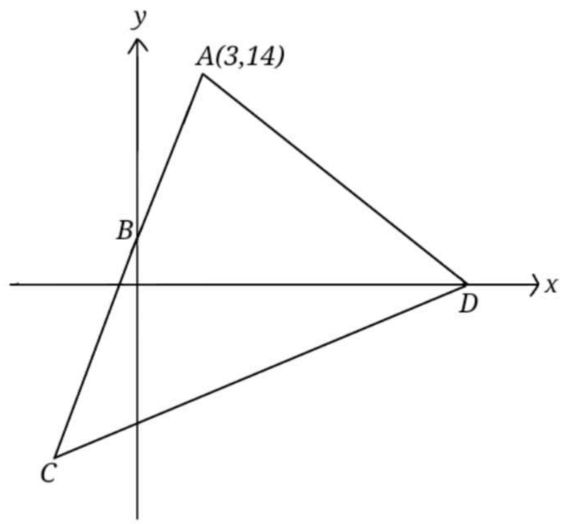

The diagram shows points $A(3, 14)$, B, C and D. B lies on the y-axis while D lies on the x-axis. D is equidistant from A and B and the area of $ABCD$ is $492.75\ units^2$.

(a) Given that AB is perpendicular to the line $4y + x = 3$, find the coordinates of B. [2]

$4y + x = 3$

$y = -\frac{1}{4}x + \frac{3}{4}$

gradient of $AB = -\frac{1}{-\frac{1}{4}}$

$= 4$ M1

Let B be $(0, y)$

$\frac{y-14}{0-3} = 4$

$y - 14 = -12$

$y = 2$

$\therefore B$ is $(0, 2)$ // A1

(b) Find the coordinates of D. [3]

Let D be $(x, 0)$

$AD = BD$

$\sqrt{(x-3)^2+(0-14)^2} = \sqrt{(x-0)^2+(0-2)^2}$ M1 for formulae

$x^2 - 6x + 9 + 196 = x^2 + 4$ M1 for expanding

$6x = 201$

$x = \frac{67}{2}$

$\therefore D$ is $(\frac{67}{2}, 0)$ // A1

(c) The equation of BC is $y = \frac{17}{4}x + 2$. Find the coordinates of C. [4]

Let C be (p, q)

$q = \frac{17}{4}p + 2$ —— ①

$\frac{1}{2}\begin{vmatrix} 0 & p & \frac{67}{2} & 3 & 0 \\ 2 & q & 0 & 14 & 2 \end{vmatrix} = 492.75$ M1 for expression

$\left(\frac{67}{2} \times 14 + 3 \times 2\right) - \left(2p + \frac{67}{2}q\right) = 985.5$

$2p + \frac{67}{2}q = -510.5$ —— ② M1

Sub ① in ②: $2p + \frac{67}{2}\left(\frac{17}{4}p + 2\right) = -510.5$

$\frac{1155}{8}p = -\frac{1155}{2}$

$p = -4$ M1

Sub $p = -4$ in ①: $q = \frac{17}{4}(-4) + 2$

$= -15$

$\therefore C$ is $(-4, -15)$ // A1

(d) Explain why AC is **not** a straight line. [1]

gradient of AB is 4, while gradient of BC is $\frac{17}{4} \neq 4$, hence AC is not a straight line // A1

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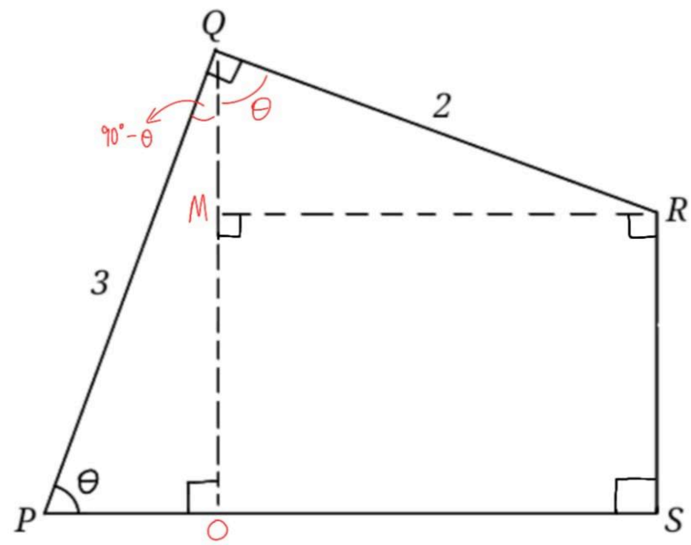

The diagram shows the cross-sectional area of a garden shed, where $PQ = 3\,metres$ and $QR = 2\,metres$. Angle $QPS = \theta\,radians$.

(a) Show that the area of $PQRS$, $A = 3 + \frac{5}{4}sin2\theta - 3cos2\theta$. [4]

$\sin\theta = \frac{QO}{3}$

$\boxed{QO = 3\sin\theta}$

$\cos\theta = \frac{PO}{3}$

$\boxed{PO = 3\cos\theta}$

$\sin\theta = \frac{MR}{2}$

$\boxed{MR = 2\sin\theta}$ $(OS = MR = 2\sin\theta)$

$\cos\theta = \frac{QM}{2}$

$\boxed{QM = 2\cos\theta}$

M1 for finding relevant trigo exp

area A = area of ΔOPQ + area of $OQRS$

$= \frac{1}{2}(3\cos\theta)(3\sin\theta) + \frac{1}{2}(3\sin\theta + 3\sin\theta - 2\cos\theta)(2\sin\theta)$ M1 for formula

$= \frac{9}{2}\sin\theta\cos\theta + 6\sin^2\theta - 2\sin\theta\cos\theta$

$= \frac{9}{4}\sin2\theta + 6\left(\frac{1-\cos2\theta}{2}\right) - \sin2\theta$

$= 3 + \frac{5}{4}\sin2\theta - 3\cos2\theta$ // (shown)

A1 for representing as $3 + \frac{5}{4}\sin2\theta - 3\cos\theta$

(b) Express $\frac{5}{4}sin2\theta - 3cos2\theta$ in the form $Rsin(2\theta - \alpha)$, where $R > 0$ and α is acute. [2]

$R = \sqrt{\left(\frac{5}{4}\right)^2 + 3^2}$

$= \frac{13}{4}$

$\alpha = \tan^{-1}\left(\frac{3}{\frac{5}{4}}\right)$

$= 1.1760$

$= 1.18$ (3sf) M1

$\therefore \frac{5}{4}\sin 2\theta - 3\cos 2\theta = \frac{13}{4}\sin(2\theta - 1.18)$ // A1

(c) Find the maximum possible area of $PQRS$ and the corresponding value of θ. [3]

$A = 3 + \frac{13}{4}\sin\left(2\theta - \tan^{-1}\left(\frac{12}{5}\right)\right)$

max area is $3 + \frac{13}{4} = \frac{25}{4}$ m^2 // A1

when A is max, $\sin\left(2\theta - \tan^{-1}\left(\frac{12}{5}\right)\right) = 1$ M1

$\boxed{0 < \theta < \frac{\pi}{2} \quad 0 < 2\theta < \pi}$

$2\theta - \tan^{-1}\left(\frac{12}{5}\right) = \frac{\pi}{2}$

$2\theta = \frac{\pi}{2} + \tan^{-1}\left(\frac{12}{5}\right)$

$\theta = \frac{\frac{\pi}{2} + \tan^{-1}\left(\frac{12}{5}\right)}{2}$

$= 1.3734$

$= 1.37$ // (3sf) A1

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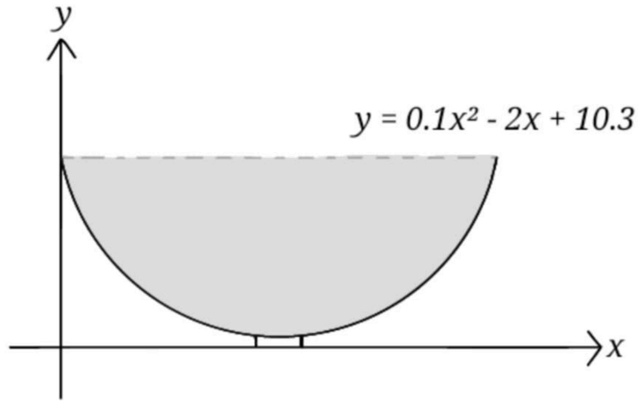

A bowl is set on a surface such that it can be modelled by the equation $y = 0.1x^2 - 2x + 10.3$, where the x-axis represents the surface and y represents the height of the bowl above the surface in centimetres.

(a) **(i)** State the highest height of the bowl above the surface. [1]

10.3cm (y-intercept) A1

(ii) Find the diameter of the bowl. [2]

$0.1x^2 - 2x + 10.3 = 10.3$

$0.1x^2 - 2x = 0$ M1

$x(0.1x - 2) = 0$

$x = 0$ or $0.1x - 2 = 0$

(found) $\underline{\underline{x = 20}}$

diameter $= 20 - 0 = 20$cm// A1

(b) Express y in the form $a(x - h)^2 + k$ and hence find the coordinates of the lowest point inside the bowl. [3]

$$y = 0.1(x^2 - 20x) + 10.3$$
$$= 0.1\left[x^2 - 20x + \left(\tfrac{20}{2}\right)^2 - \left(\tfrac{20}{2}\right)^2\right] + 10.3 \quad \text{M1}$$
$$= 0.1\left[(x-10)^2 - 100\right] + 10.3$$
$$= 0.1(x-10)^2 + 0.3 \quad \text{M1}$$

min pt. coords is $(-(-10), 0.3)$
$= (10, 0.3)$ // A1

(c) Find the cross-sectional area of the bowl (the shaded region). [4]

$$\text{area} = 20(10.3) - \int_0^{20}(0.1x^2 - 2x + 10.3)\,dx$$ M1 for area of rect. - area under curve

$$= 206 - \left[\frac{0.1x^3}{3} - x^2 + 10.3x\right]_0^{20}$$ M1 for integrating correctly

$$= 206 - \left[\frac{0.1(20^3)}{3} - 20^2 + 10.3(20) - 0 + 0 - 0\right]$$ M1 for subbing values

$$= \frac{400}{3}\text{ cm}^2$$ // A1

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