JC2 MYE Revision Package
H2 Mathematics (9758)
Vectors

1 2010Promo/RVHS/6

The position vectors of vertices *A*, *B* and *C*, relative to the origin *O*, are $-3\mathbf{i}+4\mathbf{k}$, $-2\mathbf{i}+4\mathbf{j}-3\mathbf{k}$ and $11\mathbf{i}-4\mathbf{j}-9\mathbf{k}$ respectively.

(i) Find a unit vector parallel to $\overrightarrow{OA}$. [1]

(ii) A point *P* divides *AC* in the ratio 1 : 3. Find the position vector of *P*. [2]

(iii) Show that the points *O*, *B* and *P* are collinear. [2]

(iv) *OBDC* forms a parallelogram. Find the position vector of *D*. [2]

2 2015Promo/DHS/I/2

The position vectors of the points *A* and *B* relative to the origin *O* are $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$ respectively. The point *P* lies between *A* and *B* such that $\overrightarrow{AP}=\lambda\overrightarrow{AB}$ where $0<\lambda<1$.

(i) Find the position vector of *P* in terms of λ. [1]

(ii) If $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{AB}$, find the value of λ. [2]

Given that $\lambda=\frac{1}{3}$,

(iii) Find the area of triangle *OPA*. [2]

(iv) Write down the ratio of the area of triangle *OPB* to the area of triangle *OPA*. [1]

3 2013Promo/MJC/I/4

Referred to the origin *O*, the points *A* and *B* are such that $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$, where **a** and **b** are non-zero and non-parallel vectors. The point *C* lies on *OB* such that $\overrightarrow{OC}=k\overrightarrow{OB}$, where k is a constant. *P* is on *AC* such that *AP* : *PC* = 3 : 1, and *Q* is on *AB* such that *AQ* : *AB* = 2 : 3.

(i) Find $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of **a**, **b** and k. [2]

(ii) Given that *O*, *P* and *Q* are collinear, find the value of k. [3]

4 2015Promo/NYJC/1/4

Relative to the origin *O*, the position vectors of two points *A* and *B* are $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The vector $\mathbf{a}$ is a unit vector which is perpendicular to $2\mathbf{a}+5\mathbf{b}$. The angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{2\pi}{3}$.

(i) Show that $|\mathbf{b}|=\frac{4}{5}$. [3]

(ii) The point *M* divides *AB* in the ratio $\lambda:1-\lambda$. The point *N* is such that *OMBN* is a parallelogram. By considering $\overrightarrow{ON}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, find the area of triangle *OAN* in terms of λ. [4]

5 RI Promo 9758/2017/2

Referred to the origin *O*, points *A* and *B* have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, such that $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. Point *C* lies on line *AB*, such that the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OB}$ is 5 units. Given that $|\mathbf{b}|=2$ and $\mathbf{a}\bullet\mathbf{b}=1$, find the possible position vectors of *C* in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

6 2014 Promo/SAJC/5

(a) Relative to an origin *O*, the position vectors of *A* and *B* are $\mathbf{a}$ and $\mathbf{b}$ respectively, and $\mathbf{c}$ is the position vector of the point *C* on *AB* which divides *AB* in the ratio 3:1. Given that angle *AOB* is acute, show that the length *d* of the projection of $\overrightarrow{OC}$ on $\overrightarrow{OB}$ is given by $d=\frac{3}{4}|\mathbf{b}|+\frac{\mathbf{a.b}}{4|\mathbf{b}|}$. [4]

(b) Three vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are such that $\mathbf{p}\times\mathbf{q}=\mathbf{p}\times\mathbf{r}$, $\mathbf{p}\neq\mathbf{0}$. Show that $\mathbf{q}-\mathbf{r}=k\mathbf{p}$, where $k\in\mathbb{R}$. [2]

7 2012/YJC/II/1

Two planes π_1 and π_2 have equations $x+2y+z=2$ and $2x+y-z=3$ respectively. The point *A* has coordinates (4, –1, 2).

(i) Find the acute angle between the planes π_1 and π_2. [2]

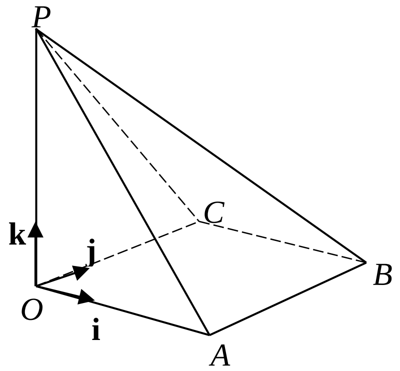
(ii) Let *B* be the foot of the perpendicular from *A* to π_1. Find the coordinates of *B*. [3]

(iii) If π_3 contains the line *AB* and is perpendicular to π_1 and π_2, find the Cartesian equation of π_3. [3]

8 2011/TPJC/I/11

The diagram shows a pyramid *POABC*. Taking unit vectors **i**, **j, k** as shown, the position vectors of *A*, *B*, *C* and *P* are given by $\overrightarrow{OA} = 3\mathbf{i}$, $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = 4\mathbf{j}$ and $\overrightarrow{OP} = 4\mathbf{k}$.

Given that point *E* lies on *PC* such that *PE* : *EC* = $\lambda : 1 - \lambda$.



(i) Write down the position vector of *E* in terms of λ. [1]

(ii) It is given that *PA* is parallel to plane *OEB*.

Show that *E* is the midpoint of *PC*. [4]

(iii) The point *D* lies on *AP* and has position vector $\frac{3}{2}\mathbf{i} + 2\mathbf{k}$. Find the coordinates of the foot of the perpendicular from *D* to plane *OEB*. [4]

(iv) Hence deduce the distance between *PA* and the plane *OEB*. [4]

9 2010/SRJC/I/9

The position vectors of the points *A*, *B*, *C* and *D* are given as **i** + 3**j**, 2**j** + 4**k**, **i** + **j** + **k** and 4**i** + 5**k** respectively.

(i) Find the vector equation of plane π in the form $\mathbf{r} \cdot \mathbf{n} = p$, that contains the points *A*, *B* and *C*. [3]

(ii) Find the foot of the perpendicular of the point *D* to the plane π. Hence find the shortest distance from the point *D* to the plane π. [4]

A line *l* parallel to the vector **j** + **k** passes through point *D* and it meets the plane π at the point *N*.

(iii) Find the position vector of the point *N* and hence find the vector equation of the reflection of line *l* about the plane π. [5]

10 2010/SAJC/I/7

The equations of two planes π_1, π_2 are given by

$$\pi_1: \quad 2x + 4y - z = 8$$

$$\pi_2: \quad x + 2z = 6$$

(i) Find the vector equation of the line of intersection *l* between the planes π_1 and π_2. [2]

(ii) Find the foot of perpendicular, F_1 from the point (6, 9, −2) to the plane π_1. [3]

Another plane π_3 contains the points F_1 and F_2 and is parallel to *l*.

(iii) Given that $\overrightarrow{OF_2} = \begin{pmatrix} \frac{26}{5} \\ 9 \\ \frac{2}{5} \end{pmatrix}$, show that the Cartesian equation of the plane π_3 is given by $15x - 8y + 40z = 22$. [3]

11 2009/AJC/I/13

The points *P* and *Q* have position vectors $\mathbf{i} - \mathbf{j}$ and $3\mathbf{i} + 13\mathbf{j} + 6\mathbf{k}$ respectively. The plane π_1 contains the point *P* and the line $\frac{x}{2} = -1 - z,\ y = 0$.

(i) Find a vector equation of the plane π_1 in scalar product form. [3]

(ii) Find the position vector of the foot of the perpendicular from *Q* to π_1. [2]

The line l_1 passes through the points *P* and *Q*.

(iii) The line l_2 is the reflection of the line l_1 about the plane π_1. Find a vector equation of l_2**.** [3]

The plane π_2 has the equation $\mathbf{r} \bullet \begin{pmatrix} a \\ 6 \\ 4 \end{pmatrix} = b$.

Find the values of *a* and *b* such that

(iv) π_1 and π_2 are parallel and at a distance of $\sqrt{224}$ apart. [3]

(v) π_1 and π_2 are intersecting. [1]

12 2009/RI/II/4

In this question, give each of your answers in exact form.

The plane Π have equation $\mathbf{r} = (2 + \lambda - 3\mu)\mathbf{i} + (3 - 2\lambda)\mathbf{j} + (\mu - 2)\mathbf{k}$ where $\lambda, \mu \in \mathbb{R}$ and the point *A* has position vector 2**j**.

(i) Express the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$ where **n** is a vector perpendicular to Π. [3]

(ii) Find the position vector of the foot of perpendicular, *B*, from *A* to Π. Deduce the perpendicular distance from *A* to Π. [5]

(iii) By using a vector product, find the length of projection of $\overrightarrow{OA}$ on Π. [2]

(iv) By using **(ii)** and **(iii)**, find the area of triangle *OAB*. [2]

Answer Key

No	Year	JC/CI	Answers
1	**2010 Promo**	**RVHS**	**(i)** $-\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{k}$ **(ii)** $\frac{1}{2}\mathbf{i}-\mathbf{j}+\frac{3}{4}\mathbf{k}$ **(iv)** $9\mathbf{i}-12\mathbf{k}$
2	**2015 Promo**	**DHS**	**(i)** $\overrightarrow{OP}=\begin{pmatrix}2\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}-4\\6\\0\end{pmatrix}$; **(ii)** $\lambda=\frac{7}{26}$; **(iii)** $\frac{\sqrt{29}}{3}$ units^2 or 1.80 units^2; **(iv)** 2:1
3	**2013 Promo**	**MJC**	**(i)** $\overrightarrow{OP}=\frac{\mathbf{a}+3k\mathbf{b}}{4}$, $\overrightarrow{OQ}=\frac{\mathbf{a}+2\mathbf{b}}{3}$ **(ii)** $\therefore k=\frac{2}{3}$
4	**2015 Promo**	**NYJC**	$\frac{(1-\lambda)\sqrt{3}}{5}$
5	**2017 Promo**	**RI**	$-2\mathbf{a}+3\mathbf{b}$ or $\frac{14}{3}\mathbf{a}-\frac{11}{3}\mathbf{b}$
7	**2012**	**YJC**	(i) $\theta=\frac{\pi}{3}$ (ii) $\left(\frac{11}{3},-\frac{5}{3},\frac{5}{3}\right)$ (iii) $x-y+z=7$
8	**2011**	**TPJC**	(i) $\begin{pmatrix}0\\4\lambda\\4-4\lambda\end{pmatrix}$ (iii) $\left(\frac{3}{34},\frac{18}{17},\frac{16}{17}\right)$ (iv) $\frac{6\sqrt{34}}{17}$
9	**2010**	**SRJC**	(i) $\mathbf{r}\cdot\begin{pmatrix}7\\1\\2\end{pmatrix}=10$ (ii) $\frac{14\sqrt{6}}{9}$ (iii) $\overrightarrow{ON}=\begin{pmatrix}4\\-\frac{28}{3}\\-\frac{13}{3}\end{pmatrix}$; $l':\mathbf{r}=\begin{pmatrix}4\\-\frac{28}{3}\\-\frac{13}{3}\end{pmatrix}+\beta\begin{pmatrix}-7\\8\\7\end{pmatrix}$
10	**2010**	**SAJC**	(i) $\mathbf{r}=\begin{pmatrix}6\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1.25\\1\end{pmatrix}$, $\lambda\in\mathbb{R}$ (ii) (2, 1, 0)

11	**2009**	**AJC**	(i) $r \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = -2$ (ii) $\overrightarrow{ON} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ (iii) $\underset{\sim}{r} = \begin{pmatrix} -5 \\ -11 \\ -10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix}, \lambda \in \mathbb{R}$ (iv) $a = 2$, $b = 108$ or -116 (v) $a \neq 2, b \in \mathbb{R}$
12	**2009**	**RI**	(i) $\mathbf{r} \bullet \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix} = -5$ (ii) $\overrightarrow{OB} = \frac{1}{41}\begin{pmatrix} -14 \\ 75 \\ -42 \end{pmatrix}$, $\frac{7\sqrt{41}}{41}$ units (iii) $4\sqrt{\frac{10}{41}}$ units (iv) $\frac{14\sqrt{10}}{41}$ units^2