Section A: Pure Mathematics [40 marks]

1. Find the exact range of values of k for which $(1-k)x^2 - \sqrt{24}x - k$ is negative for all values of x. [4]

2. **(i)** Differentiate $(1-2x)^{\frac{3}{2}}$ with respect to x. [1]

(ii) Integrate $\dfrac{1}{\sqrt{1-2x}}$ with respect to x. [2]

(iii) Express $6x-5$ in the form $a(1-2x)+b$, where a and b are constants. Hence, using the results in parts **(i)** and **(ii)**, find $\displaystyle\int \frac{6x-5}{\sqrt{1-2x}}\,\mathrm{d}x$. [4]

3. The curve C has equation $y = \dfrac{x}{4} - \mathrm{e}^{\frac{x}{2}} + 8$.

(i) Use differentiation to find the exact x-coordinates of the turning point(s) of C. [3]

(ii) Sketch C, stating the coordinates of any points of intersection with the axes and the turning point(s). [2]

(iii) By adding an appropriate graph on the same diagram in part **(ii)**, solve $\dfrac{x}{4} - \mathrm{e}^{\frac{x}{2}} + \ln(-x) = 0$. [3]

4. The curve C has equation $y = 3 + 2\mathrm{e}^{1-2x}$.

(i) Sketch C, stating clearly the equation of any asymptotes and the exact coordinates of any points of intersection with the axes. [2]

(ii) Find the exact equation of the tangent to C at $x = 2$, giving your answer in the form $y = mx + c$. [4]

(iii) Find the area of the region bounded by C, the tangent to C at $x = 2$ and the y-axis. [2]

5. Mr Dodo is the manager of Todo Fishball Company which relies solely on manual production of fishballs. Mr Dodo wants to keep track of the monthly production of fishballs in the fiscal year 2021. The rate, x kg per month, at which fishballs are produced and the time, m months, can be modelled by the equation

$$x = 30m^3 - 585m^2 + 1980m + 12000, \text{ for } 0 \le m \le 12.$$

(i) Using differentiation, determine the stationary points of the graph of the above equation. State the nature of the stationary points. [5]

(ii) Sketch the graph of x against m, stating clearly the coordinates of the turning points and axial intercepts (if any). [2]

(iii) Find the area of the region bounded by the above graph, the x-axis, and the lines $m = 0$ and $m = 12$. Give an interpretation of the value of this area. [3]

To improve the efficiency of the company, Mr Dodo implemented a compulsory upgrading programme for all the employees. Each employee will be able to produce y kg of fishball on the dth day after the completion of the programme. The model of the production by each employee is

$$y = 40 - 5e^{-0.2d}.$$

(iv) Determine the amount of fishball produced by an employee immediately after the training programme. [1]

(v) Sketch the graph of y against d and comment on its suitability. [2]

Section B: Statistics [60 marks]

6. In a darts shooting competition, each competitor throws 2 darts onto a target board. The probability of a randomly chosen competitor hitting the bull's-eye with his first throw is 0.8. The probability of hitting the bull's-eye on his second throw is

- 0.9 if the first throw hits the bull's-eye,
- the same as the probability of the first throw if the first throw does not hit the bull's-eye.

(i) Draw a tree diagram to represent the above information. [2]

(ii) Find the probability that a randomly chosen competitor hits the bull's-eye in his second throw. [1]

(iii) If a randomly chosen competitor hits the bull's-eye on his second throw, find the probability that he hits the bull's eye on his first throw. [3]

7. A 7-letter codeword is formed using the letters *A, U, X, Y, Z.* Repetition of letters are allowed. Find the number of possible codewords if

(i) there are no restrictions, [1]

(ii) letter *Z* occurs only once in the centre. [1]

The codeword is of the form *AUXY****, such that * is any of the letters *A, U, X, Y, Z.*

(iii) Show that if *** is in alphabetical order and consists of exactly two identical letters, the number of possible codewords is 20. [1]

(iv) Hence find the number of possible codewords if *** is in alphabetical order. [3]

8. In a particular region, a person is either left-handed or right-handed. On average, 8% of the people are left-handed. A school in the region has 50 classes, each with 30 students.

(i) State, in the context of the question, one assumption needed for the number of left-handed students in the school to be well modelled by a binomial distribution. [1]

Assume now that the number of left-handed students in the school follows a binomial distribution.

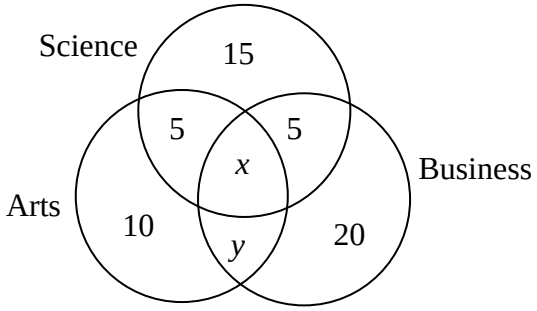
(ii) State the expected number of students who are right-handed in the school. [2]

(iii) Find the probability that there are at least 4 left-handed students in a randomly chosen class. [3]

The school has 4 lecture theatres each with a maximum capacity of 250. Following feedback from students, the school now wants to include chairs with swivel boards that are custom-made for left-handed students in the 4 lecture theatres.

(iv) Determine the minimum number of such chairs needed for a randomly chosen lecture theatre in order to be 90% certain of meeting the needs of left-handed students. [3]

9. An institution offers classes in three different subjects: Sciences, Arts and Business. The number of students studying different combinations of these subjects are shown in the Venn diagram below.



All students study at least one subject. The number of students who study all three subjects is x, and the number of students who study Arts and Business but not Science is y. One of the students is chosen at random. S, A and B are events defined as follows.

- S is the event that the student studies Science.
- A is the event that the student studies Arts.
- B is the event that the student studies Business.

A and B are **not** mutually exclusive. Write down a relationship satisfied by x and y. [1]

A and S are independent, find an expression for y in terms of x. [3]

It is further given that $\mathrm{P}(B \mid S) = \frac{3}{8}$.

(i) Find the value of x and y. [2]

(ii) Find $\mathrm{P}\left((S \cap A) \cup B'\right)$. [1]

10. The ages x (in months) and the resale prices \$$y$ of a random sample of 9 bicycles of a certain model are given in the following table.

Age (x)	11	17	24	33	37	43	48	57	63
Price (y)	108	102	88	73	74	63	65	50	43

(i) Sketch a scatter diagram of the data. [2]

(ii) Determine the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the regression line of y on x, in the form $y = mx + c$, leaving the values of m and c correct to 2 decimal places. Interpret the value of c in the context of the question. Sketch the regression line on the scatter diagram drawn in part **(i)**. [3]

(iv) Estimate the price of a 72-month old bicycle and comment on the reliability of this estimate. [3]

11. A mobile application states that the mean time taken to travel from Town A to Park B is 14.5 minutes. A data analyst would like to test if the mean travel time is more than 14.5 minutes. The data analyst gathers a random sample of 35 travel times of various people. The travel times, x minutes, are summarized below.

$$\sum x = 539 \qquad \sum x^2 = 8647.4$$

(i) Calculate the unbiased estimates of the population mean and variance of the travel time from Town A to Park B. [2]

(ii) Test, at the 5% level of significance, whether the mean travel time from Town A to Park B is more than 14.5 minutes. You should state your hypotheses clearly and define any symbols used. [4]

(iii) Explain, in the context of the question, the meaning of 'at the 5% level of significance'. [1]

It is now known that the variance of travel times from Town A to Park B is 9.6 minutes^2. A rival mobile application claims that the data collected is incorrect and hence proceeds to collect a new set of travel times from Town A to Park B from a random sample of 100 people. The mean travel time of this new sample is m minutes. A test at the 1% level of significance indicates that there is sufficient evidence that the mean travel time is less than 14.5 minutes from this new sample.

(iv) Find the range of possible values of m. [3]

12. The mass of a "Comforta" cushion has a normal distribution with mean 170g and standard deviation 12g.

(i) Find the probability that the mass of a randomly chosen "Comforta" cushion is more than 165g. [1]

(ii) Find the probability that, of three randomly chosen "Comforta" cushions, the mass of the heaviest cushion is less than 165g. [2]

Each "Comforta" cushion comes with a cushion cover. The mass of a cushion cover has a normal distribution with mean 20g and standard deviation σ.

(iii) It is given that more than 90% of "Comforta" cushions with their covers have mass at least 170g. Show that $\sigma < 9.98$. [3]

(iv) Given that $\sigma = 8$. A "Comforta" cushion costs \$0.05 per gram to produce while a cushion cover costs \$0.02 per gram to produce. A "Comforta" cushion with cover is sold at \$10 each. Find the probability that profit will be made with the sale of a "Comforta" cushion with cover. [3]

The company manufacturing "Comforta" cushions decided to launch a new product called "Serena" cushions. The mean mass of a "Serena" cushion is 250g. A sample of 50 "Serena" cushions were collected for quality check and it was found that the sample standard deviation is 17g. Find the probability that the sample mean mass of a "Serena" cushion is at most 245g. [3]

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