INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION 2
in preparation for General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME

Civics Group   INDEX NUMBER

Mathematics **9740/01**

Paper 1 **16 September 2014**

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **1** blank page.

1 Adam bought a total of 50 fruits consisting of apples, oranges and pineapples. The apples, oranges and pineapples cost \$0.80, \$0.60 and \$1.20 each respectively. The total cost of all the fruits he bought is \$40. If the cost of apples is doubled and that of oranges is halved, then the total cost of all the fruits that he bought would be \$53. Find the number of each type of fruit bought by Adam. [3]

2 It is given that x, y, z are the first three terms of a geometric progression. When the three terms are arranged in the order of z, x, y, they form three consecutive terms of an arithmetic progression.

(i) Show that $\left(\frac{z}{y}\right)^2+\left(\frac{z}{y}\right)-2=0$. [4]

(ii) Hence determine if the sum to infinity of the geometric progression exists. [2]

3 **(i)** Without the use of a calculator, solve the inequality $\frac{\left(x^2-2x+4\right)\left(x-3\right)}{\left(x+2\right)}\geq 0$. [5]

(ii) Hence solve the inequality $\frac{\left(x^2-2|x|+4\right)\left(|x|-3\right)}{\left(|x|+2\right)}\geq 0$. [2]

4 **(i)** Solve the equation $z^4=\sqrt{3}-\mathrm{i}$, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r>0$ and $-\pi<\theta\leq\pi$. [4]

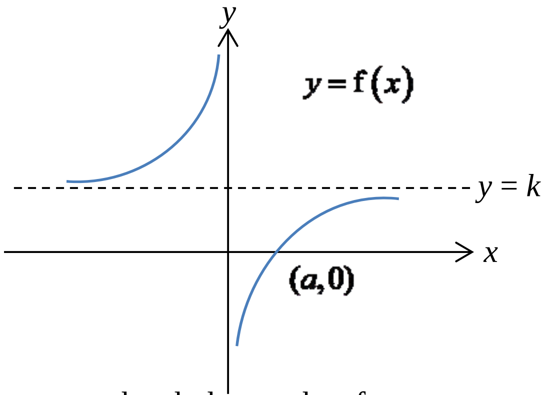
(ii) Show the roots on an Argand diagram and state the cartesian equation of a geometrical shape that the roots lie on. [3]

5 **(i)** Express $\frac{5+x^2}{(2+x)(1-x)^2}$ in the form of $\frac{A}{(2+x)}+\frac{B}{(1-x)^2}$, where A and B are constants to be found. [3]

(ii) Hence, expand $\frac{5+x^2}{(2+x)(1-x)^2}$ as a series of ascending powers of x up to and including the x^2 term. [4]

(iii) State the range of values of x for which the expansion is valid. [1]

6 **(a)** The diagram shows the graph of $y = \mathrm{f}(x)$ with asymptotes $y = k$, $x = 0$ and the graph cuts the x-axis at $(a, 0)$.



On separate diagrams, sketch the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$ and

(ii) $y = \sqrt{\mathrm{f}(-x)}$,

giving the equations of any asymptotes and the coordinates of any points where the curves cross the x- and y-axes. [4]

(b) The curve $y = \mathrm{g}(x)$ undergoes the transformations A, B and C in succession:

A. A stretch parallel to the x-axis with scale factor 2,
B. A reflection in the y-axis, and
C. A translation of 1 unit in the direction of the y-axis.

Find an expression for $\mathrm{g}(x)$ if the equation of the resulting curve is $y = 1 - \dfrac{1}{x}$.
[3]

7 A sequence u_1, u_2, u_3, … is such that $u_1 = \dfrac{3}{4}$ and $u_{n+1} = u_n + \dfrac{3}{4}\left(\dfrac{1}{2}\right)^{2n}$ for all $n \geq 1$.

(i) Write down the values of u_2, u_3 and u_4. [1]

(ii) By considering $1 - u_n$ or otherwise, write down a conjecture for u_n. Use the method of mathematical induction to prove the conjecture. [5]

(iii) Hence find $\displaystyle\sum_{r=2}^{N} \frac{3}{4}\left(\frac{1}{2}\right)^{2r}$ in terms of N. [2]

(iv) Find the smallest value of N such that $\displaystyle\sum_{r=2}^{N} \frac{3}{4}\left(\frac{1}{2}\right)^{2r}$ exceeds $\dfrac{3}{50}$. [2]

8 **(a)** Find the exact value of the constant p such that

$$\int_{5}^{p+9} \frac{1}{\sqrt{9-x}}\, dx = \int_{0}^{\frac{1}{4}} \frac{1}{\sqrt{1-4x^2}}\, dx. \qquad [4]$$

(b) Use the substitution $x = \cos^2\theta$ to find the exact value of $\int_{0}^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\, dx$. [5]

9 Relative to the origin O, the position vectors of points A and B are **a** and **b** respectively, where **a** and **b** are non-zero and non-parallel vectors. The point P on OA is such that $OP : PA = 2 : 3$. The point Q is such that $OPQB$ is a parallelogram.

(i) Find $\overrightarrow{OQ}$ in terms of **a** and **b.** [3]

(ii) Show that the area of the triangle OAQ can be written as $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be found. [2]

(iii) State the ratio of the area of triangle OPB to area of triangle OAB. [1]

(iv) Given $\mathbf{a}\times\mathbf{b}$ is a unit vector, $|\mathbf{a}| = 2$ and the angle between **a** and **b** is 60°, find the exact value of $|\mathbf{b}|$. [3]

10 **(i)** On a single Argand diagram, sketch the locus of points representing the complex number z such that

$$|z-4-2\mathrm{i}| \le 2 \qquad \text{and} \qquad |z-3| \le |z-5| . \qquad [3]$$

(ii) Find the greatest and least possible values of

(a) $|z|$, [4]

(b) $\arg(z)$. [3]

11 The curve C has equation $y = x\cos 2x$, where $0 \le x \le \pi$.

(i) Find the exact x-coordinates of the points where C crosses the x-axis. [3]

(ii) Sketch C, stating the coordinates of any points where the curve crosses the x- and y-axes. [2]

(iii) Find the exact value of $\int_{\frac{\pi}{4}}^{\pi} |x\cos 2x|\, dx$. [4]

(iv) Find the volume of revolution when the region bounded by the curve C, the x-axis and the line $x = \pi$ is rotated completely about the x-axis. [2]

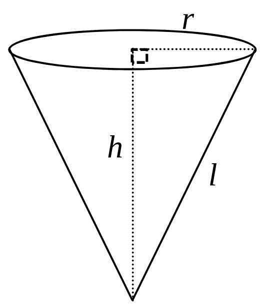
12 [It is given that a cone with radius r, height h and slant height l has curved surface area πrl.]

(a) A drinking cup is manufactured in the shape of a cone. It has a volume of $50\pi\ \text{cm}^3$. Show that

$$A^2 = \frac{22500\pi^2}{r^2} + \pi^2 r^4,$$

where A is the curved surface area of the cone.

Use differentiation to find the height h cm and radius r cm of the cup that will require the least amount of material. [8]



(b) Another drinking cup of the same shape is manufactured. At the instant when the depth of water in the drinking cup is h cm, the volume V cm^3 of the water is given by $V = \frac{\pi h^3}{12}$. The cup is filled and it is discovered that there is a leak at the vertex of the cup and the volume of water in the cone is decreasing at the constant rate of $3\,\text{cm}^3\text{s}^{-1}$. Calculate,

(i) the rate at which the depth is decreasing at the instant when the depth is 2 cm, [3]

(ii) the time taken in seconds for the depth to decrease from 6 cm to 3 cm. [2]

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