Mathematics - Important Notes and Formulas

Numbers

Type	Definition
Natural numbers	All whole numbers except 0 eg: 1, 2, 3, 4, 5...
Even numbers	0, 2, 4, 6, 8, 10...
Odd numbers	1, 3, 5, 7, 9...
Integers	whole numbers that can be positive, negative, or zero eg: -1, -2, -3, 1, 2, 3...
Prime number	a natural number which has only 2 different factors eg: 2, 3, 5, 7, 11, 13...
Composite number	a natural number that has more than 2 different factors eg: 4, 6, 8, 9...
Real number	Include rational and irrational numbers, fractions, and integers
Rational number	a number that can be expressed as a fraction or as a ratio
Irrational number	a number that cannot be expressed as a fraction or a ratio of 2 integers. eg: pi and roots

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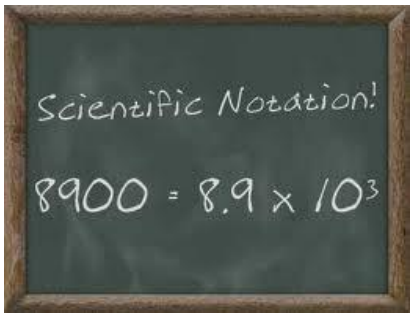
Test of Divisibility

Divisible by	Test
2	if the number is even
3	if the sum of the digits is divisible by 3
4	if the number formed by the last 2 digits is divisible by 4
5	if the last digit is 0 or 5
9	if the sum of its digits is divisible by 9
10	if the last digit is 0
11	if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is equal to 0 or is a multiple of 11

Standard form

This is a convenient way to write very large or very small numbers, using the from a x 10^n, where n is a positive or negative integer, and a s between 1 to 10 inclusive.



More examples:
123 400 written as standard form is 1.234 x 10^5
0.0000987 written as standard form is 9.87 x 10^{-5}

Multiplying numbers in standard form

$$(3.6\times10^{7})\times(9\times10^{-5})=(3.6\times9)\times(10^{7}\times10^{-5})$$
$$=32.4\times10^{2}$$

Dividing numbers in standard form

$$(5.8\times10^{9})\div(3.2\times10^{-3})=(5.8\div3.2)\times(10^{9}\div10^{-3})$$
$$=1.81\times10^{12}$$

Adding and Subtracting numbers in standard form

- Make the index between the 2 numbers the same so that it is easier to factorise the numbers before adding
eg

$$7.5\times10^{8}+8\times10^{3}=7.5\times10^{8}+0.00008\times10^{8}$$
$$=10^{8}(7.5+0.00008)$$
$$=7.50008\times10^{8}$$

Scales and Maps

Given that a map has a scale of 1:10 000, this means that 1cm on the map represents 10,000cm on the actual ground.

1cm : 200m = 1cm : 0.2km = $1cm^2$: $0.04km^2$

Proportion

A. Direct Proportion

This means that when y increases, x increases, and vice versa.

Use this equation: y = kx

B. Indirect Proportion

This means that when y increases, x decreases, and vice versa.

Use this equation: y=k/x

Percentage Change

$$\% \text{ Change} = \frac{\text{new value - original value}}{\text{original value}} \times 100\%$$

Percentage Profit and Loss

$$\% \text{ Profit} = \frac{\text{selling price - cost price}}{\text{cost price}} \times 100\%$$

$$\% \text{ loss} = \frac{\text{cost price - selling price}}{\text{cost price}} \times 100\%$$

Simple Interest and Compound Interest

A. Simple Interest Formula

$$I = \frac{PRT}{100}$$

where I is the interest,

P is the principle sum,

R is the interest rate per annum,

T is the number of years,

$$A = P + \frac{PRT}{100} = P(1 + \frac{RT}{100})$$

where A is the total amount of money earned plus interest

B. Compound Interest Formula

$$A = P(1 + \frac{R}{100})^T$$

$$I = A - P = P(1 + \frac{R}{100})^T - P$$

C. Compound interest compounded MONTHLY

Formula:
$S = P(1 + r/k)^n$

S = final value
P = principal
r = interest rate (expressed as decimal eg 4% = 0.04)
k = number of compounding periods

Note:

- if compounded monthly, number of periods = 12
- if compounded quarterly, number of periods = 4

Example:

If $4000 is invested at an annual rate of 6.0% compounded monthly, what will be the final value of the investment after 10 years?

Since the interest is compounded monthly, there are 12 periods per year, so, k = 12.
Since the investment is for 10 years, or 120 months, there are 120 investment periods, so, n = 120.

$S = P(1 + r/k)^n$

$S = 4000(1 + 0.06/12)^{120}$
$S = 4000(1.005)^{120}$
$S = 4000(1.819396734)$
$S = \$7277.59$

Coordinate Geometry Formulas

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$

The slopes of parallel lines are equal.

The slopes of perpendicular lines are opposite reciprocals of one another.

Midpoint formula: $(x_{mp}, y_{mp}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope-intercept form of the equation of a line:
$y = mx + b$

Point-slope form of the equation of a line:
$y - y_1 = m(x - x_1)$

Horizontal line: $y = b$

Vertical line: $x = a$

Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

From: http://www.dummies.com/how-to/content/coordinate-geometry-formulas.html

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Algebraic Manipulation

x = y+z	y = x-z
x = y-z	y = x+z
x = yz	y = x/z ; z = x/y
x = y/z	y = xz ; z = y/x
wx = yz	w = yz/x ; x=yz/w ; y = wx/z ; z = wx/y
$x = y^2$	y = +/-sqrt.x
x = sqrt.y	$y = x^2$
$x = y^3$	y = cuberoot.x
x = cuberoot.y	$y = x^3$

$ax + bx = x(a+b)$

$ax + bx + kay + kby = x(a+b) + ky(a+b) = (a+b)(x+ky)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a + b)(a - b)$

Solving algebraic fractional equations

Avoid these common mistakes!

$$\frac{a}{b} \neq \frac{a+c}{b+c}$$

$$\frac{x+2}{x} \neq \frac{1+2}{1} \rightarrow \text{cannot cancel out the x!}$$

$$\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d} \rightarrow \text{must change to same denominator!}$$

Solution of Quadratic Equations

The roots of $ax^2 + bx + c = 0$ are given by

$$\frac{-6 \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the Square

Step 1: Take the number or coefficient before x and square it
Step 2: Divide the square of the number by 4

Eg. to make $x^2 + 5x$ into a perfect square, add $\frac{5^2}{4}$

Since $\frac{5^2}{4} = \frac{25}{4}$, the result is $x^2 + 5x + \frac{25}{4} = (x + \frac{5}{2})^2$

Eg. $y = x^2 + 6x - 11$

$y = x^2 + 2x(6/2) + (6/2)^2 - 11 - (6/2)^2$

$y = (x + 3)^2 - 20$

Sketching Graphs of Quadratic Equations

A. eg. $y = +/-(x - h)^2 + k$

Steps:
1. Identify shape of curve

- look at sign in front of(x - h) to determine if it is "smiley face" or "sad face".

2. Find turning point

- (h, -k)

3. Find y-intercept

- sub x = 0 into the equation --> (0, y)

4. Line of symmetry reflect

- x = h, reflect to get (2x, y)

B. eg. y = +/-(x - a)(x - b)

Steps:
1. Identify shape of curve

- look at the formula ax2 + bx + c.
- if a>1, it is positive; otherwise, it is negative

2. Find turning point

- (a + b)/2, sub answer into equation --> (a,b)

3. Find y-intercept

- sub x = 0 into the equation --> (0, y)

4. Line of symmetry reflect

- x = a, reflect to get (2a, y)

Inequalities

$x < y$ means x is less than y

$x \leq y$ means x is less than or equal to y

$x > y$ means x is greater than y

$x \geq y$ means x is greater than or equal to y

Ways to solve equalities:

1. Add or subtract numbers from each side of the inequality
eg 10 - 3 < x - 3

2. Multiply or divide numbers from each side of the inequality by a constant
eg 10/3 < x/3

3. Multiply or divide by a negative number AND REVERSE THE INEQUALITY SIGNS
eg. 10 < x becomes 10/-3 > x/-3

Example

Solve $3(x + 4) < 5x + 9$

$3(x + 4) < 5x + 9$

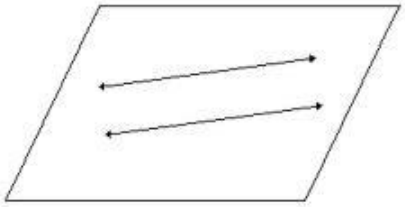
$3x + 12 < 5x + 9$

$-2x < -3$

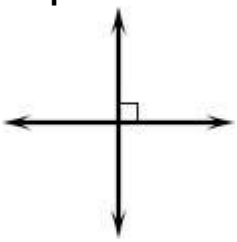
$x > \frac{3}{2}$

Geometrical terms and relationships

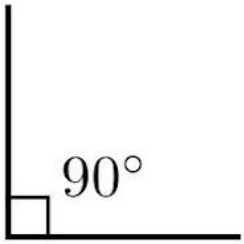
Parallel Lines



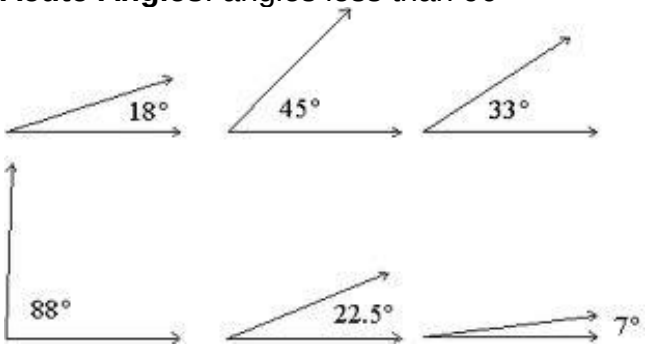
Perpendicular Lines



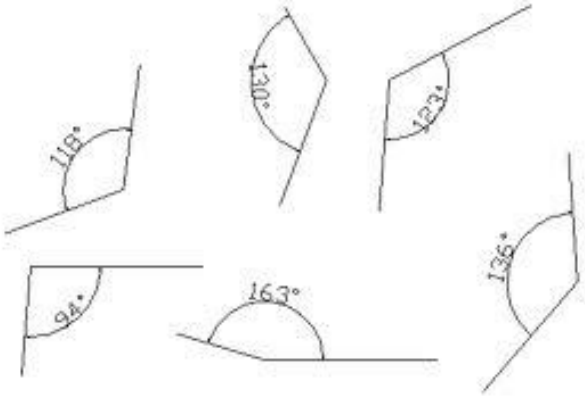
Right Angle



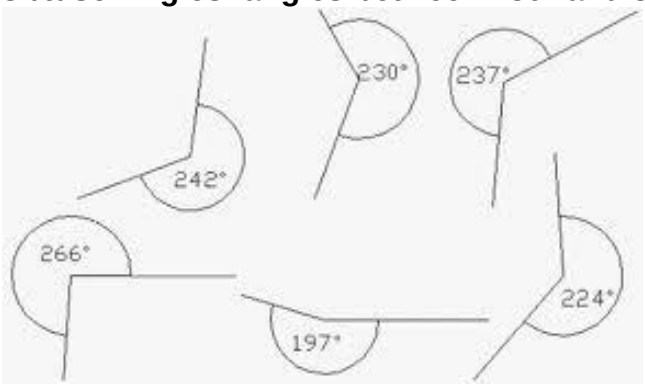
Acute Angles: angles less than 90^o



Obtuse Angles: angles between 90^o and 190^o



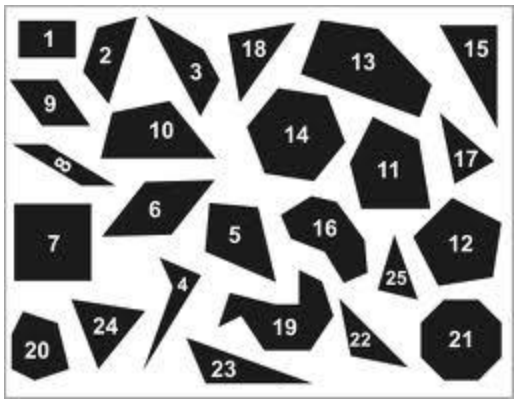
Obtuse Angles: angles between 180^o and 360^o



Polygons

Polygon: a closed figure made by joining line segments, where each line segment intersects exactly 2 others

Irregular polygon: all its sides and all its angles are not the same
Regular Polygon: all its sides and all its angles are the same



The sum of angles in a polygon with n sides, where n is 3 or more, is

$$180° \times (n-2)$$

Name of Polygons

Number of sides	Polygon
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Triangles

Triangle	Property
Equilateral	All sides of equal length All angles are equal Each angle is 60^o
Isoceles	2 sides are equal 2 corresponding angles are equal
Scalene	All sides are of unequal length
Acute	All 3 angles in the triangle are acute angles
Obtuse	1 of the 3 angles is obtuse
Right-angled	1 of the 3 angles is 90^o

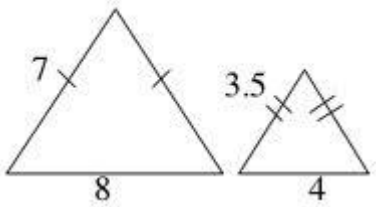
Quadrilaterals

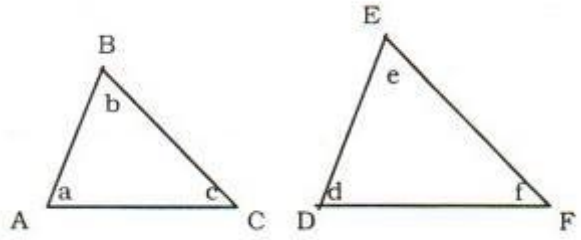
Quadrilateral	Property
Rectangle	All sides meet at 90^o
Square	All sides meet at 90^o All sides are of equal length
Parallelogram	2 pairs of parallel lines
Rhombus	All sides are of equal length 2 pairs of parallel lines
Trapezium	Exactly 1 pair of parallel sides

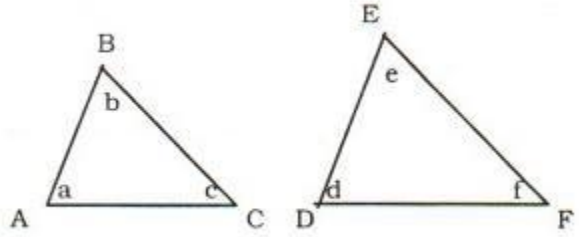
Similar Plane Figures

Figures are similar only if

- their corresponding sides are proportional
- their corresponding angles are equal







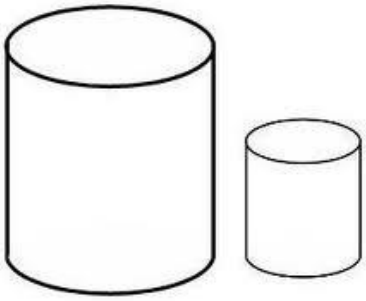
$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = k$$

$$\frac{\text{Area of ABC}}{\text{Area of DEF}} = k^2$$

k is the scale factor

Similar Solid Figures

Solids are similar if their corresponding linear dimensions are proportional.



$$\frac{r_1}{r_2} = \frac{h_1}{h_2} = k \rightarrow \text{k is the scale factor}$$

$$\frac{\text{surface area of A}}{\text{surface area of B}} = k^2$$

$$\frac{\text{volume of A}}{\text{volume of B}} = k^3$$

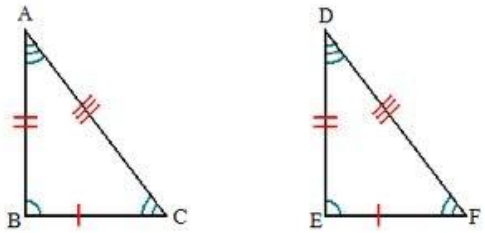
r is the radius, h is the height

Congruent Figures

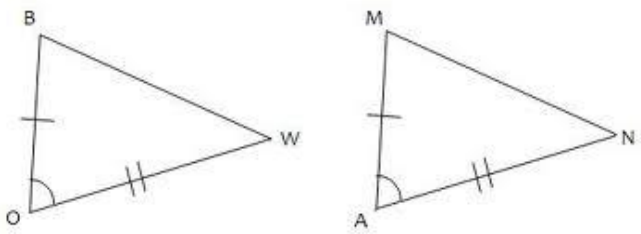
Congruent figures are exactly the same size and shape.

2 triangles are congruent if they satisfy any of the following:

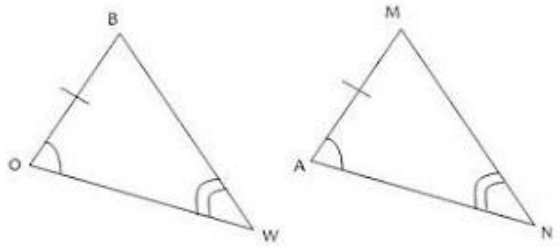
a. SSS property: All 3 sides of one triangle are equal to the corresponding sides of the other triangle.



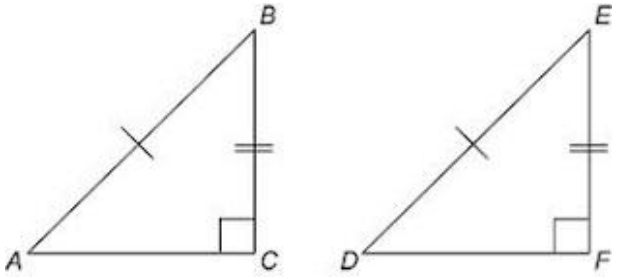
b. SAS property: 2 given sides and a given angle of one triangle are equal to the corresponding sides and angle of the other triangle.



c. AAS property: 2 given angles and a given side of one triangle are equal to the corresponding angles and side of the other triangle.

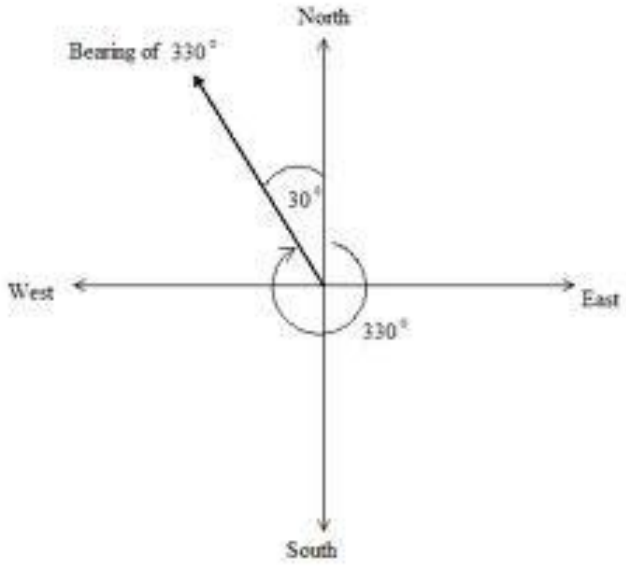


d. RHS property: The hypothenuse and a given side of a right-angled triangle are equal to the hypothenuse and the corresponding side of the other right-angled triangle.



Bearings

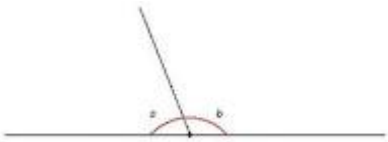
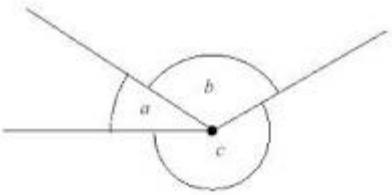
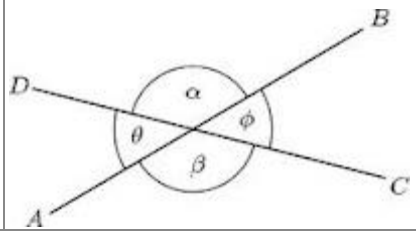
A bearing is an angle, measured clockwise from the north direction.

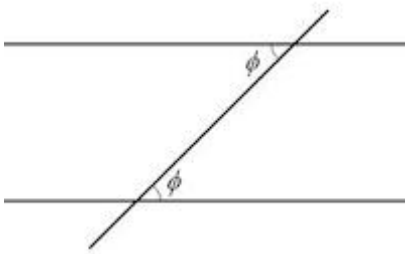
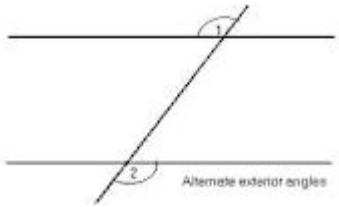
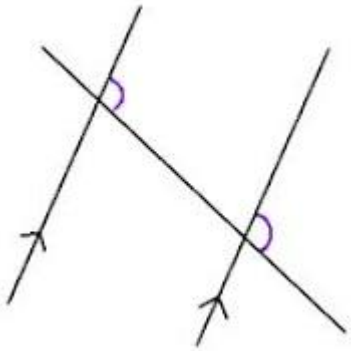
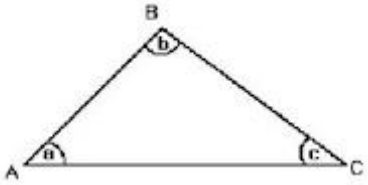
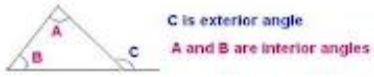


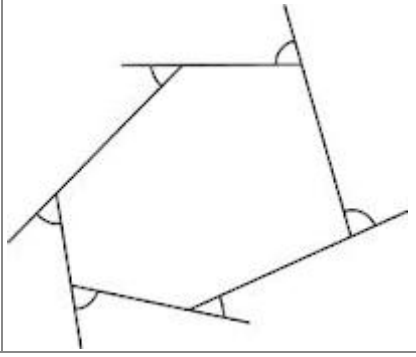
Symmetry

Shape	Number of lines of symmetry	Order of rotational symmetry	Centre of point symmetry
Equilateral triangle	3	3	Yes
Isosceles triangle	1	1	None
Square	4	4	Yes
Rectangle	2	2	Yes
Kite	1	1	None
Isosceles trapezium	1	1	None
Parallelogram	0	2	Yes
Rhombus	2	2	Yes
Regular pentagon	5	5	Yes
Regular hexagon	6	6	Yes

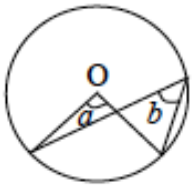
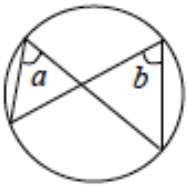
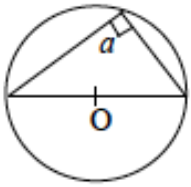
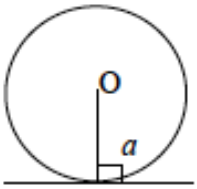
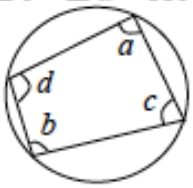
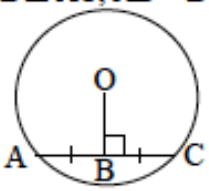
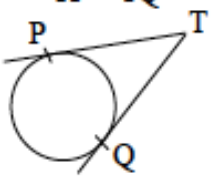
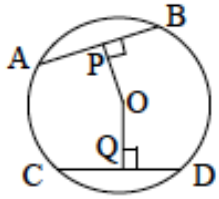
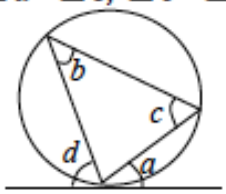
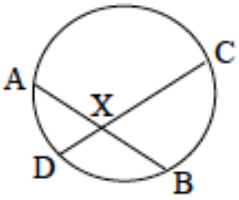
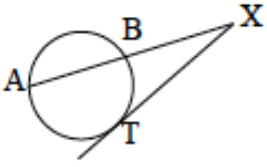
Angle properties

No.	Property	Explanation	Example
1	Angles on a straight line	• Angles on a straight line add up to 180^o • 2 angles are **complementary** is they add up to 90^o • 2 angles are called supplementary if they add up to 180^o	
2	Angles at a point	Angles at a point add up to 360^o	
3	Vertically opposite angles	Vertically opposite angles are equal	

4	Angles formed by parallel lines	Alternate interior angles are equal	
5	Angles formed by parallel lines	Alternate exterior angles are equal	
6	Angles formed by parallel lines	Corresponding angles are equal	
7	Angle properties of triangles	The sum of angles in a triangle adds up to 180^o	
8	Angle properties of triangles	The sum of 2 interior opposite angles is equal to the exterior angle	
9	Angle properties of polygons	• sum of **interior** angles of an n-sided polygon = (n-2) x 180^o • each **interior** angle of a regular n-sided polygon = (n-2) x 180o / n	

10	Angle properties of polygons	• sum of **exterior** angles of an n-sided polygon is 360^o • each **exterior** angle of a regular n-sided polygon = 360^o / n	

Angle Properties of Circles

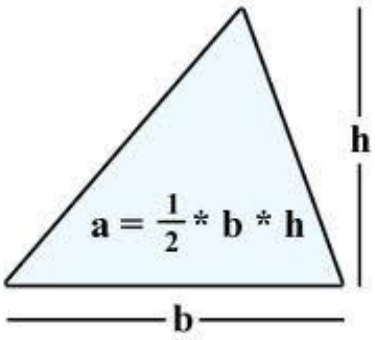
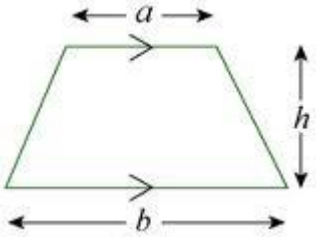
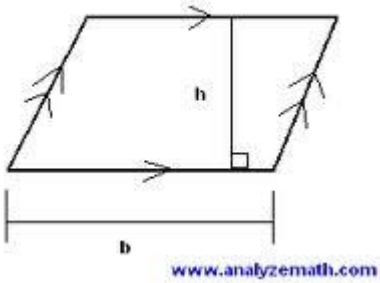
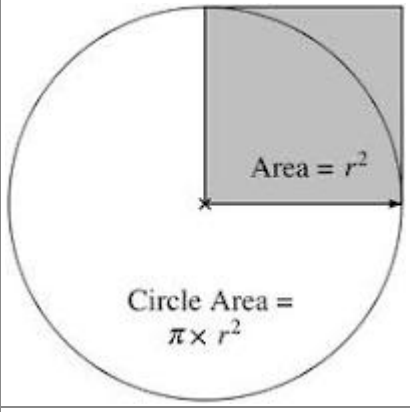
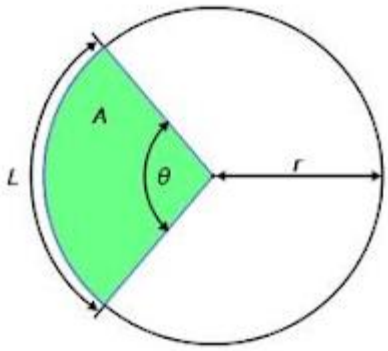
∠ at Centre $\angle a = 2\angle b$ 	**∠s in Same Segment** $\angle a = \angle b$ 	**∠ in Semi-Circle** $\angle a = 90°$ 	**Radius ⊥ Tangent** $\angle a = 90°$ 
Opp. ∠s of Cyclic Quadrilateral $\angle a + \angle b = 180°$ $\angle c + \angle d = 180°$ 	**⊥ bisector of chord passes through centre** OB ⊥ AC, AB = BC 	**Tangents from external point** TP = TQ 	**Equal chords equidistant from centre** AB = CD ↔ OP = OQ 
Alternate Segment Theorem $\angle a = \angle b,\ \angle c = \angle d$ 	**Intersecting Chords Theorem** AX·XB = CX·XD 	**Tangent-Secant Theorem** $AX \cdot BX = TX^2$ 	

Mensuration

All the mensuration formulas you'll ever need can by found here...
http://oscience.info/math-formulas/mensuration-formulas/

But here's a quick reference for the important ones...

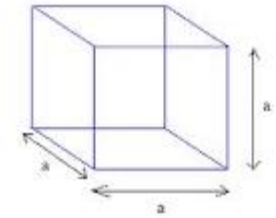
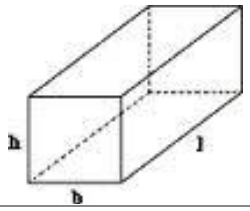
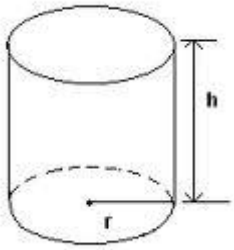
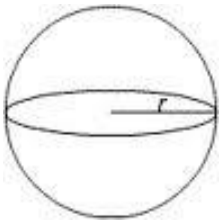
Area of Figures

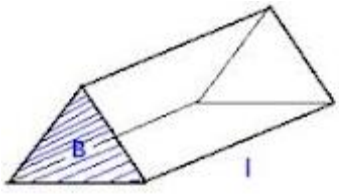
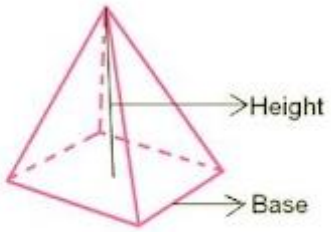
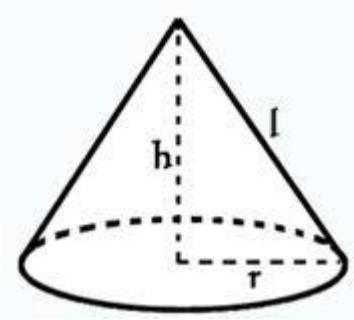
Triangle		$A = \frac{1}{2} \times b \times h$
Trapezium		$A = \frac{1}{2} \times h \times (a+b)$
Parallelogram		**A=b x h**
Circle		$A = \pi r^2$ $Circumference = 2\pi r = \pi d$
Sector		$A = \frac{\theta}{360} \times \pi r^2$ $\text{Arc length} = \frac{\theta}{360} \times 2\pi r$ $Perimeter = \frac{\theta}{360} \times 2\pi r + 2r$

Radian Measure

- Radian is another common unit to measure angles.
- A radian is a measure of the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.
- To convert radians to degrees and vice versa, use these formulas:
  - π rad = 180º
  - 1 rad = 180º/π
  - 1º = π/180 rad

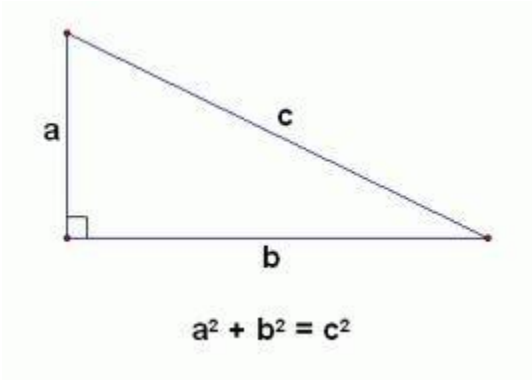
Volume of Figures

Cube		$V = a^3$ $SA = 6a^2$
Cuboid		**V = l x b x h** **SA = 2bl + 2hb + 2hl**
Cylinder		$V = \pi r^2 h$ $SA = 2\pi rh + 2\pi r^2$
Sphere		$V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$

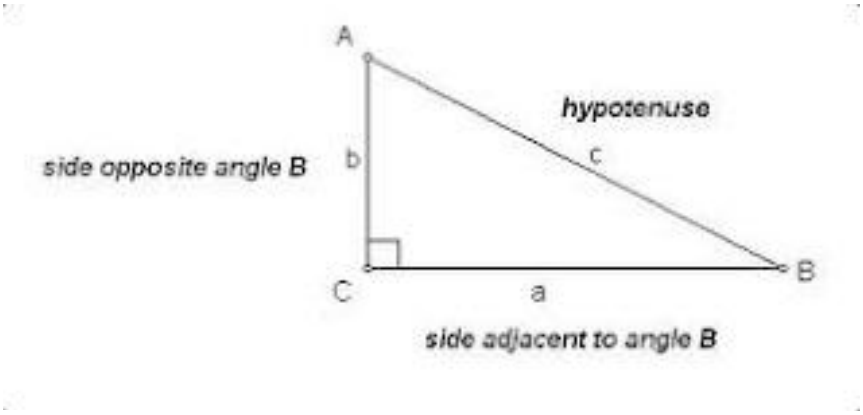
Prism	 Base - Base area l - Distance between the bases	**V = base area x height**
Pyramid	 Pyramid	$V = \frac{1}{3} \times \text{ base area } \times \text{ height}$
Cone		$V = \frac{1}{3}\pi r^2 h$ $SA = \pi rl + \pi r^2$

Trigonometry

Pythagora's theorem



Trigonometrical Ratio



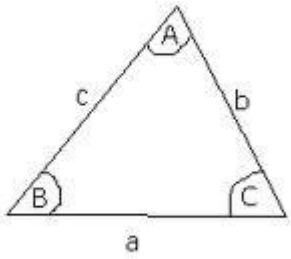
$$\sin B = \frac{b}{c}$$

$$\cos B = \frac{a}{c}$$

$$\tan B = \frac{b}{a}$$

Remember: TOA CAH SOH

SINE RULE



$$\frac{\mathbf{A}}{\mathbf{Sin\ a}} = \frac{\mathbf{B}}{\mathbf{Sin\ b}} = \frac{\mathbf{C}}{\mathbf{Sin\ c}}$$

To find an angle, can write as follows:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

COSINE RULE

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of Triangle

$$\text{Area of } \Delta ABC$$

$$= \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}bc\sin A$$

$$= \frac{1}{2}ac\sin B$$

Statistics

Mean

$$Mean = \bar{x} = \frac{x_1 + x_2 + \ldots + x_N}{N}$$

N is the number of items

Mode

The mode is the most **frequent** value.

Median

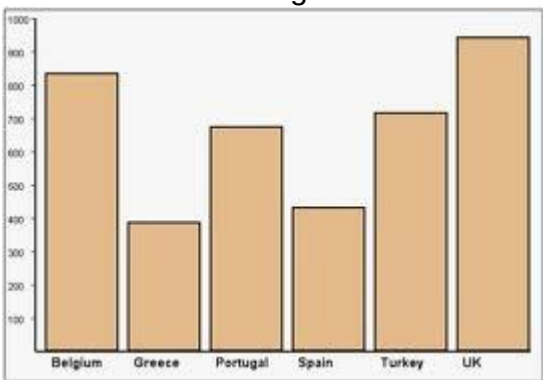
The median of a group of numbers is the number in the middle, when the numbers are **in order of magnitude** (in increasing order).

If you have n numbers in a group, the median in:

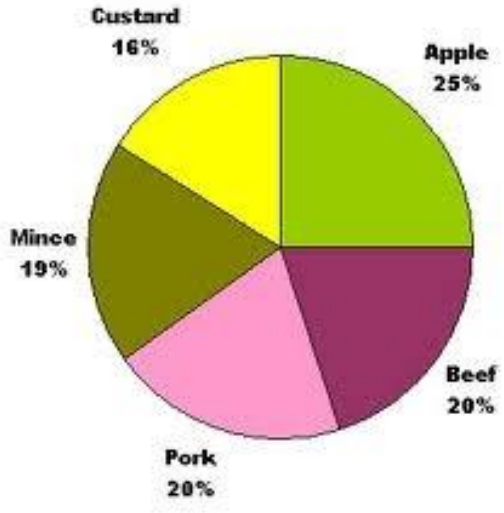
$$\frac{n+1}{2}th \text{ value}$$

Types of Chart

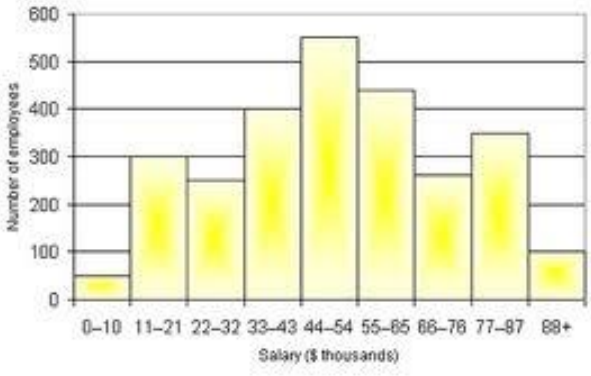
1. Bar chart: the heights of the bars represent the frequency. The data is **discrete**.



2. Pie chart: the angles formed by each part adds up to 360^{o}



3. Histogram: it is a vertical bar graph with no gaps between the bars. The area of each bar is proportional to the frequency it represents.

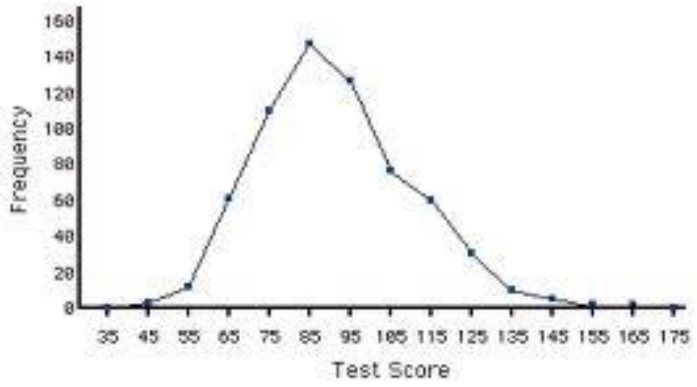


4. Stem-and-leaf diagram: a diagram that summarises while maintaining the individual data point. The stem is a column of the unique elements of data after removing the last digit. The final digits (leaves) of each column are then placed in a row next to the appropriate column and sorted in numerical order.

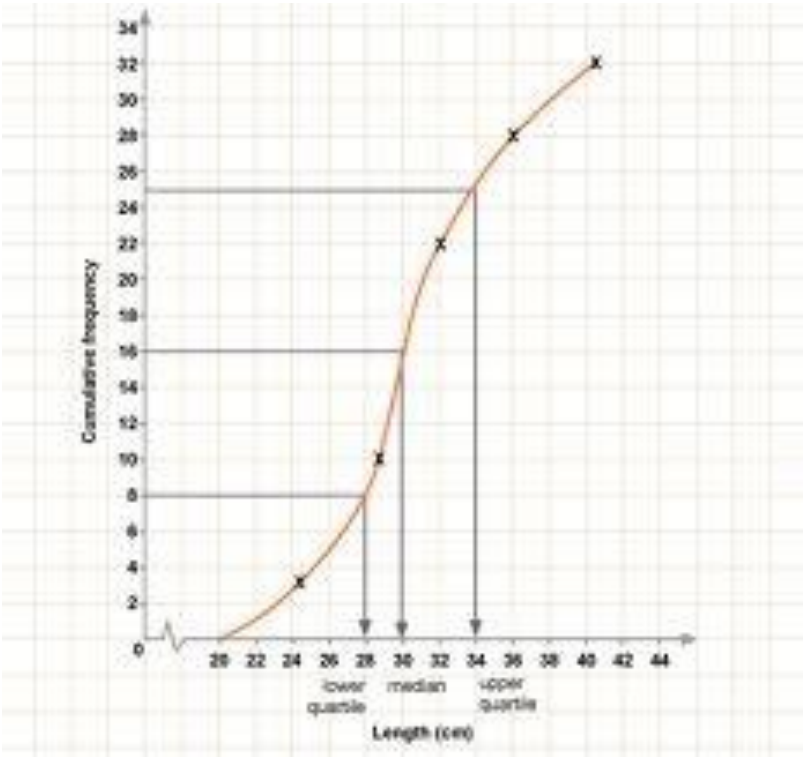
Key	Stem = 10s.
9	0 6
8	3 5 7
7	1 6 6 7 8
6	0 2 2 4 4 5 6 8 8
5	1 1 2 2 4 5 7
4	3 4 7 8
3	5 7
2	
1	6

Key	Stem = 10s.
9	0 6
8	3 5 7
7	1 6 6 7 8
6	0 2 2 4 4 5 6 8 8
5	1 1 2 2 4 5 7
4	3 4 7 8
3	5 7
2	
1	6

5. Simple frequency distribution and frequency polygons: a plot of the cumulative frequency against the upper class boundary with the points joined by line segments.



6. Quartiles



$$\text{Interquartile range} = Q_3 - Q_1$$

$$\text{Semi-interquartile range} = \frac{1}{2}(Q_3 - Q_1)$$

Probability

Probability is the likelihood of an event happening

$$\text{Probability} = \frac{\text{the number of ways of achieving success}}{\text{the total number of possible outcomes}}$$

- The probability that a certain event happening is 1
- The probability that a certain event cannot happen is 0

- The probability that a certain event not happening is 1 minus he probability that it will happen

2 events are **independent** if the outcome of one of the events does not affect the outcome of another
2 events are **dependent** if the outcome of one of the events depends on the outcome of another

- If 2 events A and B are independent of each other, then the probability of both A and B occurring is found by P(A) x P(B)
- If it is impossible for both events A and B to occur, then the probability of A or B occurring is P(A) and P(B)

Set Notation

$\in$: Element of
$\notin$: Not a element of
$n(a)$: The **number** of elements in a Set
$\subset$: A proper subset
$\subseteq$: A subset
$\varnothing$ or {} : Empty Set
ε : Universal Set
A': Complement of a Set
$\cup$: Union of 2 Sets (dont repeat element)
$\cap$: Intersection of sets