YISHUN JUNIOR COLLEGE
2015 JC2 PRELIMINARY EXAMINATION

MATHEMATICS **9740/02**

Higher 2

Paper 2

25 August 2015
TUESDAY 0800h – 1100h

Additional materials :
Answer paper
Graph paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

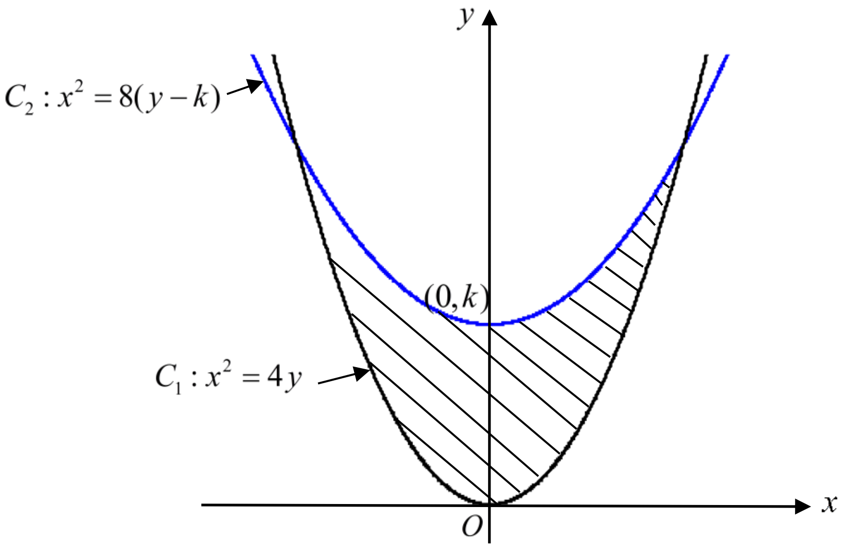
1 **(a)** Using an Argand diagram, show that any complex number $a+\mathrm{i}b$, where a and b are real numbers, may be expressed in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, where $r>0$ and $-\pi<\theta\le\pi$. State the geometrical meaning of r and θ. [3]

Write down the cartesian equation of the locus of the point representing $3(\cos\theta+\mathrm{i}\sin\theta)$ as θ varies. [1]

(b) Given that $z=\cos\alpha+\mathrm{i}\sin\alpha$, show that $z+\frac{1}{z}=2\cos\alpha$ and $z-\frac{1}{z}=2\mathrm{i}\sin\alpha$.

Hence, find $\frac{z^2+1}{z^2-1}$ in terms of α. [5]

2 In the given diagram, the curve C_1 has equation $x^2=4y$ and the curve C_2 has equation $x^2=8(y-k)$ where $k>0$. The region bounded by C_1 and C_2 is shaded.



A bowl is generated by rotating the shaded region through π radians about the y-axis.

(i) Show that the diameter of the rim of the bowl is $4\sqrt{(2k)}$. [2]

(ii) Show that the volume of the material used in making the bowl is equal to the capacity of the bowl. [5]

(iii) If k = 4, find the area of the shaded region, giving your answer correct to 3 decimal places. [2]

3 The curve C has equation $y=\frac{x^2+qx-7}{x-2}$, where q is a negative constant.

(i) Obtain the equations of the asymptotes of C. [2]

(ii) Show that $q=-2$ if $y=x$ is an asymptote to C, and sketch C in this case, stating the coordinates of any points of intersection of C with the axes. [4]

(iii) Hence, by sketching another suitable curve on the same diagram, solve the inequality $\frac{x^2-2x-7}{(x-2)\sqrt{16-x^2}}\le 1$. [5]

4 The functions g and h are defined by

$$\text{g} : x \mapsto \ln x, \quad x \in \mathbb{R},\ x > 0,$$

$$\text{h} : x \mapsto \frac{1}{x}, \quad x \in \mathbb{R},\ x > 0.$$

(i) Explain why the function g^{-1} exists. [1]

Find $\text{g}^{-1}(x)$ and write down the domain and range of g^{-1}. [2]

(ii) Explain why the composite function hg does not exist. [1]

(iii) Find the composite function gh in a similar form and write down its range. [3]

(iv) Find $\text{h}^2(x)$ and hence $\text{h}^9(x)$ in simplified form. [2]

(v) State a relationship between the graphs of

(a) g and gh, **(b)** g and g^{-1}. [2]

Section B: Statistics [60 marks]

5 A school has 600 male and 400 female students. 5% of the students are to be sampled to find out their opinions about the PE programme.

(i) Describe how the sample could be obtained using stratified sampling. [2]

(ii) State one reason why a stratified sample is preferable in this context. [1]

6 The voltage of a battery is a random variable with the distribution $\text{N}\left(\mu, \sigma^2\right)$. The mean voltage of 4 randomly selected batteries is denoted by $\overline{X}$. It is given that $\text{P}\left(\overline{X} < 22.2\right) = \text{P}\left(\overline{X} > 26.2\right) = 0.0128$. State the value of μ and find the value of σ. [4]

7 A box contains 5 black balls and 3 red balls. 3 balls are drawn randomly from the box, one at a time, without replacement. The colour of the ball that is drawn is noted.

(i) Find the probability that at least 2 red balls are drawn. [2]

(ii) Given that the third ball drawn is red, find the probability that the first ball drawn is also red. [3]

(iii) Justifying your conclusion, determine whether the events “The first ball drawn is red.” and “At least 2 red balls are drawn.” are independent. [2]

8 Under ordinary conditions, the mean time required by a person to complete an arithmetic sum is 50 s. Sixty people were randomly selected to do the sum in a room with a temperature set at $15\,^{\circ}\text{C}$. The time taken for this sample had an average of 55 s and a standard deviation of 19.3 s.

(i) Test, at the 5% significance level, whether there has been an increase in the mean time required by a person to complete the sum. [5]

(ii) Explain whether any assumptions about the population are needed in order for the test to be valid. [1]

(iii) The same sample is now used to carry out a test at the 5% significance level, of whether the mean time required by a person to complete the sum has changed. State with a reason using **(i)**, the conclusion of this test. [2]

9 Five-letter codewords are formed using the letters of the word MATHEMATICS.

(a) Find the number of codewords that can be formed if there are no restrictions. [4]

(b) Find the probability that a codeword

(i) contains both M's and both A's, [1]

(ii) has both A's separated, given that it contains all the four vowels. [3]

10 A weight is suspended in mid-air by means of a string of length x cm. In an experiment, the weight was displaced slightly and the time, y s, taken to complete 10 oscillations was recorded. The results are shown in the table.

x	10	30	50	100	150	200
y	7.05	9.85	15.53	18.86	25.91	28.31

(i) Draw a scatter diagram to illustrate the data. [1]

It is thought that the time taken y can be modelled by one of the formulae

$$y = a + bx \qquad \text{or} \qquad y = c + d\sqrt{x}$$

where a, b, c and d are constants.

(ii) Find the value of the product moment correlation coefficient between

(a) y and x,

(b) y and $\sqrt{x}$. [2]

(iii) Use your answers to **(i)** and **(ii)** to explain which formula is the better model. [1]

(iv) Find the equation of a suitable regression line and use it to estimate the value of y when $x = 45$. Comment on the reliability of the estimate. [3]

(v) The addition of a data point will not change the equation of the regression line found in **(iv)**. State the coordinates of such a point. [1]

11 It is found on average that 2 people will arrive at a bus stop in a period of 1 minute.

(i) State in this context, two conditions that must be met for the number of arrivals at the bus stop to be well modelled by a Poisson distribution. Explain why each of your two conditions may not be met. [3]

For the remainder of this question, assume that these conditions are met.

(ii) Find the probability that a total of 5 people arrive at the bus stop in a period of 2 minutes. [2]

(iii) Given that a total of 5 people arrive at a bus stop in a period of 2 minutes, find the probability that at most 1 of them arrive in the first minute, giving your answer correct to 4 significant figures. [3]

(iv) Use a suitable approximation, which should be stated together with its parameter(s), to find the probability that not more than 22 people arrive at the bus stop in a period of 10 minutes. [3]

12 In a basketball game, a two-point shot is a shot that earns the team 2 points if it is successful. Likewise, a three-point shot is a shot that earns the team 3 points if it is successful.

Alex has a 60% success rate for two-point shots and a 40% success rate for three-point shots. Assume that the outcome of each shot is independent of the outcome of any other shot.

(i) Find the least number of two-point shots that Alex needs to attempt for the probability of having at least one successful shot to be more than 0.99. [3]

(ii) Alex attempted 60 three-point shots. Use a suitable approximation to find the probability that between 21 and 28 shots inclusive are successful shots. State the parameter(s) of the distribution that you use. [4]

(iii) Alex played 50 games. Given that he attempted 10 two-point shots and 5 three-point shots for each game, find the probability that he scored an average of at least 17 points for the 50 games. [4]

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