Anglo - Chinese School
(Independent)

FINAL EXAMINATIONS 2015

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

Wednesday **7 October 2015** **1 hour 30 minutes**

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. ***Please start each question on a new page.***

1 [*Maximum mark* : 8]

(a) The curve $y=(p+2)x^2-4x+(p-1)$ has a minimum point and cuts the *x*- axis at two points. Find the range of values of p. [5 marks]

(b) Show that $4x^2-8x+11$ is positive for all real values of x. [3 marks]

2 [*Maximum mark* : 8]

(a) Solve $\frac{135}{x^2}-3=\frac{25}{x}$, giving your answer correct to 2 decimal places. [4 marks]

(b) Find the range of values of *x* which satisfy $-x \le 4x-5 < \frac{8x+1}{3}$. [4 marks]

3 [*Maximum mark* :7]

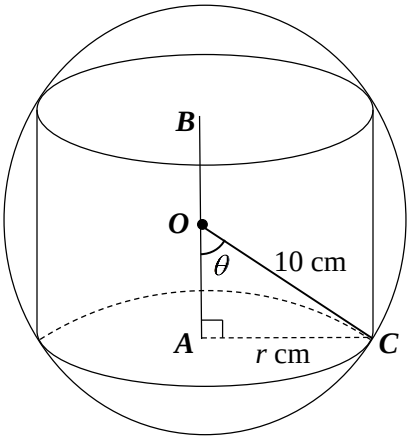
The figure shows a circular cylinder inscribed in a sphere with centre *O* and radius of 10 cm. The height of the cylinder is *AB* and *AC* = *r* cm is the base radius.

(a) Given that $\angle AOC=\theta$ radians, express *r* in terms of θ. [1 mark]

(b) Express *OA* in terms of r only. [1 mark]

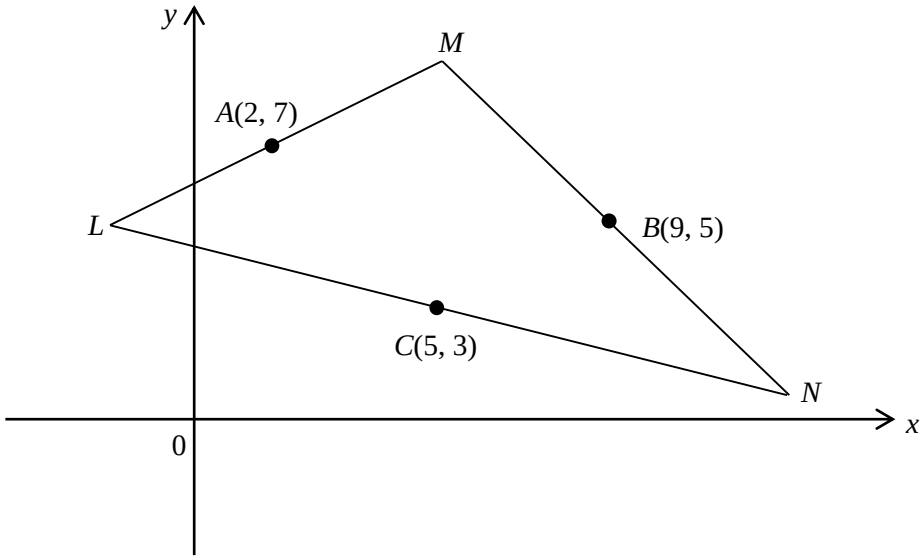
(c) Show that the height of the cylinder, $AB=20\sqrt{1-\sin^2\theta}$ cm [3 marks]

(d) Hence, express the volume of the cylinder in terms of θ. [2 marks]



4 [*Maximum mark* : 9]

Solutions to this question by accurate drawing will not be accepted.



In the diagram above, the points $A(2, 7)$, $B(9, 5)$ and $C(5, 3)$ are the midpoints of the straight lines LM, MN and LN respectively. Given also that $ABNC$ is a parallelogram, find

(a) the coordinates of N, [2 marks]

(b) the area of the parallelogram $ABNC$. [3 marks]

S is a point which lies on the x-axis such that it is equidistant from L and from N.

(c) Find the coordinates of S. [4 marks]

5 [*Maximum mark* :11]

(a) **(i)** Sketch the graph of $y = e^x$, indicating the *y*-intercept clearly. [2 marks]

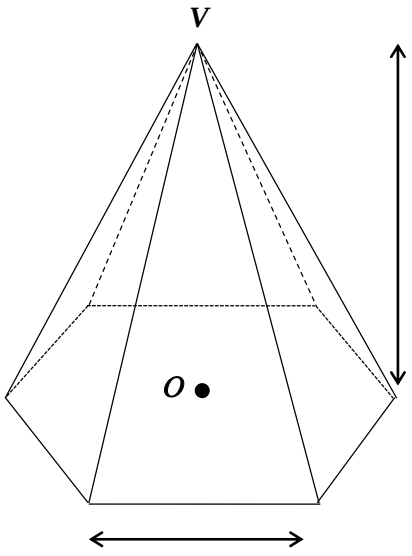
(ii) Insert on your sketch the additional graph required to obtain a graphical solution of the equation $x(e^x) = 1$. [2 marks]

(b) The diagram below shows a regular hexagonal pyramid which has a height of $\left(2\sqrt{3}+3\right)$ m.
The centre, *O*, of the hexagonal base is directly below the vertex, *V*.
Each side of the hexagonal base measures $\left(\sqrt{3}-1\right)$ m .

(i) Using the following information, as is necessary, show that the area of the hexagonal base is $3\left(2\sqrt{3}-3\right)$ m^2. [4 marks]

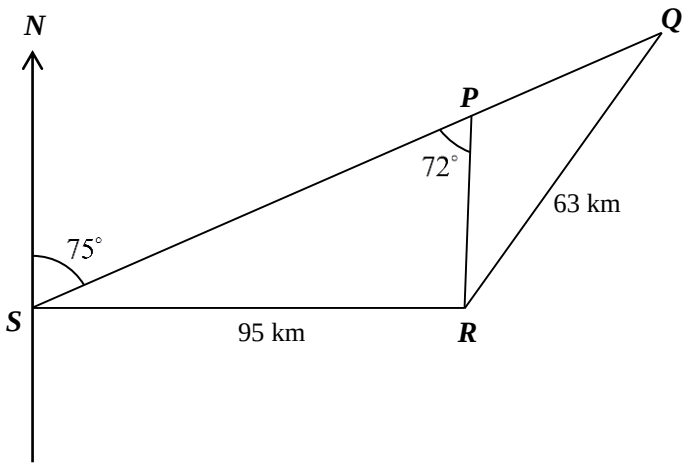
	30°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$

(ii) Hence, or otherwise, find the volume of the hexagonal pyramid. [3 marks]



6 [*Maximum mark* :13]

The diagram below represents a map showing a seaport *S* and three oil production platforms, *P*, *Q* and *R*, along the Gulf of Mexico, where *R* is due east of *S*. *SPQ* is a straight line which lies on a bearing of 075° from *S* and angle *SPR* = 72°. It is given that *SR* = 95 km and *RQ* = 63 km.



(a) Calculate the bearing of *R* from *Q*. [4 marks]

(b) A ship carrying supplies for the crew members is travelling from *S* to *P* at a constant speed of 30 km/h. Calculate

(i) the time it will take to reach *P*, [4 marks]

(ii) the shortest distance between this supply ship and the oil platform *R*. [3marks]

(c) A helicopter, *H*, is hovering at a point vertically above *R*. If the largest angle of elevation of *H* from the supply ship is 11°, find the height of the helicopter from *R*. [2 marks]

7 [*Maximum mark* : 9]

Answer the whole of Question 7 on a sheet of graph paper.

It is known that variables *x* and *y* are related by the equation $y = a\,x + b\sqrt{x}$, where *a* and *b* are unknown Constants. Observed values of the two variables are shown in the following table.

x	0.5	1.0	1.5	2.0	2.5	3.0
y	1.33	3.20	5.16	7.16	9.19	11.25

(a) On a sheet of graph paper, draw a straight line graph of $\frac{y}{\sqrt{x}}$ against $\sqrt{x}$, using a scale of 1 cm to represent 0.1 units on the $\sqrt{x}$ - axis, and 2 cm to represent 1 unit on the $\frac{y}{\sqrt{x}}$ - axis. [4 marks]

(b) Use the graph to estimate the value of *a* and of *b*. [3 marks]

(c) By drawing a suitable straight line, find the value of *x* for which $y = 5\sqrt{x}$. [2 marks]

8 [*Maximum mark* :15]

(a) Without using a calculator, solve the equation $(\ln a)^{\ln a} = a$. [5 marks]

(b) Given that $\log_a 4 = x$ and $\log_b 2 = y$, find $\log_2(a\,b^5)$ in terms of *x* and of *y*. [4 marks]

(c) Solve the simultaneous equations

$$5^x - 2^y = -7$$
$$2^{y+1} = 59 + 5^{x-1}.$$ [6 marks]

----------End of Paper----------

Answers :

1(a) $-2 < p < 2$

1(b) $b^2 - 4ac = (8)^2 - 4(4)(11) = -112 < 0$

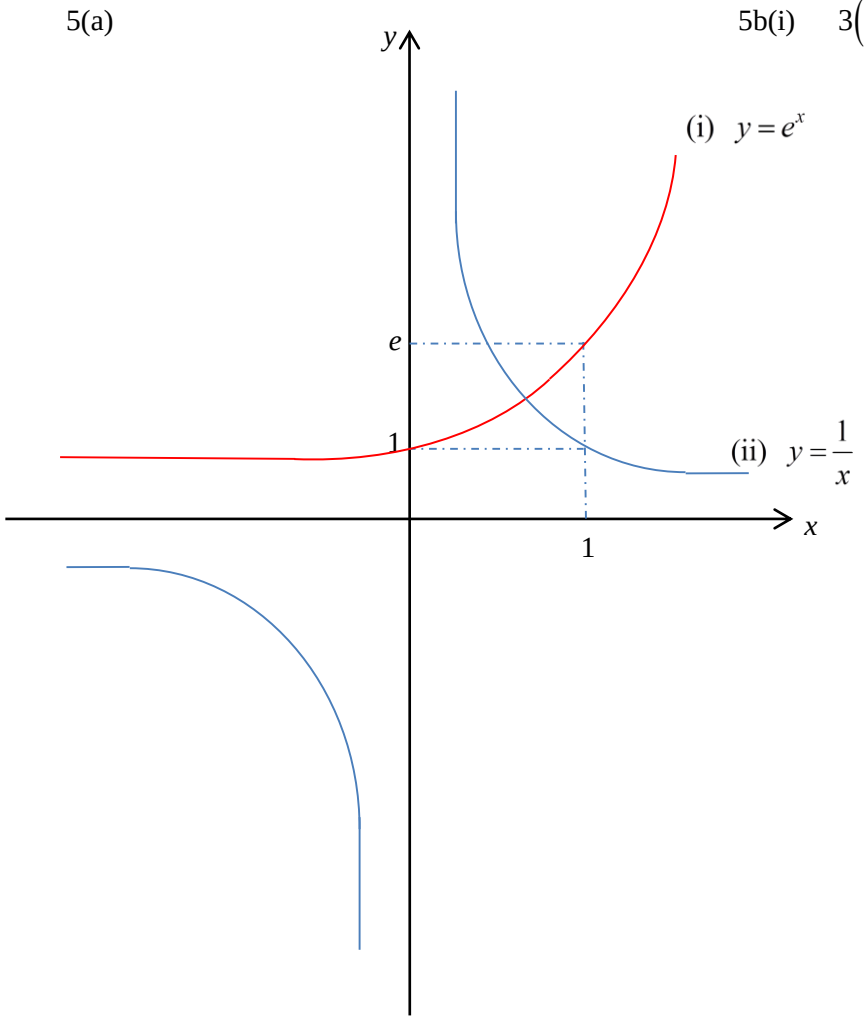
Since the discriminant of this upward-facing parabola (coefficient of $x^2 > 0$) is negative, this parabola has no real roots ie always lying above the x-axis. Hence the quadratic function is always positive.

2(a) 3.73 or -12.06 2(b) $1 \le x < 4$

3(a) $r = 10\sin\theta$ 3(b) $OA = \sqrt{100 - r^2}$ 3(d) $2000\pi \sin^2\theta\sqrt{1-\sin^2\theta}$

4(a) $N(12, 1)$ 4(b) $22\ \text{u}^2$ 4(c) $S(\frac{29}{7}, 0)$

5(a) 5b(i) $3\left(2\sqrt{3} - 3\right)$ 5b(ii) $3\ \text{m}^3$



6(a) 232.1° 6b(i) 3.33 hours 6b(ii) 24.6 km (to 3 sf) 6(c) 4.78 km (to 3 sf)

8(a) $a = e^e$ 8(b) $\frac{2}{x} + \frac{5}{y}$ 8(c) $x = 2, y = 5$