ST ANDREW'S JUNIOR COLLEGE
PRELIMINARY EXAMINATION
HIGHER 2

Candidate Name

CLASS 2 3

H2 MATHEMATICS 9758/01

Paper 1 **26 AUGUST 2024 (Monday)**

Candidates answer on the Question Paper. **3h**
Additional Materials: MF 26

Number of pieces of additional writing paper : ____________________ (N.A. if none)

Q	1	2	3	4	5	6	7	8	9	10	11	TOTAL
M												
	4	4	7	5	8	8	11	12	12	13	16	100

READ THESE INSTRUCTIONS FIRST

Write your name, civics group, index number and calculator models on the cover page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions. Total marks : **100**

Write your answers in the spaces provided in the question paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 Show that x^2+2x+4 is always positive for all real values of x.

Hence solve the inequality $\dfrac{x^2+2x+4}{3+2x-x^2}<0$. [4]

2 A function is defined as $\mathrm{f}(\theta)=\sqrt{3}\sin\theta+\cos\theta$.

(i) Show that $\mathrm{f}(\theta)=\sqrt{3}\sin\theta+\cos\theta$ can be written in the form $R\sin(\theta+\alpha)$ where exact values of R and α are to be found. [1]

(ii) Hence, state a sequence of transformations that will transform the graph with equation $y=\sin\theta$ on to the graph with equation $y=\sqrt{3}\sin\theta+\cos\theta+5$. [3]

3 **(i)** Show that $\dfrac{1}{(r-1)!}-\dfrac{2}{r!}+\dfrac{1}{(r+1)!}=\dfrac{r^2-r-1}{(r+1)!}$, where $r\in\mathbb{Z}^+$. [1]

(ii) Hence find an expression for $\displaystyle\sum_{r=2}^{n}\frac{r^2-r-1}{(r+1)!}$. [4]

(iii) Show that $\displaystyle\sum_{r=2}^{\infty}\frac{r^2-r-1}{(r+1)!}$ is convergent. [2]

4 A curve has equation $3x^2-5xy+y^3=2$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. Hence, deduce the number of tangent(s) to the curve that is/are parallel to the y-axis. [5]

5 **(a)** Using the substitution $u=\mathrm{e}^x$, find $\displaystyle\int\frac{2}{\mathrm{e}^x-\mathrm{e}^{-x}}\,\mathrm{d}x$. [3]

(b) **(i)** Find $\displaystyle\int\cos x\sin 3x\,\mathrm{d}x$. [2]

(ii) Without the use of a calculator, evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{3\pi}{4}}|\cos x|\sin 3x\,\mathrm{d}x$. [3]

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6 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) The point C lies on OB such that $\overrightarrow{OC} = k\overrightarrow{OB}$, where k is a constant. X is on AC such that $AX : XC = 1 : 2$ and Y is on AB produced such that $AY : BY = 5 : 4$.

(i) Find $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of **a**, **b** and k. [2]

(ii) Given that O, X and Y are collinear, find k. [3]

(b) It is given that $|\mathbf{a}| = 2|\mathbf{b}|$ and $\cos\angle AOB = -\frac{1}{4}$. Give the geometrical meaning of $|(\mathbf{a} + m\,\mathbf{b}) \times \mathbf{b}|$, for some real value of m, and evaluate the value of $|(\mathbf{a} + m\,\mathbf{b}) \times \mathbf{b}|$, leaving your answer in terms of $|\mathbf{a}|$. [3]

7 An inverted right pyramid is inscribed in a sphere of fixed radius R and center O, where the vertices A, B, C, D and T are touching the surface of the sphere as shown in **Figure 1**. The pyramid has a square base $ABCD$ and height, ST, where $ST = h$ units. S is the point where the diagonals AC and BD of the square intersect.

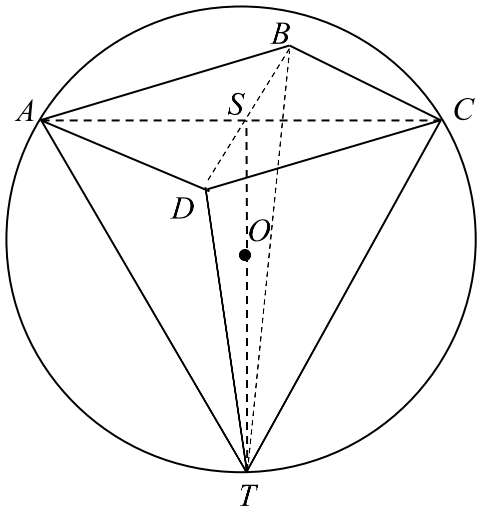

Figure 1

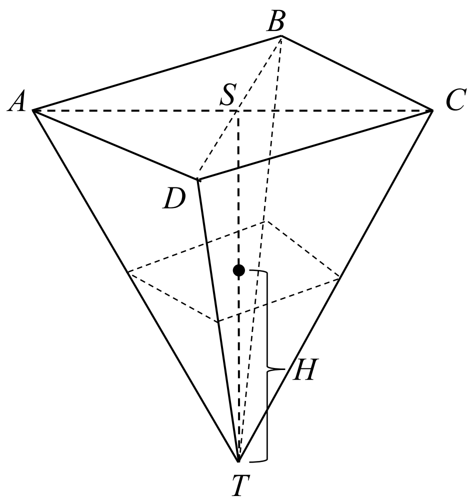

Figure 2

[It is given that the volume of a pyramid is $\frac{1}{3} \times \text{base area} \times \text{height}$.]

(i) Show that the volume of the pyramid, V is $\frac{2}{3}h\left(2Rh - h^2\right)$. [3]

(ii) Given that h varies, show that the maximum volume of pyramid, V, is obtained when the length of the square base is equal to the height of the pyramid. [5]

[Turn over

(iii) A container in the form of an inverted right pyramid, shown in **Figure 2**, with maximum volume V as described in **(ii)** is made. Water is poured into this container at a rate of 10 units^3 per second. Find the rate of increase of the water level, H, when the height of the water is 5 units. [3]

8 **Do not use a calculator in answering this question.**

(i) Given that $z = 1 + \mathrm{i}$, find z^2, z^3 and z^4 in cartesian form. Given also that

$$z^4 + az^3 - 2z^2 + 8z - 8 = 0,$$

where a is real, find the value of a. [4]

(ii) Using the value of a in **(i)**, express $z^4 + az^3 - 2z^2 + 8z - 8$ as the product of two quadratic factors. [3]

(iii) Hence, deduce the two quadratic factors of $1 - a\mathrm{i}w + 2w^2 + 8\mathrm{i}w^3 - 8w^4$. [2]

(iv) A second complex number is given by $v = 2\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}$. Find $\arg\left[(zv)^3\right]$. [3]

9 The curve C_1 has equation $y = \dfrac{3x^2 - 2x + 3}{x - 1}$. The curve C_2 has the equation $y = -\mathrm{e}^{2x} - 5$.

(a) Sketch C_1, stating the equation(s) of any asymptote and the coordinates of any turning point(s) and point(s) where the curve crosses the axes. [4]

(b) C_1 and C_2 intersect at a point $P(a, b)$ where $x < 0$. Find the values of a and b, giving your answer correct to 4 decimal places. [1]

The region R is bounded by C_1, C_2, x-axis, y-axis and the line $x = k$ where $k < a$.

(c) Given that the area of region R is at least 10 units^2, show that k satisfies the inequality $\dfrac{1}{2}\mathrm{e}^{2k} + 5k + p \le 0$ where p is a constant to be determined. Hence find the maximum value of k. [4]

(d) Now, given that $k = -4$, find the volume of the solid formed when R is rotated completely about the x-axis, correct to 2 decimal places. [3]

10 Scientists are investigating the growth in length of a particular species of fish.

(i) The scientists used the von Bertalanffy growth model to predict the length L cm of the fish at a particular time, t years after birth. It is said that the rate of growth in length of the fish is directly proportional to the difference between its theoretical maximum length of the fish, 60 cm and its length at the time t years. The constant of proportionality, k, is known as the growth coefficient which is always positive. At birth, it is known that the rate of growth when the fish is 10 cm long is 5 cm per year.

(a) Write down a differential equation for this situation. Solve this differential equation to get L as an exact function of t. [7]

(b) Find the length of the fish 5 years after it is born. [1]

(ii) Based on the data collected, one of the scientists proposed a second model to predict the length L cm of the fish at a particular time, t years after birth. The proposed model is as follows:

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \frac{50}{L}.$$

(a) Given that the length of the fish is 10 cm long at birth, find L in terms of t. [4]

(b) Comment on the validity of the second model in modelling the length of fish, L cm, at a particular time, t years after birth as compared to the model in **(i)**. [1]

11 A group of drone operators are preparing for an aerial display at a community celebration. They used a computer simulation program to design the flight paths of the drones to ensure that none of the drones will cross into the flight path of another drone for safety reasons. Points (x, y, z) are defined relative to a main control base located at $(0, 0, 0)$. The flight path of any drone is taken to be a straight line and the mass of the drone can be neglected in the simulation program.

Drone A is launched from the main control base and the flight path is in the direction $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Another drone, drone B is launched from a point $(2, 1, 1)$ and its flight path is in the direction $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(i) Show that the flight paths of the two drones are skew lines. [3]

Drone A is expected to hover at a point $P(5, 5, 5)$ which lies on its flight path. While drone A is hovering at point P, drone B is expected to fly to point Q, where it will be at its shortest distance from drone A to prepare for an aerial formation.

(ii) Find the coordinates of point Q and the distance PQ. [5]

Plane Π, containing the point P, is parallel to both $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and the flight path of drone B.

(iii) Find the vector equation of the plane Π. [3]

To complete the aerial formation, a third drone, C, is launched. The computer simulator program uses the plane Π to determine the location of the point T where drone C is expected to be. Point T is the point of reflection of the position of Q in the plane Π.

(iv) Find the position vector of point T. [5]

End of Paper

ST ANDREW'S JUNIOR COLLEGE
PRELIMINARY EXAMINATION
HIGHER 2

Candidate Name	
CLASS	2 3

H2 MATHEMATICS 9758/02

Paper 2 **10 SEPTEMBER 2024 (Tuesday)**

Candidates answer on the Question Paper. **3h**
Additional Materials: MF 26

Number of pieces of additional writing paper : ____________________ (N.A. if none)

Q	1	2	3	4	5	6	7	8	9	10	11	TOTAL
M												
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Section A: Pure Mathematics [40 marks]

1 A geometric series has common ratio r, and an arithmetic series has first term a and common difference d, where a and d are non-zero. The first three terms of the geometric series are equal to the seventh, fourth and second terms respectively of the arithmetic series.

(i) Find d in terms of a. [3]

(ii) Deduce that the geometric series is convergent and find, in terms of a, the sum to infinity. [4]

It is now known that a is positive.

(iii) Without the use of a calculator, find the set of values of n such that the sum of the first n terms of the geometric progression differs from the sum to infinity of the geometric progression by less than 1% of the sum to infinity of the geometric progression. [3]

2 A curve C has parametric equations

$$x = t\ln t^2, \qquad y = 4(t-2)^2, \qquad \text{for } 0.5 \le t \le 3.5.$$

(i) Find the equation of the normal to C at point P where $t = 3$ in the form $ax + by = c$, where a, b and c are exact constants. [4]

The normal to curve C at point P meets the curve again at point N.

(ii) Find the coordinates of N. [3]

(iii) Find the area bounded by curve C and the normal at point P. [3]

3 A function g is said to be self-inverse if $\mathrm{g}^{-1}(x) = \mathrm{g}(x)$ for all x in the domain of g.

The function f is defined by

$$\mathrm{f}(x) = \frac{ax+b}{x-2}, \; x \in \mathbb{R}, \; x \neq 2,$$

where a and b are real constants.

It is given that f is self-inverse.

(i) Find a and the value(s) that b cannot take. [4]

(ii) Explain why the composite function f^n always exists for all integers $n > 1$. [1]

(iii) Hence find $\mathrm{f}^{2025}(1)$ in terms of b. [2]

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The function h is defined for all $x \in \mathbb{R}$ by

$$\mathrm{h}(x) = \begin{cases} 3x+2 & x \geq 0, \\ 2x^2 & x < 0. \end{cases}$$

(iv) Sketch the graph of h. [2]

(v) Using the value of a found in **(i)** and given that $b=1$, find $\mathrm{hf}\left(\frac{1}{2}\right)$. [1]

4 It is given that $y = \ln(\cos x)$.

(i) Prove that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 1 = 0$. [2]

(ii) By differentiating further, find the first two non-zero terms of the expansion of y. [4]

(iii) Using the answer in **(ii)** and an appropriate expansion from MF26 up to and including the term in x^4, find an approximate value for $\int_0^{0.08} \ln\left(\frac{\cos^3 x}{1-x}\right) \mathrm{d}x$, leaving your answer correct to 7 decimal places. [2]

The calculated value of $\int_0^{0.08} \ln\left(\frac{\cos^3 x}{1-x}\right) \mathrm{d}x$ is given to be 0.0030328 correct to 7 decimal places.

(iv) Comparing the calculated value of $\int_0^{0.08} \ln\left(\frac{\cos^3 x}{1-x}\right) \mathrm{d}x$ given above to your answer in **(iii)**, and with reference to the value of x, comment on the accuracy of your approximation. [2]

Section B: Probability and Statistics [60 marks]

5 A dart player, Mike, in Smart Andrew's College Darts Team is having a practice session of dart throwing in the hall. The probability that Mike hits his target on the first throw is 0.7.

For each throw after his first throw, the probability that Mike hits the target, given that he hit the target on his preceding throw, is 0.77.

For each throw after his first throw, the probability that Mike does not hit the target, given that he did not hit the target on his preceding throw, is 0.33.

(i) Draw a fully labelled tree diagram to represent Mike's first two throws. [2]

(ii) Find the probability that Mike hits his target on the second throw of the dart. [2]

It is now known that Mike threw the dart three times.

(iii) Given that Mike did not hit his target on the second throw, find the probability that he hit the target twice. [3]

6 Timothy runs a game stall at a funfair. In a particular card game, there is a deck of $(n+4)$ cards consisting of four cards numbered 5 and n cards numbered 3, where $n \geq 2$. To play a game, a player draws two cards at random and without replacement from the deck of cards. The random variable M represents a player's score, which is the product of the scores of the two cards drawn in one game.

(i) Determine the probability distribution of a player's score in one game. [3]

For the card game, the payout of one game in dollars, W, is such that $W = |M - 15|$.

(ii) Given that $\mathrm{E}(W) = \frac{100}{23}$, find the value of n. [3]

(iii) Using the value of n found in **(ii)**, find the probability that the mean payout of 40 independent games is less than \$4.50. [3]

7 In a small company, there are four departments, namely, Administrative, Marketing, Sales and Human Resource (HR). There is a total of 7 staff in the four departments and the number of staff in each department is shown in the table below.

Administrative	Marketing	Sales	HR
1	2	3	1

At an annual company dinner, the organiser would like to draw up a seating arrangement for a round table with 7 seats, consisting of all 7 staff from these four departments.

To plan the seating arrangement, the organiser prepared card labels with the respective department names. Each card label is indistinguishable if they represent the same department.

Find the number of different arrangements that can be drawn up, using the card labels, where staff members from the Marketing department are not seated together and the staff members from the Sales department are not seated together. [4]

8 Gilbert is thinking of buying a new car of a particular model with an engine capacity of 1 000 cc or less and is trying to find a good time to purchase his car. He collects some information from a website stating the number of cars, y thousand, in Singapore with an engine capacity of 1 000 cc or less for certain years from 2011 in the table below. The variable x is the number of years after a base year of 2010.

Year	2011	2012	2015	2016	2018	2021	2023
Number of years after base year, x	1	2	5	6	8	11	13
Number of cars, y thousand, in Singapore with an engine capacity of 1 000 cc or less	6.62	6.49	5.80	4.82	6.44	10.42	20.28

Source: Land Transport Authority, Singapore , Data last updated: 4 Mar 2024
(https://tablebuilder.singstat.gov.sg/table/TS/M650681)

(a) Draw a scatter diagram of these data. [1]

After looking at the data, Gilbert thinks that the data should be modelled by an equation of either of the following two forms:

$$y = ax^2 + b \quad \text{or} \quad y = ce^x + d$$

where a, b, c and d are constants.

(b) Calculate the product moment correlation coefficients for the models $y = ax^2 + b$ and $y = ce^x + d$ respectively. Hence, determine whether the relationship between x and y is modelled better by $y = ax^2 + b$ or $y = ce^x + d$. Explain how you decide which model is better, and state the equation in this case. [5]

(c) Use the equation of the regression line to estimate the number of cars in Singapore with an engine capacity of 1 000 cc or less in 2025. Explain whether your estimate is reliable. [2]

(d) Explain if the product moment correlation coefficient will be affected if the number of years after base year, x, is given as number of months after a base year of 2010 instead. [1]

9 Eggs are packed in trays of n eggs each and sold at Aung Moo Supermarket. A trainee truck driver is employed to transport a large delivery of eggs to the supermarket. On average, 3% of eggs crack during the transportation to the supermarket.

(i) State, in context, two assumptions needed for the number of cracked eggs in a tray during transportation to the supermarket to be well modelled by a binomial distribution. [2]

It is now assumed that the number of cracked eggs during the transportation to the supermarket follows a binomial distribution.

(ii) Given that the probability of a randomly selected tray containing at most 2 cracked eggs during the transportation to the supermarket is at least 0.91, find the largest possible value of n. [3]

For the rest of the question, let n = 30.

For delivery, 40 trays of eggs are packed into a carton. A carton will be rejected by the supermarket if it contains at least 3 trays of eggs with more than two cracked eggs each.

(iii) Find the probability that a carton of eggs will be rejected by the supermarket. [2]

10 A factory produces wooden toy cars. Each toy car is made up of 1 rectangular block and 4 circular blocks. The masses of the rectangular blocks have the distribution $N(100, 4^2)$ and the masses of the circular blocks have the distribution $N(15, 2^2)$. The units for mass are grams.

(i) Find the probability that the total mass of a randomly chosen toy car is between 150 and 170 grams. [2]

(ii) The probability that the total mass of n randomly chosen circular blocks is more than 65 grams is at least 0.105. Find the least value of n. [3]

(iii) State an assumption needed for your calculations in parts **(i)** and **(ii)**. [1]

The circular blocks are given a coating which increases the mass of each circular block by 10%. The rectangular blocks are not given any coating.

(iv) Find the probability that the mass of a randomly chosen rectangular block is heavier than the total mass of 4 randomly chosen coated circular blocks by 40 grams. [3]

(v) A factory inspector decides to randomly choose 5 coated circular blocks for inspection. A coated circular block will pass the inspection if it has a mass of more than 18 grams, otherwise it will fail the inspection. Find the probability that two coated circular blocks pass the inspection. [3]

11 Ms Lam, a teacher from Shelton College, claims that students spend an average of 4 hours studying in the college per week. A survey was conducted and the amount of time, x hours, spent by 50 randomly chosen students studying in the college were recorded. It was found that the amount of time that the students spent studying in the college per week, in hours, had a mean of 3.8 hours and a standard deviation of 0.46 hours.

(i) Test, at the 5% level of significance, whether the average amount of time spent by students studying in the college per week is 4 hours. [5]

(ii) Explain the meaning of the p-value obtained in **(i)** in the context of the question. [1]

(iii) Explain if the test in part **(i)** would still be valid if the amount of time spent by students studying in the college was collected from a random sample of 15 students instead. [1]

A publicity campaign on the night study program is held. Ms Lam claims that due to the publicity campaign, the mean amount of time that students spend studying in the college is now μ_0 hours. Another survey on 8 students was conducted after the publicity campaign, and the amount of time, in hours, spent studying in the college for each student were recorded as follows:

4.3	4.5	3.8	4.4	4.9	3.5	4.6	3.7

It is now known that the amount of time that students spend studying in Shelton College after the publicity campaign follows a normal distribution with a standard deviation of 0.6 hours.

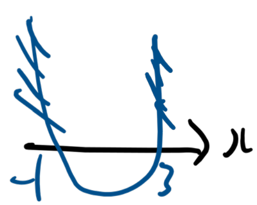
(iv) Calculate the sample mean amount of time spent studying in Shelton College of the 8 students. Hence, find the largest possible value of μ_0, such that at 5% level of significance, there is insufficient evidence that Ms Lam has overstated the mean amount of time that the students spent studying in Shelton College. [5]

End of Paper

St Andrew's Junior College
2024 Preliminary Examinations H2 Mathematics Paper 1 (9758/01) Solutions

<table>
<tr><th>Q</th><th>Solutions</th></tr>
<tr><td>1</td><td><u>Method 1 (completing the square)</u>
$x^2+2x+4=(x+1)^2+3$
For all real values of x,
$(x+1)^2 \ge 0$
$\Rightarrow \quad (x+1)^2+3>0$
$\Rightarrow \quad x^2+2x+4>0$
$\therefore \quad x^2+2x+4$ is always positive for all real values of x.

<u>Method 2 (Using discriminant – NOT recommended)</u>
Discriminant = $(2)^2-4(1)(4)=-12<0$
Since discriminant is < 0 and coefficient of x^2 is 1 > 0, x^2+2x+4 is always positive for all real values of x.</td></tr>
<tr><td></td><td>$\dfrac{x^2+2x+4}{3+2x-x^2} \le 0$

Since $x^2+2x+4>0 \quad \forall\, x \in \mathbb{R}$,
$3+2x-x^2<0$
$x^2-2x-3>0$
$\Rightarrow \quad (x-3)(x+1)>0$
$\Rightarrow \quad x>3 \quad \text{or} \quad x<-1$

</td></tr>
</table>

Q	Solutions
2 (i)	$R=\sqrt{3+1^2}=2$, $\alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$ $f(\theta)=\sqrt{3}\sin\theta+\cos\theta$ $=2\sin(\theta+\frac{\pi}{6})$
(ii)	Let $g(\theta)=\sin\theta$. $y=\sqrt{3}\sin\theta+\cos\theta+5$ $=f(\theta)+5=2g\left(\theta+\frac{\pi}{6}\right)+5$ The graph of $y=\sin\theta$ undergoes the following transformations of: 1. Translation of $\frac{\pi}{6}$ unit in the negative θ-direction, followed by 2. Scaling parallel to the y-axis with scale factor 2 , followed by 3. Translation of 5 units in the positive y-direction to obtain the graph of $y=\sqrt{3}\sin\theta+\cos\theta+5$. The graph of $y=\sin\theta$ undergoes the transformations of: 1) Translation of $\frac{\pi}{6}$ unit in the negative θ-direction, followed by 2) Translation of $\frac{5}{2}$ units in the positive y-direction , followed by 3) Scaling parallel to the y-axis with scale factor 2 to obtain the graph of $y=\sqrt{3}\sin\theta+\cos\theta+5$.

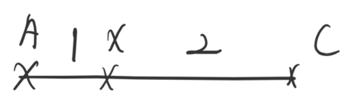
Q	**Solutions**
3 (i)	$LHS=\frac{1}{(r-1)!}-\frac{2}{r!}+\frac{1}{(r+1)!}$ $=\frac{r(r+1)}{(r-1)!r(r+1)}-\frac{2(r+1)}{r!(r+1)}+\frac{1}{(r+1)!}$ $=\frac{r^2+r-2r-2+1}{(r+1)!}$ $=\frac{r^2-r-1}{(r+1)!}$ $=RHS$ (shown)

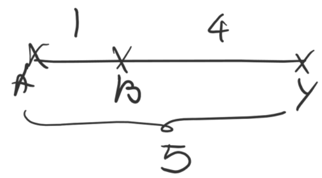
Q	Solutions
(ii)	$\sum_{r=2}^{n}\frac{r^2-r-1}{(r+1)!}\cdots(1)$ $=\sum_{r=2}^{n}\left[\frac{1}{(r-1)!}-\frac{2}{r!}+\frac{1}{(r+1)!}\right]\cdots(2)$ $=\left[\frac{1}{1!}-\frac{2}{2!}+\frac{1}{3!}\right.$ $+\frac{1}{2!}-\frac{2}{3!}+\frac{1}{4!}$ $+\frac{1}{3!}-\frac{2}{4!}+\frac{1}{5!}$ $+\frac{1}{4!}-\frac{2}{5!}+\frac{1}{6!}$ $+\cdots$ $+\frac{1}{(n-3)!}-\frac{2}{(n-2)!}+\frac{1}{(n-1)!}$ $+\frac{1}{(n-2)!}-\frac{2}{(n-1)!}+\frac{1}{n!}$ $\left.+\frac{1}{(n-1)!}-\frac{2}{n!}+\frac{1}{(n+1)!}\right]$ $=\frac{1}{1!}-\frac{2}{2!}+\frac{1}{2!}+\frac{1}{n!}-\frac{2}{n!}+\frac{1}{(n+1)!}$ $=\frac{1}{2}-\frac{1}{n!}+\frac{1}{(n+1)!}$

Q	Solutions
(iii)	$\sum_{r=2}^{\infty}\frac{r^2-r-1}{(r+1)!}=\lim_{n\to\infty}\sum_{r=2}^{n}\frac{r^2-r-1}{(r+1)!}$ $=\lim_{n\to\infty}\left[\frac{1}{2}-\frac{1}{n!}+\frac{1}{(n+1)!}\right]$ $=\frac{1}{2}$ since $\frac{1}{n!}\to 0$ and $\frac{1}{(n+1)!}\to 0$ as $n\to\infty$ Since $\frac{1}{2}$ is a finite and unique value, $\sum_{r=2}^{\infty}\frac{r^2-r-1}{(r+1)!}$ is convergent. OR As $n\to\infty.\frac{1}{n!}\to 0$ and $\frac{1}{(n+1)!}\to 0$, $\sum_{r=2}^{n}\frac{r^2-r-1}{(r+1)!}=\frac{1}{2}-\frac{1}{n!}+\frac{1}{(n+1)!}\to\frac{1}{2}$ Since $\frac{1}{2}$ is a finite and unique value, $\sum_{r=2}^{\infty}\frac{r^2-r-1}{(r+1)!}$ is convergent.
4	$3x^2-5xy+y^3=2$ -------(*) Differentiate with respect to x $6x-5x\frac{dy}{dx}-5y+3y^2\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{-6x+5y}{-5x+3y^2}$ For tangent parallel to the y-axis, denominator=0. $-5x+3y^2=0$ $x=\frac{3}{5}y^2$ Substitute $x=\frac{3}{5}y^2$ into (*)

Q	**Solutions**
	$3\left(\frac{3}{5}y^2\right)^2-5\left(\frac{3}{5}y^2\right)y+y^3=2$ $\frac{27}{25}y^4-2y^3-2=0$ From GC, there are two real solutions : y = 2.06, -0.879 These will give two corresponding values of x=2.55,0.464 since $x=\frac{3}{5}y^2$. Hence there are two tangents parallel to the y-axis.
5 (a)	$u=\mathrm{e}^x$ $x=\ln u \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$

Q	Solutions
	$\int \frac{2}{e^{x}-e^{-x}}\,dx$ $= \int \frac{2}{u-\frac{1}{u}}\left(\frac{dx}{du}\right)du$ $= \int \frac{2u}{u^{2}-1}\left(\frac{1}{u}\right)du$ $= \int \frac{2}{u^{2}-1}\,du$ $= 2\times\frac{1}{2}\ln\left\|\frac{u-1}{u+1}\right\|+C$ $= \ln\left\|\frac{e^{x}-1}{e^{x}+1}\right\|+C,$ where C is an arbitrary constant
(b) (i)	$\int \cos x \sin 3x\,dx$ $= \frac{1}{2}\int\left(\sin 4x+\sin 2x\right)dx$ $= \frac{1}{2}\left[-\frac{1}{4}\cos 4x-\frac{1}{2}\cos 2x\right]+c$ $= -\frac{1}{8}\left[\cos 4x+2\cos 2x\right]+c,$ where c is an arbitrary constant

Q	Solutions
(b) (ii)	$\int_{\frac{\pi}{6}}^{\frac{3\pi}{4}} \lvert\cos x\rvert \sin 3x \, \mathrm{d}x$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \sin 3x \, \mathrm{d}x + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (-\cos x) \sin 3x \, \mathrm{d}x$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \sin 3x \, \mathrm{d}x - \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos x \sin 3x \, \mathrm{d}x$ $= -\frac{1}{8}\left[\cos 4x + 2\cos 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \frac{1}{8}\left[\cos 4x + 2\cos 2x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$ $= \frac{1}{8}\left\{ \begin{array}{l} -\left[\cos 2\pi + 2\cos \pi - \cos\left(\frac{2\pi}{3}\right) - 2\cos\left(\frac{\pi}{3}\right)\right] \\ +\left[\cos 3\pi + 2\cos \frac{3\pi}{2} - \cos 2\pi - 2\cos \pi\right] \end{array} \right\}$ $= \frac{1}{8}\left[-\left(1 - 2 - \left(-\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)\right) + (-1 + 0 - 1 + 2)\right]$ $= \frac{1}{8}\left[-\left(-\frac{3}{2}\right)\right]$ $= \frac{3}{16}$
6(a)(i)	Using Ratio Theorem, $\overrightarrow{OX} = \frac{2\overrightarrow{OA} + \overrightarrow{OC}}{3} = \frac{2\mathbf{a} + k\mathbf{b}}{3}$ 

Q	Solutions
	$\overrightarrow{OB}=\dfrac{4\overrightarrow{OA}+\overrightarrow{OY}}{5}$ $\mathbf{b}=\dfrac{4\mathbf{a}+\overrightarrow{OY}}{5}$ $\overrightarrow{OY}=5\mathbf{b}-4\mathbf{a}$ 
(ii)	Since O, X, Y are collinear, $\overrightarrow{OX}=m\overrightarrow{OY}$, where m is some real constant $\dfrac{2\mathbf{a}+k\mathbf{b}}{3}=m\left(5\mathbf{b}-4\mathbf{a}\right)$ $\dfrac{2}{3}\mathbf{a}+\dfrac{k}{3}\mathbf{b}=-4m\mathbf{a}+5m\mathbf{b}$ Since **a** and **b** are non-zero and non-parallel vectors, $\Rightarrow\dfrac{2}{3}=-4m\Rightarrow m=-\dfrac{1}{6}$ $\Rightarrow\dfrac{k}{3}=5m\Rightarrow k=15\left(-\dfrac{1}{6}\right)=-\dfrac{5}{2}$ OR

Q	Solutions
	Since O, X, Y are collinear, $\overrightarrow{OX} = m\overrightarrow{OY}$, where m is some real constant $\dfrac{2\mathbf{a}+k\mathbf{b}}{3} = m(5\mathbf{b}-4\mathbf{a})$ $\dfrac{2}{3}\mathbf{a}+\dfrac{k}{3}\mathbf{b}+4m\mathbf{a}-5m\mathbf{b}=\mathbf{0}$ $\left(\dfrac{2}{3}+4m\right)\mathbf{a}+\left(\dfrac{k}{3}-5m\right)\mathbf{b}=0\mathbf{a}+0\mathbf{b}$ Since **a** and **b** are non-zero and non-parallel vectors, $\Rightarrow \dfrac{2}{3}+4m=0 \Rightarrow \dfrac{2}{3}=-4m \Rightarrow m=-\dfrac{1}{6}$ $\Rightarrow \dfrac{k}{3}-5m=0 \Rightarrow \dfrac{k}{3}=5m \Rightarrow k=15\left(-\dfrac{1}{6}\right)=-\dfrac{5}{2}$
(b)	$\lvert(\mathbf{a}+k\mathbf{b})\times\mathbf{b}\rvert = \lvert\mathbf{a}\times\mathbf{b}+k\mathbf{b}\times\mathbf{b}\rvert$ $= \lvert\mathbf{a}\times\mathbf{b}+\mathbf{0}\rvert$ $= \lvert\mathbf{a}\times\mathbf{b}\rvert$ $\lvert(\mathbf{a}+k\mathbf{b})\times\mathbf{b}\rvert$ represents the area of the parallelogram with **adjacent sides *OA* and *OB***.

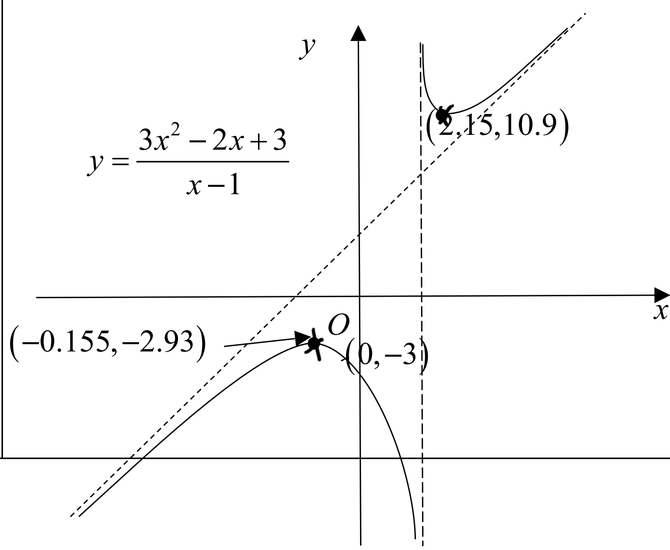
Q	Solutions
	$\left\|(\mathbf{a}+k\mathbf{b})\times\mathbf{b}\right\| = \left\|\mathbf{a}\times\mathbf{b}\right\|$ $= \|\mathbf{a}\|\|\mathbf{b}\|\sin\theta$ $= \|\mathbf{a}\|\|\mathbf{b}\|\sqrt{1-\cos^2\theta}$ $= \|\mathbf{a}\|\left(\frac{1}{2}\|\mathbf{a}\|\right)\sqrt{1-\left(-\frac{1}{4}\right)^2}$ $= \frac{\sqrt{15}}{8}\|\mathbf{a}\|^2$
7(i)	$AS = \sqrt{R^2-(h-R)^2}$ Using triangle ABC, Length of square top = AB $\cos\left(\frac{\pi}{4}\right) = \frac{AB}{AC} = \frac{AB}{2AS}$ $AB = 2AS\cos\left(\frac{\pi}{4}\right)$ $AB = 2\cos\left(\frac{\pi}{4}\right)\sqrt{R^2-(h-R)^2}$ $= 2\left(\frac{\sqrt{2}}{2}\right)\sqrt{R^2-(h-R)^2}$ $= \sqrt{2}\sqrt{R^2-(h-R)^2}$ Volume of pyramid, V

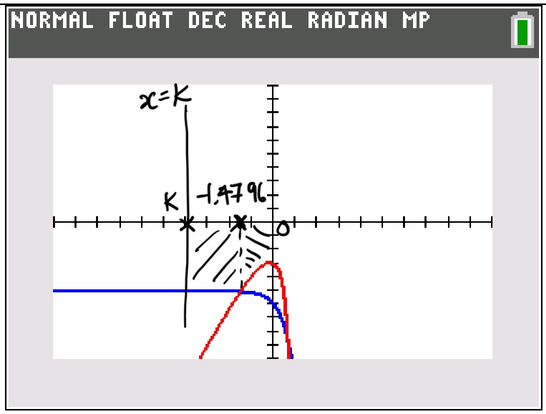
Q	Solutions
	$=\frac{1}{3}h\left(\sqrt{2}\sqrt{R^2-(h-R)^2}\right)^2$ $=\frac{2}{3}h\left[R^2-(h-R)^2\right]$ $=\frac{2}{3}h\left[2Rh-h^2\right]$ (shown)
(ii)	$V=\frac{2}{3}\left(2Rh^2-h^3\right)$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{2}{3}\left(4Rh-3h^2\right)$ For stationary point, $\frac{\mathrm{d}V}{\mathrm{d}h}=0$ $\frac{2}{3}\left(4Rh-3h^2\right)=0$ $h(4R-3h)=0$ $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=\frac{2}{3}(4R-6h)$ $h=0$ (rejected since $h>0$) or $h=\frac{4}{3}R$ When $h=\frac{4}{3}R$, $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=\frac{2}{3}\left(4R-6\left(\frac{4}{3}R\right)\right)=-\frac{8}{3}R<0$ $h=\frac{4}{3}R$ gives maximum V. At max V, length of the square base = $\sqrt{2}\sqrt{R^2-\left(\frac{4}{3}R-R\right)^2}=\sqrt{2}\sqrt{\frac{8}{9}R^2}=\frac{4}{3}R=h$ (Shown) since $R>0$

Q	Solutions
(iii)	Let the depth of water by H and length of square water surface by l. Using the result in **(ii)**, $l = H$ Volume of water, $V_w = \frac{1}{3}H\left(H^2\right) = \frac{1}{3}H^3$ $\frac{dV_w}{dH} = H^2$ When $H = 5$, $\frac{dV_w}{dt} = \frac{dV_w}{dH} \times \frac{dH}{dt}$ $10 = 5^2 \times \frac{dH}{dt}$ $\frac{dH}{dt} = 0.4$ units/s The water level H is increasing at 0.4 units/s.
8 (i)	$z^2 = (1+i)^2$ $= 1^2 + i^2 + 2i$ $= 1 - 1 + 2i$ $= 2i$ $z^3 = (2i)(1+i)$ $= 2i + 2i^2$ $= -2 + 2i$ $z^4 = (-2+2i)(1+i)$ $= -2 - 2i + 2i + 2i^2$ $= -2 - 2 = -4$

Q	Solutions
	or $z^4 = \left(z^2\right)^2 = \left(2\mathrm{i}\right)^2$ $= -4$ Since one of the root of the equation is $1+\mathrm{i}$, $\Rightarrow \left(1+\mathrm{i}\right)^4 + a\left(1+\mathrm{i}\right)^3 - 2\left(1+\mathrm{i}\right)^2 + 8\left(1+\mathrm{i}\right) - 8 = 0$ $\left(-4\right) + a\left(-2+2\mathrm{i}\right) - 2\left(2\mathrm{i}\right) + 8\left(1+\mathrm{i}\right) - 8 = 0$ $\Rightarrow -4 - 2a + 2a\mathrm{i} - 4\mathrm{i} + 8 + 8\mathrm{i} - 8 = 0$ $\left(-4-2a\right) + \left(2a+4\right)\mathrm{i} = 0$ Compare real **and** imaginary parts: [learning – consistent solution] $2a = -4$ and $2a = -4$ $\therefore a = -2$
(ii)	Since the coefficients of the equation are all real, the other root is $1-\mathrm{i}$ given that $1+\mathrm{i}$ is a root of the equation. $z^4 - 2z^3 - 2z^2 + 8z - 8 = 0$ $z^4 - 2z^3 - 2z^2 + 8z - 8 = (z-1-\mathrm{i})(z-1+\mathrm{i})(z^2+bz+c)$ By comparing constant terms, $c = -4$ $z^4 - 2z^3 - 2z^2 + 8z - 8 = (z^2-2z+2)(z^2+bz-4)$ Compare coefficient of z^2: $\Rightarrow -2b + 2 - 4 = -2$ $\Rightarrow b = 0$ $z^4 - 2z^3 - 2z^2 + 8z - 8 = (z^2-2z+2)(z^2-4)$

Q	Solutions
(iii)	$1-aiw+2w^2+8iw^3-8w^4=w^4\left[\frac{1}{w^4}-\frac{ai}{w^3}+\frac{2}{w^2}+\frac{8i}{w}-8\right]$ Let $z=\frac{i}{w}$ from (ii), $\frac{1}{w^4}-\frac{ai}{w^3}+\frac{2}{w^2}+\frac{8i}{w}-8=\left[\left(\frac{i}{w}\right)^2-2\left(\frac{i}{w}\right)+2\right]\left[\left(\frac{i}{w}\right)^2-4\right]$ $1-aiw+2w^2+8iw^3-8w^4=w^4\left[\left(\frac{i}{w}\right)^2-2\left(\frac{i}{w}\right)+2\right]\left[\left(\frac{i}{w}\right)^2-4\right]$ $=\left[i^2-2iw+2w^2\right]\left[-1-4w^2\right]$ $=\left[-1-2iw+2w^2\right]\left[-1-4w^2\right]$
(iv)	$\arg(z)=\frac{\pi}{4}$ $\arg(v)=\frac{\pi}{2}$ $\arg(zv)^3=3\arg(zv)$ $=3\left[\arg(z)+\arg(v)\right]$ $=3\left[\left(\frac{\pi}{4}\right)+\frac{\pi}{2}\right]$ $=\frac{9\pi}{4}$ $=\frac{\pi}{4}$, adjusted to principal argument range

Q	Solutions
9 (a)	$y=\dfrac{3x^2-2x+3}{x-1}=3x+1+\dfrac{4}{x-1}$ **Asymptotes:** $x=1$ (vertical asymptote) $y=3x+1$ (oblique asymptote) **Axial Intercepts:** $x=0, y=-3$ (-0.155, -2.93) is a maximum point and (2.15, 10.9) is a minimum point. 

Q	Solutions
	$y=3x+1$ $\qquad\qquad\qquad$ $x=1$
(b)	Intersection between C_1 and C_2: Using GC (graph) $(-1.4796,-5.0519)$
(c)	 $y_1=\left(-e^{2x}-5\right)$; $y_2=\dfrac{3x^2-2x+3}{x-1}$ Area of R

Q	Solutions
	$=\int_{k}^{-1.4796}\left[0-y_1\right]dx+\int_{-1.4796}^{0}\left[0-y_2\right]dx$ $=\int_{k}^{-1.4796}-\left(-e^{2x}-5\right)dx+\int_{-1.4796}^{0}-\left(\frac{3x^2-2x+3}{x-1}\right)dx$ $=\left[\frac{1}{2}e^{2x}+5x\right]_{k}^{-1.4796}+5.4366$ $=-7.3721-\frac{1}{2}e^{2k}-5k+5.4366$ $=-1.9355-\frac{1}{2}e^{2k}-5k$ $-1.9355-\frac{1}{2}e^{2k}-5k\geq 10$ $\frac{1}{2}e^{2k}+5k+11.9355\leq 0$ $(p=11.9355)$ $k\leq -2.39$ (using GC) Max $k=-2.39$
(d)	$V_R=\pi\int_{-1.4796}^{0}\left(\frac{3x^2-2x+3}{x-1}\right)^2dx+\pi\int_{-4}^{-1.4796}\left(-e^{2x}-5\right)^2dx$ $=263.5682$ $=263.57$ unit^3 (to 2 dec place)

10 (i) (a)	Given that L is the length of a fish at a particular time at t years, $\frac{dL}{dt} = \frac{dL_{in}}{dt} - \frac{dL_{out}}{dt}$ where $\frac{dL_{out}}{dt} = 0$ $\frac{dL}{dt} = k(60 - L), k > 0, \; 0 < L \leq 60$ At $L = 10$, $\frac{dL}{dt} = 5$ $5 = k(60 - 10)$ $k = \frac{5}{50} = \frac{1}{10}$ $\frac{1}{60 - L}\frac{dL}{dt} = \frac{1}{10}$ Integrating both sides with respect to t, $\int \frac{1}{60 - L}\,dL = \frac{1}{10}\int dt$ $-\int \frac{-1}{60 - L}\,dL = \frac{1}{10}\int dt$ $-\ln\lvert 60 - L\rvert = \frac{t}{10} + C$, where C is an arbitrary constant $-\ln(60 - L) = \frac{t}{10} + C$, since $L \leq 60$

	$\ln(60-L)=-\frac{t}{10}-C$ $60-L=\mathrm{e}^{-\frac{t}{10}-C}$ $=\mathrm{e}^{-C}\mathrm{e}^{-\frac{t}{10}}$ $=A\mathrm{e}^{-\frac{t}{10}}$, where $A=\mathrm{e}^{-C}$ $L=60-A\mathrm{e}^{-\frac{t}{10}}$ At $t=0$, $L=10$ $10=60-A\mathrm{e}^{(0)}=60-A$ $A=50$ $L=60-50\mathrm{e}^{-\frac{t}{10}}$
(i) (b)	At $t=5$, $L=60-50\mathrm{e}^{-\frac{5}{10}}=29.7$ cm (to 3 sig fig)
(ii) (a)	$\frac{\mathrm{d}L}{\mathrm{d}t}=\frac{50}{L}$ $L\frac{\mathrm{d}L}{\mathrm{d}t}=50$ Integrating both sides with respect to t: $\int L\,\mathrm{d}L=\int 50\,\mathrm{d}t$ $\frac{L^2}{2}=50t+D$, where D is an arbitrary constant $L^2=100t+E,\ E=2D$ When $t=0$, $L=10$, $100=E$ $L=\sqrt{100t+100}$, $L\geq 0$

(ii) (b)	*Either:* The second model is similar in validity to the model in (i) provided that the life span of the species of fish has a short life span, say between 0 – 2 years. [comparison of graph] *Or*: The second model is **not valid** compared to the model in (i) as the second model increases indefinitely which is not possible in real fishes.

11(i)

For the first drone: $l_A : \mathbf{r} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$

For the second drone: $l_B : \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \neq k \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ for any real values of k. Hence l_1 is not parallel to l_2 .

Assuming that the two lines intersect,

$\begin{pmatrix} \lambda \\ \lambda \\ \lambda \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 1-\mu \\ 1+\mu \end{pmatrix}$ for some $\lambda, \mu \in \mathbb{R}$

$\lambda = 2 + \mu$ --(1)

$\lambda = 1 - \mu$ --(2)

$\lambda = 1 + \mu$ --(3)

From (1) and (3), there will not be any values of λ, μ that will satisfy both questions. Hence the two lines will not meet. Combining the two conditions, the two flight paths are skew lines.

(ii)

Since point Q is on $l_B : \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$,

$\overrightarrow{OQ} = \begin{pmatrix} 2+\mu \\ 1-\mu \\ 1+\mu \end{pmatrix}$, for some $\mu \in \mathbb{R}$

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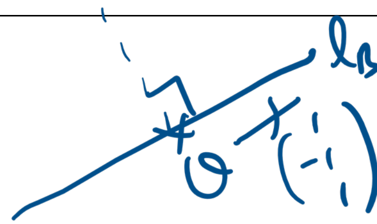
$\overrightarrow{OP}=\begin{pmatrix}5\\5\\5\end{pmatrix}$



$\overrightarrow{PQ}=\begin{pmatrix}2+\mu\\1-\mu\\1+\mu\end{pmatrix}-\begin{pmatrix}5\\5\\5\end{pmatrix}=\begin{pmatrix}-3+\mu\\-4-\mu\\-4+\mu\end{pmatrix}$

For B to be at the shortest distance from A, $\overrightarrow{PQ}$ is perpendicular to l_B .

$\begin{pmatrix}-3+\mu\\-4-\mu\\-4+\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\1\end{pmatrix}=0$

$-3+\mu+4+\mu-4+\mu=0$

$-3+3\mu=0$

$\mu=1$

Hence, the minimum value for $\left|\overrightarrow{PQ}\right|$ occurs when $\mu=1$.

$\overrightarrow{OQ}=\begin{pmatrix}2+1\\1-1\\1+1\end{pmatrix}=\begin{pmatrix}3\\0\\2\end{pmatrix}$

Hence, Q (3 , 0 , 2).

Minimum $\left|\overrightarrow{PQ}\right|=\sqrt{38}$ units

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Since point Q is on $l_B:\mathbf{r}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$, $\mu\in\mathbb{R}$,

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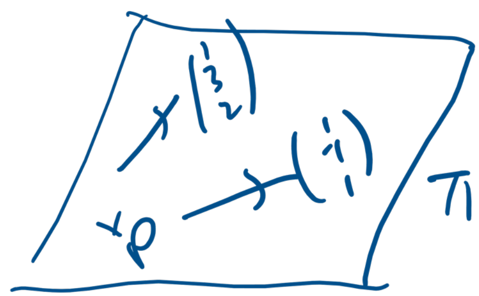
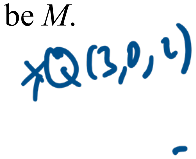
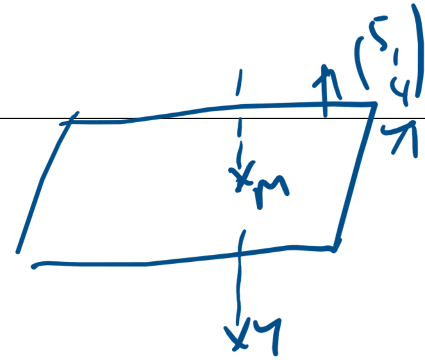
	$\overrightarrow{OQ}=\begin{pmatrix}2+\mu\\1-\mu\\1+\mu\end{pmatrix}$, for some $\mu\in\mathbb{R}$ $\overrightarrow{OP}=\begin{pmatrix}5\\5\\5\end{pmatrix}$ $\overrightarrow{PQ}=\begin{pmatrix}2+\mu\\1-\mu\\1+\mu\end{pmatrix}-\begin{pmatrix}5\\5\\5\end{pmatrix}=\begin{pmatrix}-3+\mu\\-4-\mu\\-4+\mu\end{pmatrix}$ $\left\|\overrightarrow{PQ}\right\|=\left\|\begin{pmatrix}-3+\mu\\-4-\mu\\-4+\mu\end{pmatrix}\right\|$ $=\sqrt{(-3+\mu)^2+(-4-\mu)^2+(-4+\mu)^2}$ $=\sqrt{9-6\mu+\mu^2+16+8\mu+\mu^2+16-8\mu+\mu^2}$ $=\sqrt{3\mu^2-6\mu+41}$ $=\sqrt{3(\mu^2-2\mu)+41}$ $=\sqrt{3\left[(\mu-1)^2-1\right]+41}$ $=\sqrt{3(\mu-1)^2+38}$ For the distance between *P* and *Q* to be the shortest, we can minimize the function $3(\mu-1)^2+38$. Hence, the minimum value for $\left\|\overrightarrow{PQ}\right\|$ occurs when $\mu=1$.

	$\overrightarrow{OQ}=\begin{pmatrix}2+1\\1-1\\1+1\end{pmatrix}=\begin{pmatrix}3\\0\\2\end{pmatrix}$ Hence, $Q\,(3\,,0\,,2)$. Minimum $\left\lvert\overrightarrow{PQ}\right\rvert=\sqrt{38}$ units
(iii)	$\begin{pmatrix}1\\3\\2\end{pmatrix}\times\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}3-(-2)\\-(1-2)\\-1-3\end{pmatrix}$ $=\begin{pmatrix}5\\1\\-4\end{pmatrix}$ $\Pi:\mathbf{r}\bullet\begin{pmatrix}5\\1\\-4\end{pmatrix}=\begin{pmatrix}5\\5\\5\end{pmatrix}\bullet\begin{pmatrix}5\\1\\-4\end{pmatrix}$ $\mathbf{r}\bullet\begin{pmatrix}5\\1\\-4\end{pmatrix}=25+5-20$ $\mathbf{r}\bullet\begin{pmatrix}5\\1\\-4\end{pmatrix}=10$ 
(iv)	Let the foot of perpendicular of Q on the plane Π be M. $l_{QM}:\mathbf{r}=\begin{pmatrix}3\\0\\2\end{pmatrix}+s\begin{pmatrix}5\\1\\-4\end{pmatrix},\ s\in\mathbb{R}$ 

$\overrightarrow{OM} = \begin{pmatrix} 3+5s \\ s \\ 2-4s \end{pmatrix}$ for some $s \in \mathbb{R}$



Since M lies on the plane Π ,

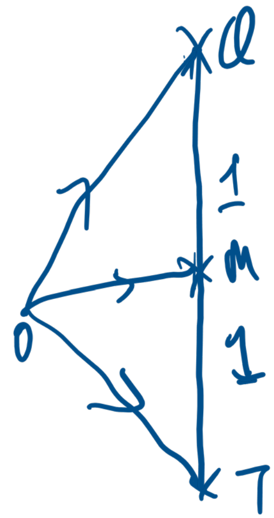
$$\begin{pmatrix} 3+5s \\ s \\ 2-4s \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} = 10$$

$$15 + 25s + s - 8 + 16s = 10$$

$$42s + 7 = 10$$

$$42s = 3$$

$$s = \frac{1}{14}$$



$$\overrightarrow{OM} = \begin{pmatrix} 3+5\left(\frac{1}{14}\right) \\ \left(\frac{1}{14}\right) \\ 2-4\left(\frac{1}{14}\right) \end{pmatrix} = \begin{pmatrix} \frac{47}{14} \\ \frac{1}{14} \\ \frac{12}{7} \end{pmatrix}$$

By Ratio Theorem,

	$\overrightarrow{OM}=\dfrac{\overrightarrow{OQ}+\overrightarrow{OT}}{2}$ $\overrightarrow{OT}=2\overrightarrow{OM}-\overrightarrow{OQ}$ $=2\begin{pmatrix}\frac{47}{14}\\\frac{1}{14}\\\frac{12}{7}\end{pmatrix}-\begin{pmatrix}3\\0\\2\end{pmatrix}$ $=\begin{pmatrix}\frac{26}{7}\\\frac{1}{7}\\\frac{10}{7}\end{pmatrix}$

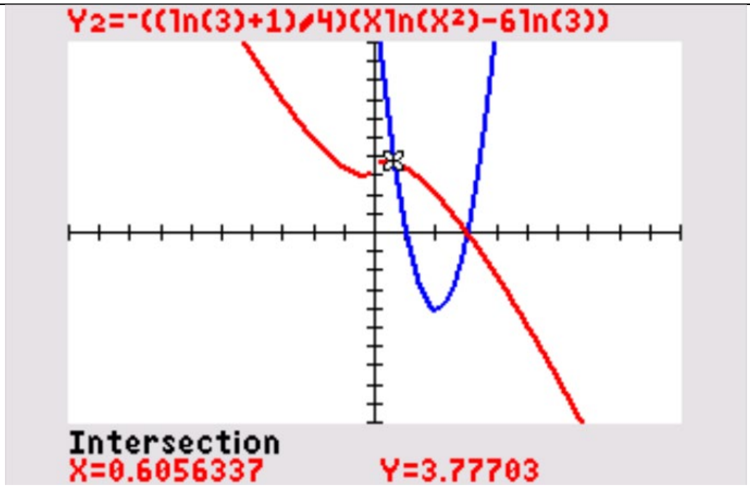
St Andrew's Junior College
2024 Preliminary Examinations H2 Mathematics Paper 2 (9758/02)

Q	Solutions
1 (i)	Terms of the G.P. $a+6d, \quad a+3d, \quad a+d, \cdots$ Since the G.P. has a constant common ratio r, $\dfrac{a+d}{a+3d}=\dfrac{a+3d}{a+6d}=r$ $\Rightarrow (a+3d)^2=(a+d)(a+6d)$ $\Rightarrow a^2+6ad+9d^2=a^2+7ad+6d^2$ $\Rightarrow 3d^2-ad=0$ $\Rightarrow d(3d-a)=0$ $\Rightarrow d=\dfrac{a}{3}$ or $d=0$ (rejected since $d \neq 0$)
(ii)	$r=\dfrac{a+d}{a+3d}$ $=\dfrac{a+\left(\frac{a}{3}\right)}{a+3\left(\frac{a}{3}\right)}$ $=\dfrac{2}{3}$ Since $\lvert r\rvert=\dfrac{2}{3}<1$, the geometric series is convergent.

Q	Solutions
	Sum to infinity $= \dfrac{a+6d}{1-\frac{2}{3}}$ $= \dfrac{a+6\left(\frac{a}{3}\right)}{1-\frac{2}{3}} = 9a$
(iii)	Given $\left\|S_n - S_\infty\right\| < \frac{1}{100}S_\infty$ $\left\| \dfrac{3a\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}} - 9a \right\| < \frac{1}{100}(9a)$ $\left\|\left(\frac{2}{3}\right)^n\right\| = \left(\frac{2}{3}\right)^n < \frac{1}{100}$ $n\ln\left(\frac{2}{3}\right) < \ln\left(\frac{1}{100}\right)$ $n > 11.358$ $\left\{n \in \mathbb{Z}^+ : n \geq 12\right\}$

Q	Solutions
2(i)	$x = t\ln t^2$, $\quad y = 4(t-2)^2$ $\dfrac{dx}{dt} = \ln t^2 + t\left(\dfrac{1}{t^2}\right)(2t)$ $\quad \dfrac{dy}{dt} = 8(t-2)$ $= \ln t^2 + 2$ $\therefore \dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt} = \dfrac{8t-16}{\ln t^2 + 2}$ When $t = 3$, we have $P(6\ln 3, 4)$ $\dfrac{dy}{dx} = \dfrac{8(3)-16}{\ln(3)^2+2} = \dfrac{8}{\ln 9 + 2} = \dfrac{8}{2\ln 3 + 2} = \dfrac{4}{\ln 3 + 1}$ Gradient of normal at $P = -1 \div \dfrac{4}{\ln 3 + 1}$ $= -\dfrac{\ln 3 + 1}{4}$ Equation of normal at P: $y - 4 = -\dfrac{\ln 3 + 1}{4}(x - 6\ln 3)$ $4y - 16 = -(\ln 3 + 1)x + (6\ln 3)(\ln 3 + 1)$ $4y + (\ln 3 + 1)x = 6(\ln 3)^2 + 6\ln 3 + 16$
(ii)	Substitute $x = t\ln t^2$, $y = 4(t-2)^2$ into $y - 4 = -\dfrac{\ln 3 + 1}{4}(x - 6\ln 3)$ We have $4(t-2)^2 - 4 = -\dfrac{\ln 3 + 1}{4}(t\ln t^2 - 6\ln 3)$ Using G.C.,

Q	Solutions



$t = 0.60563$ or $t = 3$

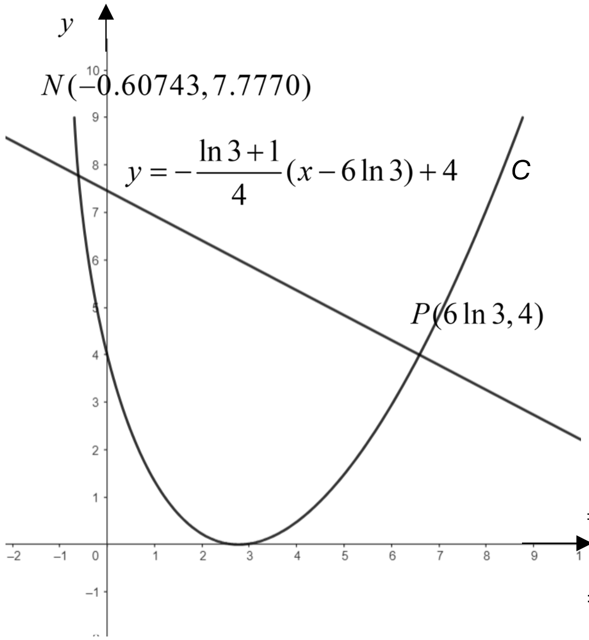
(point N) (point P)

At point N,

$x = 0.60563\ln(0.60563^2)$ $y = 4(0.60563 - 2)^2$

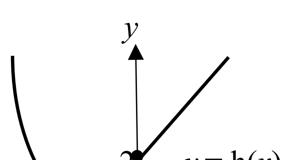
$= -0.60743$ $= 7.7770$

$\therefore N(-0.607, 7.78)$

Q	Solutions
(iii)	 Area bounded by curve C and normal $=\int_{-0.60743}^{6\ln 3} y\ \mathrm{d}x-\int_{-0.60743}^{6\ln 3} y_C\ \mathrm{d}x$ $=\int_{-0.60743}^{6\ln 3} -\frac{\ln 3+1}{4}(x-6\ln 3)+4\ \mathrm{d}x-\int_{0.60563}^{3} 4(t-2)^2\ \frac{\mathrm{d}x}{\mathrm{d}t}\ \mathrm{d}t$ $=\int_{-0.60743}^{6\ln 3} -\frac{\ln 3+1}{4}(x-6\ln 3)+4\ \mathrm{d}x-\int_{0.60563}^{3} 4(t-2)^2\ (\ln t^2+2)\ \mathrm{d}t$ $=42.392-11.979$ $=30.4$ units^2 (3 s.f.)

Q	Solutions
3 (i)	Let $y=\dfrac{ax+b}{x-2}$ $xy-2y=ax+b$ $xy-ax=2y+b$ $x=\dfrac{2y+b}{y-a}$ $f^{-1}(y)=\dfrac{2y+b}{y-a}$ $f^{-1}(x)=\dfrac{2x+b}{x-a}, x\neq a$ Since f is self-inverse, $\dfrac{ax+b}{x-2}=\dfrac{2x+b}{x-a}$ $(ax+b)(x-a)=(x-2)(2x+b)$ $ax^2+(-a^2+b)x-ab=2x^2+(b-4)x-2b$ Comparing coefficient of x^2 : $a=2$ Since range of f^{-1} = domain of f , $\dfrac{2x+b}{x-2}\neq 2$ $2x+b\neq 2x-4$ $b\neq -4$ Alternatively: when $b=-4$, $f(x)=\dfrac{2x-4}{x-2}=\dfrac{2(x-2)}{x-2}=2$, f^{-1} does not exist. Hence $b\neq -4$.

Q	**Solutions**
(ii)	$R_f = D_{f^{-1}} = D_f$, ff exists. $R_f = D_f = D_{ff}$, f^3 exists. $R_f = D_f = D_{fff}$, f^4 exists. ... $R_f = D_f = D_{fff...f}$, f^n exists for all integers $n > 1$.
(iii)	$f^{2025}(1)$ $= f^{2023}\, f\, f^{-1}(1)$ since $f(1) = f^{-1}(1)$ $= f^{2023}(1)$ since $f\, f^{-1}(1) = 1$ $= f^{2021}(1)$ ... $= f(1)$ $= -2 - b$ Alternatively: $f^{2025}(1)$ $= f^{2021}\, f^2\, f^2(1)$ since $f(1) = f^{-1}(1) \Rightarrow f^2(1) = 1$ $= f^{2023}\, f^2(1)$ $= f^{2021}(1)$ ... $= f(1)$ $= -2 - b$
(iv)	

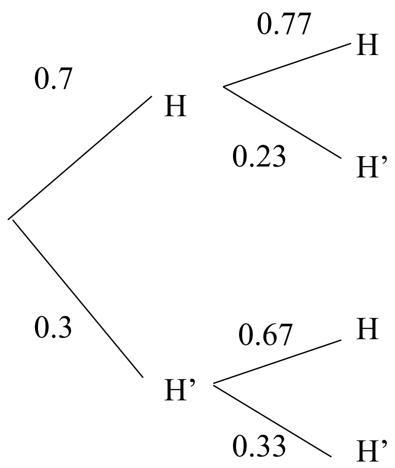


Q	Solutions
(v)	$\mathrm{f}\left(\frac{1}{2}\right)=\dfrac{2\left(\frac{1}{2}\right)+1}{\frac{1}{2}-2}=-\frac{4}{3}$ Since $\mathrm{f}\left(\frac{1}{2}\right)=-\frac{4}{3}<0,\ \ \mathrm{h}\left(\mathrm{f}\left(\frac{1}{2}\right)\right)=\mathrm{h}\left(-\frac{4}{3}\right)=2\left(-\frac{4}{3}\right)^{2}=\frac{32}{9}$
4 (i)	$y=\ln\left(\cos x\right)----(1)$ Therefore, $e^{y}=\cos x$ Differentiate with respect to x, $e^{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=-\sin x----(2)$ Differentiate with respect to x, $e^{y}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^{2}e^{y}=-\cos x$ $e^{y}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^{2}e^{y}=-e^{y}$

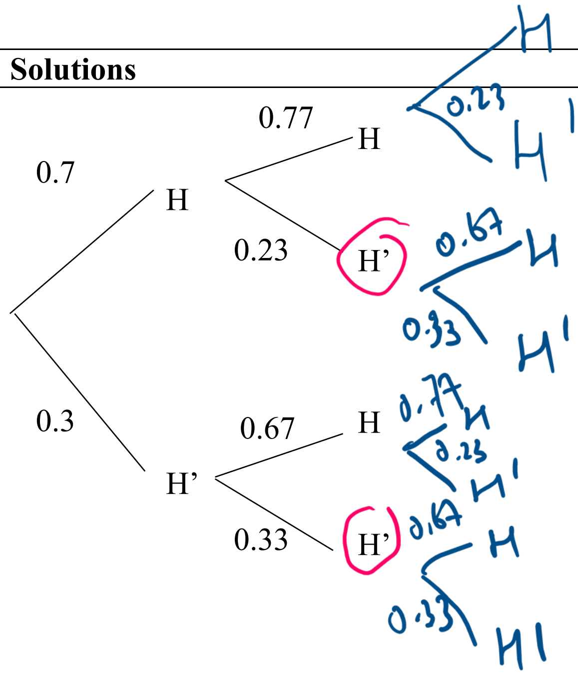
Q	Solutions
	$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-1----(3)$ $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$ (shown) <u>Alternative Method</u> $y=\ln(\cos x)$ $\frac{dy}{dx}=\frac{1}{\cos x}(-\sin x)$ $=-\frac{\sin x}{\cos x}$ $\frac{d^2y}{dx^2}=-\frac{\cos x(\cos x)-\sin x(-\sin x)}{\cos^2 x}$ $=-\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$ $=-1-\frac{\sin^2 x}{\cos^2 x}$ $=-1-\left(\frac{dy}{dx}\right)^2$ $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-1$ $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$ (shown)
(ii)	Differentiate with respect to x, $\frac{d^3y}{dx^3}+2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)=0$ (shown) $----(4)$

Q	Solutions
	Differentiate with respect to x, $\frac{d^4y}{dx^4}+2\left[\frac{d^2y}{dx^2}\bullet\frac{d^2y}{dx^2}+\frac{d^3y}{dx^3}\bullet\frac{dy}{dx}\right]=0$ $\frac{d^4y}{dx^4}+2\left(\frac{dy}{dx}\right)\left(\frac{d^3y}{dx^3}\right)+2\left(\frac{d^2y}{dx^2}\right)^2=0-----(5)$ Let $y=\ln(\cos x)=f(x)$ When $x=0$, From (1), $y=f(0)=\ln(\cos 0)=0$ From (2), $\frac{dy}{dx}=f'(0)=0$ From (3), $\frac{d^2y}{dx^2}=f''(0)=-(0)^2-1=-1$ From (4), $\frac{d^3y}{dx^3}=f'''(0)=-2(0)x(-1)=0$ From (5), $\frac{d^4y}{dx^4}=f^4(0)=-2\left[(-1)^2+(0)-(0)\right]=-2$ Using Maclaurin's expansion, $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^4(0)}{4!}x^4+\ldots$ $=-\frac{1}{2}x^2-\frac{1}{12}x^4+\ldots$ $\ln(\cos x)=-\frac{1}{2}x^2-\frac{1}{12}x^4+\ldots$

Q	Solutions
(iii)	$\int_0^{0.08} \ln\left(\frac{\cos^3 x}{1-x}\right) \mathrm{d}x$ $= \int_0^{0.08} \left[\ln(\cos x)^3 - \ln(1-x)\right] \mathrm{d}x$ $= \int_0^{0.08} \left[3\ln(\cos x) - \ln(1-x)\right] \mathrm{d}x$ $\approx \int_0^{0.08} \left[3\left(-\frac{1}{2}x^2 - \frac{1}{12}x^4\right) - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\right)\right] \mathrm{d}x$ $= 0.0030327$ (to 7 decimal places)
(iv)	The two values are the same up to 6 decimal places, and the x value is between 0 and 0.08, which is relatively small and close to 0. Hence, the approximation is relatively accurate for the integral.
5 (i)	Let H be the event that Mike hits his target on that throw. First throw Second throw

<table>
<tr><th>Q</th><th>Solutions</th></tr>
<tr><td></td><td>
</td></tr>
<tr><td>(ii)</td><td>$\text{P}\left(\text{Mike hits his target on the second throw of the dart}\right)$
$= P(H \cap H) + P(H' \cap H)$
$= (0.7)(0.77) + (0.3)(0.67)$
$= 0.539 + 0.201$
$= 0.74$</td></tr>
<tr><td>(iii)</td><td>0.77</td></tr>
</table>

Q	Solutions



Given that Mike did not hit his target on the second throw, find the probability that he hit the target twice

$\mathrm{P}(\text{hitting target twice} \mid \text{did not hit target on second throw})$

$= \dfrac{\mathrm{P}(H \cap H' \cap H)}{1 - 0.74}$

$= \dfrac{0.7 \times 0.23 \times 0.67}{0.26}$

$= 0.41488$

$= 0.415$ (to 3 sig fig)

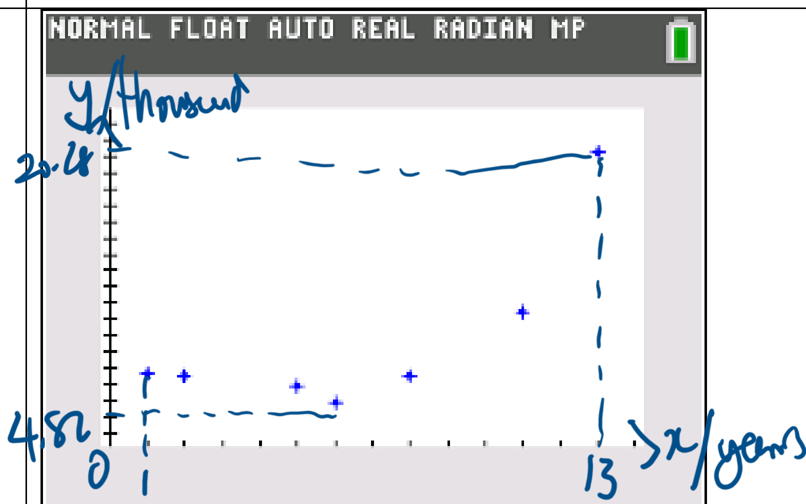
6 (i) Let M be the random variable denoting Timothy's score in a game.

$2^{nd} \backslash 1^{st}$ card	3	5
3	9	15
5	15	25

Q	Solutions
	$\mathrm{P}(M=9)$ $=\mathrm{P}(\text{both cards drawn are 3})$ $=\dfrac{n}{n+4}\times\dfrac{n-1}{n+3}=\dfrac{n(n-1)}{(n+4)(n+3)}$ $\mathrm{P}(M=25)$ $=\mathrm{P}(\text{both cards drawn are 5})$ $=\dfrac{4}{n+4}\times\dfrac{3}{n+3}=\dfrac{12}{(n+4)(n+3)}$ Given that M is a discrete random variable, $\sum_{\text{all } m}\mathrm{P}(M=m)=1$ $\mathrm{P}(M=15)$ $=1-\dfrac{12}{(n+4)(n+3)}-\dfrac{n(n-1)}{(n+4)(n+3)}$ $=\dfrac{(n+4)(n+3)-12-n(n-1)}{(n+4)(n+3)}$ $=\dfrac{n^2+7n+12-12-n^2+n}{(n+4)(n+3)}$ $=\dfrac{8n}{(n+4)(n+3)}$
(ii)	Given $W=\lvert M-15\rvert$, <table><tr><td>m</td><td>9</td><td>15</td><td>25</td></tr><tr><td>w</td><td>6</td><td>0</td><td>10</td></tr></table>

Q	Solutions
	<table><tr><td>$P(W=w)$</td><td>$\frac{n(n-1)}{(n+4)(n+3)}$</td><td>$\frac{8n}{(n+4)(n+3)}$</td><td>$\frac{12}{(n+4)(n+3)}$</td></tr></table> $E(W)=\frac{100}{23}$ $\sum_{\text{all } w} wP(W=w)=\frac{100}{23}$, where $W=\lvert M-15\rvert$ $\frac{6n(n-1)}{(n+4)(n+3)}+\frac{10(12)}{(n+4)(n+3)}=\frac{100}{23}$ $138n(n-1)+2760=100(n+4)(n+3)$ $138n^2-138n+2760=100(n^2+7n+12)$ $38n^2-838n+1560=0$ From G.C., Since $n\in\mathbb{Z}^+$, $n=20$
(iii)	$\text{Var}(W)$ $=E(W^2)-[E(W)]^2$ $=\sum_{\text{all } w} w^2P(W=w)-\left[\frac{100}{23}\right]^2$ $=(6)^2\left(\frac{20(19)}{24(23)}\right)+(10)^2\left(\frac{12}{24(23)}\right)-\left[\frac{100}{23}\right]^2$ $=\frac{4260}{529}$ Given that n = 40 is sufficiently large, by Central Limit Theorem,

Q	Solutions
	$\overline{W} \sim \mathrm{N}\left(\frac{100}{23}, \frac{213}{1058}\right)$ approximately. $\mathrm{P}\left(\overline{W} < 4.50\right) = 0.63275 = 0.633$ (to 3 sig fig)
7	Method 1 : Let the event M be such that the Marketing staff are seated together and E be such that the Sales staff are seated together. $n(M \cup E) = n(M) + n(E) - n(M \cap E)$ $= \frac{(6-1)!}{3!} + \frac{(5-1)!}{2!} - (4-1)!$ $= 26$ Number of ways $= n(M' \cap E')$ $=$ Total no of ways without restrictions $-n(M \cup E)$ $= \frac{(7-1)!}{2!3!} - 26$ $= 34$ Method 2: Case 1a : 2 marketing staff together and 3 sales staff together Number of ways=$(4-1)!$=6 Case 1b : 2 marketing staff together and 3 sales staff separated Number of ways=$(3-1)!$=2 Case 1c : 2 marketing staff together and any 2 sales staff together but separated from the third sales staff

Q	Solutions
	Number of ways=$(4-1)!\times 2$ or $(3-1)!\times {}^3C_2\times 2$ $=12$ Casel 2 : 3 sales staff together and 2 marketing staff separated Number of ways= $(3-1)!\times {}^3C_2=6$ Total number of ways $=\dfrac{(7-1)!}{2!3!}-6-2-12-6$ $=34$
8 (a)	
(b)	Product moment correlation coefficient, r, between x^2 and y is 0.878. Product moment correlation coefficient, r, between e^x and y is 0.978. (both to 3 sig fig) (From the scatter diagram, y does not increase at a constant rate.)

<table>
<tr><th>Q</th><th>Solutions</th></tr>
<tr><td></td><td>Based on the product moment correlation coefficients, the $|r|$ value between e^x and y is closer to 1 compared to the one between x^2 and y, indicating a stronger positive linear correlation between e^x and y.

Hence, $y = ce^x + d$ is a better model.
From GC:
$y = \left(3.2067\times10^{-5}\right)e^x + 6.3785$
$\quad = \left(3.21\times10^{-5}\right)e^x + 6.38$ (to 3 sig fig)</td></tr>
<tr><td>(c)</td><td>When $x = 15$,
$y = \left(3.2067\times10^{-5}\right)e^{15} + 6.3785$
$\quad = 111.206 = 111.21$
Therefore, the number of cars is 111.21 thousands.
As $x = 15$ is outside of the data range $1 \le x \le 13$, the value obtained is an extrapolation. Hence, the estimation will not be a reliable one.</td></tr>
<tr><td>(d)</td><td>No, the product moment correlation will not be affected since it is only a linear transformation of the data for x.</td></tr>
<tr><td>9
(i)</td><td>The probability of an egg cracked during transportation is <u>constant</u> at <u>0.03</u> for each egg.

The <u>event</u> of an egg cracking during transportation is independent of the <u>event</u> of any other egg cracking during transportation.</td></tr>
<tr><td>(ii)</td><td>Let X be the random variable "the number of eggs cracked during transportation out of n on one tray".
$X \sim B(n, 0.03)$
Given $P(X \le 2) \ge 0.91$
using GC,
<table><tr><th>n</th><th>$P(X \le 2)$</th></tr><tr><td>34</td><td>$0.9188 > 0.91$</td></tr><tr><td>35</td><td>$0.9131 > 0.91$</td></tr><tr><td>36</td><td>$0.9072 < 0.91$</td></tr></table></td></tr>
</table>

Q	Solutions
	Hence the largest possible value of n is 35.
(iii)	Let Y be the random variable denoting the number of trays with more than 2 cracked eggs out of 30 eggs on 40 trays in 1 carton. $Y \sim \mathrm{B}(40,\ \mathrm{P}(X \geq 3))$ $Y \sim \mathrm{B}(40, 0.060069)$ P(a carton of eggs being rejected by supermarket) $= \mathrm{P}(Y \geq 3)$ $= 1 - \mathrm{P}(Y \leq 2)$ $= 0.43426$ $= 0.434$ (to 3 sf)
10 (i)	Let X be the mass of **a** randomly chosen circular block in grams. $X \sim \mathrm{N}\left(15, 2^2\right)$ Let Y be the mass of **a** randomly chosen rectangular block in grams. $Y \sim \mathrm{N}\left(100, 4^2\right)$ $\mathrm{E}\left(X_1 + X_2 + X_3 + X_4 + Y\right) = 60 + 100 = 160$ $\mathrm{Var}\left(X_1 + X_2 + X_3 + X_4 + Y\right) = 16 + 4^2 = 32$ Let $T = X_1 + X_2 + X_3 + X_4 + Y \sim \mathrm{N}\left(160, 32\right)$ $\mathrm{P}(150 < T < 170) = 0.92290 \approx 0.923$
(ii)	$\mathrm{E}(X_1 + X_2 + ... + X_n) = n(15) = 15n$ $\mathrm{Var}(X_1 + X_2 + ... + X_n) = n(2^2) = 4n$ Let $S = X_1 + X_2 + ... + X_n \sim \mathrm{N}\left(15n, 4n\right)$ $\mathrm{P}(S > 65) \geq 0.105$

n	$\mathrm{P}(S > 65)$

Q	Solutions

3	3.89×10^{-9}
4	0.10565
5	0.98733

Least $n=4$

Alternative Method

$P(S>65)\geq 0.105$

$P(\frac{S-15n}{\sqrt{4n}}>\frac{65-15n}{\sqrt{4n}})\geq 0.105$

$P(Z>\frac{65-15n}{\sqrt{4n}})\geq 0.105$, $Z\sim N(0,1)$

$\frac{65-15n}{\sqrt{4n}}\leq 1.2536$

$65-15n\leq 2.5072\sqrt{n}$

n	65-15n	$2.5072\sqrt{n}$
3	20	4.3426
4	5	5.0144
5	-10	5.6063

Least $n=4$

(iii) Assume that the distribution of the masses of all the blocks are independent of one another

(iv) $A=1.1X$

$E(1.1X)=1.1E(X)=1.1(15)=16.5$

$Var(1.1X)=(1.1)^2\,Var(X)$

$=(1.1)^2(2^2)=4.84$

$A\sim N(16.5,4.84)$

Q	Solutions
	$\mathrm{E}\left[Y-\left(A_1+A_2+A_3+A_4\right)\right]$ $=\mathrm{E}(Y)-(4)\mathrm{E}(A)$ $=100-(4)(16.5)=34$ $\mathrm{Var}\left[Y-\left(A_1+A_2+A_3+A_4\right)\right]$ $=\mathrm{Var}(Y)+(4)\mathrm{Var}(A)$ $=16+(4)(4.84)=35.36$ $Y-\left(A_1+A_2+A_3+A_4\right)\sim \mathrm{N}(34,35.36)$ Required probability $=\mathrm{P}\left(Y\geq\left(A_1+A_2+A_3+A_4\right)+40\right)$ $=\mathrm{P}\left(Y-\left(A_1+A_2+A_3+A_4\right)\geq 40\right)$ $=0.156$ (to 3 sig fig)
(v)	$A\sim N(16.5,4.84)$ $\mathrm{P}(A>18)=0.24768$ Require probability $=\left[\mathrm{P}\left(A>18\right)\right]^2\left[\mathrm{P}\left(A<18\right)\right]^3\times\frac{5!}{2!3!}$ $=\left(0.24768\right)^2\left(1-0.24768\right)^3\times\frac{5!}{2!3!}$ $=0.26121\approx 0.261$ <u>Alternative Method</u> $A\sim N(16.5,4.84)$ $\mathrm{P}(A>18)=0.24768$ Let C be the random variable denoting the number of circular blocks that passes the inspection, out of 5 circular blocks.

Q	Solutions
	$C \sim B(5, 0.24768)$ $P(C=2) = 0.261$
11 (i)	Let X be the time, in hours, that a randomly chosen student spends studying in Shelton College, and μ be the population mean. Unbiased estimate of the population variance $s^2 = \frac{50}{49}(0.46)^2 = 0.21592 \approx 0.216$ (3 sig. fig.) Test $H_o : \mu = 4$ against $H_1 : \mu \neq 4$ Under H_o, since sample size = 50 is large, by Central Limit Theorem, $\overline{X} \sim N\left(4, \frac{0.21592}{50}\right)$ approximately Use a two-tailed test at 5% level of significance. Using GC, with $\overline{x} = 3.8$, $s = \sqrt{0.21592}$, n =50, z value $= -3.0435$ p-value $= 0.0023387 \leq 0.05$ We **reject** H_o and conclude that there is **sufficient evidence** at 5% significance level to conclude that students **do not spend an average of 4 hours** studying in college.
(ii)	The p-value of 0.00234 is twice the probability that sample mean time spent by students studying in the college per week is at most 3.8 hours when the (population) mean time spent by students studying in the college per week is 4 hours.
(iii)	The test would no longer be valid because the sample size 15 is small (and the distribution of the time spent by students studying in the college is not given be to normal), therefore Central Limit Theorem could not be used . [cf N2021/2/Q8]
(iv)	Unbiased estimate of population mean, $\overline{x} = 4.2125$ (exact, from GC) Test $H_o : \mu = \mu_0$ against $H_1 : \mu < \mu_0$ at 5% level of significance

Q	Solutions
	Under H_o, $\overline{X} \sim N\left(\mu_0, \frac{0.6^2}{8}\right)$ $Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$ Since there is insufficient evidence that Ms Lam has overstated the average time, we do not reject H_0 at 5% (z-calc does not lie in critical region) $z_{calc} = \frac{4.2125 - \mu_0}{0.6/\sqrt{8}} > -1.6449$ $4.2125 - \mu_0 > -1.6449\left(\frac{0.6}{\sqrt{8}}\right)$ $\mu_0 < 4.56144$ hrs = 4.56 h Largest possible value of μ_0 is 4.56 h.