Name	Register Number	Class	Calculator Model

MANJUSRI SECONDARY SCHOOL

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智行慈願

PRELIMINARY EXAMINATION 2021

Subject:	Additional Mathematics
Paper:	4049/01
Level:	Secondary 4 Express
Date:	26 August 2021
Duration:	2 hours 15 minutes
Setter:	Mdm Neo Chai Meng

READ THESE INSTRUCTIONS FIRST

Write your Name, Register Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Marks Obtained
/90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

$$\Delta = \frac{1}{2}bc\sin A$$
.

Answer **all** the questions.

1 Given that $\frac{p^{2x}}{q^{3-4x}} \times \frac{q^{y-1}}{\left(p^{y+3}\right)^2} = p^2q^7$, find the value of *x* and of *y*. [4]

2 The volume of a right square pyramid of length $\left(3+\sqrt{2}\right)$ cm is $\frac{1}{3}\left(29-2\sqrt{2}\right)$ cm^3. Without using a calculator, find the height of the pyramid in the form $\left(a+b\sqrt{2}\right)$ cm, where a and b are integers. [5]

3 **(i)** Given $\frac{3\lg 3x - 2\lg x}{4} = \lg 3$, find the value of *x*. [3]

(ii) Given $\log_{(x-2)} y = 2$ and $\log_y (x+k) = \frac{1}{2}$, find the value of *k* where *k* is an integer. [3]

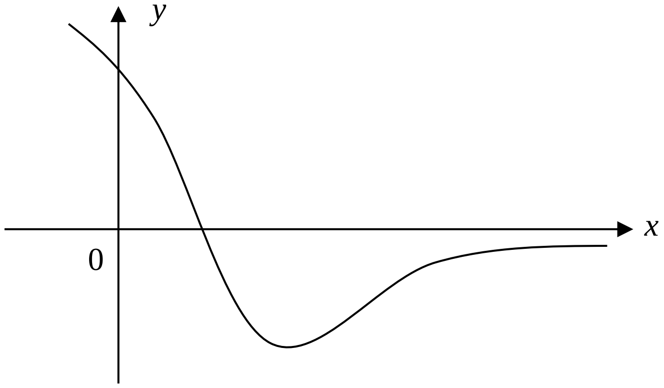
4 It is estimated that the percentage of Singapore's population (P) responding to the Save Electricity Campaign is given by the equation $P = 100\,(1 - e^{-0.05t})$, where t is the number of weeks after the launch of the campaign.

(i) Find the percentage of population responding to the Save Electricity Campaign after 8 weeks of its launch. [1]

(ii) Form an inequality to find the number of weeks it takes for at least 65% of the population to respond after the launch of the Campaign. [3]

(iii) Is it possible for the percentage of Singapore's population responding to the Campaign to reach 100%? Explain your answer by showing your working clearly. [2]

5 The diagram shows part of the graph $y = \dfrac{2-3x}{e^x}$.



Find the

(i) set of values of *x* for which *y* is a decreasing function of *x*. [4]

(ii) the gradient of the curve $y = \dfrac{2-3x}{e^x}$ at the point where it cuts the *x*-axis. [2]

6 **(i)** By expressing cos 3*x* as cos (2*x* + *x*), show that $\cos 3x = 4\cos^3 x - 3\cos x$. [3]

(ii) Hence, in radians, find angles of x between 0 and 2π which satisfies the equation $2\cos 3x - 8\cos^3 x + 5 = 0.$ [4]

7 **(a)** The acute angles A and B are such that $\cos(A - B) = \frac{7}{12}$ and $\cos A \cos B = \frac{1}{4}$.
Without using a calculator, find the exact value of

(i) $\sin A \sin B$, [2]

(ii) $\tan A \tan B$. [2]

(b) Without using a calculator, show that $\tan 15° = 2 - \sqrt{3}$. [3]

8 **(i)** Express $\dfrac{7x+27}{12+5x-2x^2}$ in partial fractions. [3]

(ii) Given that $\int_{-1}^{3} \frac{7x+27}{12+5x-2x^2}\,\mathrm{d}x = a\ln a + b\ln b,$ calculate the value of $a+b.$ [5]

9 **(a)** The term containing the highest power of x in the polynomial $\text{f}(x)$ is $-2x^4$.

The roots of the equation $\text{f}(x) = 0$ are -2, 1 and a repeated root k, where $k > 0$.

Given that $\text{f}(x)$ has a remainder of 64 when divided by $x + 1$.

(i) Show that the value of $k = 3$. [3]

(ii) Hence, find an expression for $\text{f}(x)$ in descending powers of x. [2]

(b) Solve the equation $2x^3 - 5x^2 - x + 6 = 0$. [4]

10 Given that circle *A*, whose equation is $x^2 + y^2 + 4x - 2y - 15 = 0$, has centre *C* and radius *r*.

(i) Find the coordinates of *C* and the value of *r*. [3]

(ii) Determine, with reason and working, if $(1, 3)$ is a point inside the circle. [2]

(iii) Find the equation of the tangent to the circle A at the point $(2, 3)$. [3]

(iv) The points M and N lie on the circle. Find the length of the chord MN which cuts the y-axis. [2]

11 A particle moves in the straight line such that t seconds after leaving a fixed point O, its velocity v m/s is given by $v = 5(3t-2)^2 - 45$.

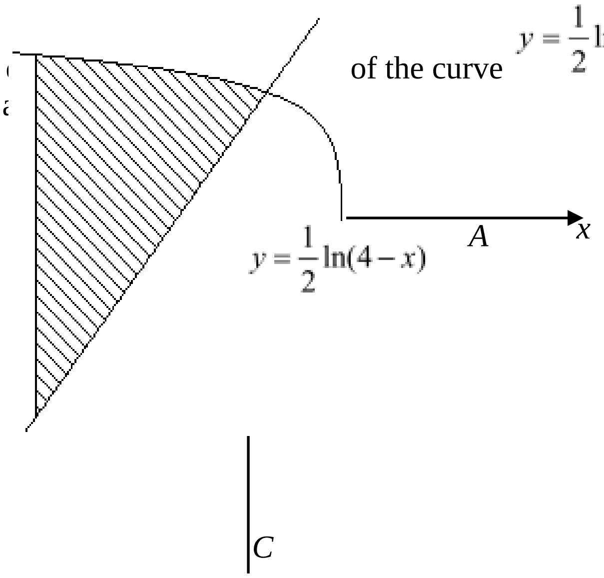
(i) Find the initial acceleration, in m/s^2. [2]

(ii) Find the minimum velocity, in m/s. [3]

(iii) Find the time, in seconds, when the particle is at instantaneous rest. [2]

(iv) Calculate the total distance, in m, travelled by the particle in the first 2 seconds. [4]

12 The (of the curve $y = \frac{1}{2}\ln(4 - x)$, which crosses the axes at *A* and a



(i) Find the coordinates of *A*. [1]

(ii) The coordinates of *B* are $(0, \ln a)$, state the value of *a*. [1]

The normal to the curve at *A* meets the *y*-axis at *C*.

(iii) Find the coordinates of *C*. [4]

Given that $y = \frac{1}{2}\ln(4 - x)$ can be expressed as $x = 4 - e^{ky}$.

(iv) Write down the value of *k*. [1]

(v) Hence, show that the area of the shaded region is $(4\ln 2 + 7.5)$ units^2. [4]

End of Paper 1

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Answer Key for MJR 4049/01 AM PRELIM 2021

Qn	Answer Key
Q1	$x = 3,\ \ y = -1$
Q2	$7 - 4\sqrt{2}$
Q3(i)	$x = 3$
(ii)	$k = -2$
Q4(i)	33.0 (3 s.f.)
(ii)	Number of weeks =21.0
(iii) Either one	● No solution for *t*, hence it is not possible for percentage to reach 100%. ● As t⍰ very large, $e^{-0.05t}$⍰0 but not 0, hence it is **not possible** for percentage to reach 100%.
Q5(i)	$x < 1\frac{2}{3}$
(ii)	-1.54 (3sf)
Q6(i)	Show Question
(ii)	x = 0.586 rad or x = 5.70 rad (3sf)
Q7(a)(i)	$\sin A \sin B = \frac{1}{3}$
(a)(ii)	$\tan A \tan B = \frac{4}{3}$
(b)	Show Question
Q8(i)	$\frac{7x+27}{(2x+3)(4-x)} = \frac{3}{2x+3} + \frac{5}{4-x}$
(ii)	$a + b = 3 + 5 = 8$
Q9(a)(i)	$k = 3$
(a)(ii)	$f(x) = -2x^4 + 10x^3 - 2x^2 - 42x + 36$
(b)	$x = -1$ or $x = 2$ or $x = \frac{3}{2}$
Q10(i)	$r = 4.47$ units (3sf)
(ii)	Since 3.60 < 4.47 (radius), point $(1, 3)$ lies inside the circle.
(iii)	$y = -2x + 7$
(iv)	*MN* = 8 units
Q11(i)	$a = -60$ m/s^2
(ii)	$v = -45$ m/s
(iii)	$t = 1\frac{2}{3}$ s

(iv)	Total distance $= 61\frac{1}{9}$ m
Q12(i)	*A* (3, 0)
(ii)	$a - 2$
(iii)	*C* (0, -6)
(iv)	*k* = 2
(v)	Show Question

Marked Solution for MJR 4049/01 AM PRELIM 2021

Q1

$$\frac{p^{2x}}{q^{3-4x}} \times \frac{q^{y-1}}{\left(p^{y+3}\right)^2} = p^2q^7$$

$$p^{2x-2y-6}q^{y-1-3+4x} = p^2q^7$$

[M1] Attempt to apply product/ quotient law of indices

$2x - 2y - 6 = 2$

$4x + y - 4 = 7$

[M1] Attempt to form 2 eqns by equating the power of *p* and *q*

$x - y = 4$ ....(eqn1)

$4x + y = 11$ ....(eqn2)

Solving eqn (1) & eqn (2),

$x = 3, \quad y = -1$ [A2]

Q2

ht of the pyramid = $\dfrac{\frac{1}{3}\left(29-2\sqrt{2}\right)}{\frac{1}{3}\times\left(3+\sqrt{2}\right)^2}$ [M1] Attempt to formulate expression

$= \dfrac{\left(29-2\sqrt{2}\right)}{11+6\sqrt{2}}$ [M1] Attempt to simplify expression

$= \dfrac{\left(29-2\sqrt{2}\right)\left(11-6\sqrt{2}\right)}{\left(11+6\sqrt{2}\right)\left(11-6\sqrt{2}\right)}$ [M1] Attempt to multiply by conjugate surd

$= \dfrac{319-174\sqrt{2}-22\sqrt{2}+24}{121-72}$ [M1] Attempt to simplify

$= \dfrac{343-196\sqrt{2}}{49}$

$= 7-4\sqrt{2}$ [A1]

Q3(i)

$\frac{3\lg 3x - 2\lg x}{4} = \lg 3$

$\lg \frac{(3x)^3}{x^2} = \lg 3^4$ [M1] Attempt to apply power/ quotient law of logarithm

[M1]

$27x = 81$

$x = 3$ [A1]

(ii)

$\log_{(x-2)} y = 2$ □ $(x-2)^2 = y$ ……(eqn1) [M1] Attempt to form eqn (1) or (2)

$\log_y (x+k) = \frac{1}{2}$ □ $y^{\frac{1}{2}} = x+k$ …….(eqn 2)

Solving eqn (1) & eqn (2),

$(x-2)^{2\times\frac{1}{2}} = x+k$ [M1] Attempt to form eqn

$x - 2 = x + k$

$k = -2$ [A1]

Q4(i)

When t =8

$P = 100\ (1 - e^{-0.05(8)})$

$= 100\ (1 - e^{-0.4})$

≈ 32.968

$= 33.0$ (3 s.f.) [B1]

33.0% responded to the Save Electricity Campaign after 8 weeks.

(ii)

For $P \geq 65$

$100\ (1 - e^{-0.05t}) \geq 65$ [M1] Attempt to form the inequality

$(1 - e^{-0.05t}) \geq 0.65$

$-e^{-0.05t} \geq -0.35$

$e^{-0.05t} \leq 0.35$

Take *ln* to both sides,

$-0.05t \leq \ln 0.35$ [M1] Attempt to simplify the inequality

$t \geq 21.0$ (3 s.f.)

Number of weeks =21.0 [A1]

(iii)

If P =100

$100 = 100\ (1 - e^{-0.05t})$ [M1]

$(1 - e^{-0.05t}) = 1$

$-e^{-0.05t} = 0$

Take ln to both sides

$-0.05t = \ln 0$ (no solution)

Hence it is **not possible** for percentage to reach 100%. [A1]

Alternate solution

As t □ very large, $e^{-0.05t}$ □0 but not 0, hence it is **not possible** for percentage to reach 100%.

Q5(i)

$y = (2-3x)e^{-x}$

$\frac{dy}{dx} = (2-3x)e^{-x}(-1) + e^{-x}(-3)$ [M1] Attempt to product/ quotient law of differentiation

$\frac{dy}{dx} = e^{-x}(3x-5)$

$\frac{dy}{dx} = \frac{3x-5}{e^x}$ [A1] also accept

For decreasing function, $\frac{dy}{dx} < 0$,

$\frac{3x-5}{e^x} < 0$, [M1]

$x < 1\frac{2}{3}$ [A1]

(ii)

When $y = \frac{2-3x}{e^x}$ cuts the x-axis, $y = 0$,

$0 = \frac{2-3x}{e^x}$ [M1] Attempt to find x-coordinate of graph

$x = \frac{2}{3}$

$\frac{dy}{dx} = \frac{3\left(\frac{2}{3}\right)-5}{e^{\frac{2}{3}}} = -1.54$ (3sf) [A1]

Q6(i)

$\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$ [M1] Attempt to apply addition formula

$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$ [M1] Attempt to apply trigonometrical identities

$= (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$ (shown) [A1]

(ii)

$2\cos 3x - 8\cos^3 x + 5 = 0$

$2(4\cos^3 x - 3\cos x) - 8\cos^3 x + 5 = 0$ [M1] Attempt to use (i) result

$8\cos^3 x - 6\cos x - 8\cos^3 x + 5 = 0$ [M1] Attempt to simplify expression

$-6\cos x = -5$

$\cos x = \frac{5}{6}$

$\alpha = 0.58568$ (1st & 4th quad) [M1] Attempt to find basic angle

x = 0.586 rad or x = 5.70 rad (3sf) [A1]

Q7(a)(i)

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\frac{7}{12}=\frac{1}{4}+\sin A\sin B$ [M1] Attempt to apply addition formula

$\sin A\sin B=\frac{1}{3}$ [A1]

(ii)

$\tan A\tan B=\frac{\sin A\sin B}{\cos A\cos B}$

$=\frac{\frac{1}{3}}{\frac{1}{4}}$ [M1]

$=\frac{4}{3}$ [A1]

(b) $\tan 15^\circ=\tan(45^\circ-30^\circ)$

$=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}$ [M1] Attempt to apply addition formula

$=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$ [M1]

$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$ x $\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$=\frac{3-2\sqrt{3}+1}{3-1}$

$=2-\sqrt{3}$ (shown) [A1]

Q8(i)

$$\frac{7x+27}{12+5x-2x^2}=\frac{7x+27}{(2x+3)(4-x)}$$

$$\frac{7x+27}{(2x+3)(4-x)}=\frac{A}{2x+3}+\frac{B}{4-x}$$

[M1] Attempt to split the fraction

$$7x+27=A(4-x)+B(2x+3)$$

Sub $x = 4$,

$$55=11B$$

$$B=5$$

[M1] Attempt find *A* or *B*

Sub $x=-\frac{3}{2}$,

$$\frac{33}{2}=A\left(\frac{11}{2}\right)$$

$$A=3$$

$$\frac{7x+27}{(2x+3)(4-x)}=\frac{3}{2x+3}+\frac{5}{4-x}$$

[A1] in correct partial fractions

(ii)

$$\int_{-1}^{3}\frac{7x+27}{12+5x-2x^2}\,dx=\int_{-1}^{3}\frac{3}{2x+3}+\frac{5}{4-x}\,dx$$

[M1] Attempt to apply integral

$$=\left[\frac{3\ln(2x+3)}{2}+\frac{5\ln(4-x)}{-1}\right]_{-1}^{3}$$

[M1]

$$=\left[\frac{3\ln(9)}{2}+\frac{5\ln(1)}{-1}\right]-\left[\frac{3\ln(1)}{2}+\frac{5\ln(5)}{-1}\right]$$

[M1]

$=3\ln 3+5\ln 5$ as compared to $a\ln a+b\ln b$

[A1]

Hence the value of $a+b=3+5=8$

[B1]

Q9(a)(i)

highest power of x in the polynomial $\mathrm{f}(x)$ is $-2x^4$, roots of the equation $f(x)=0$ are -2, 1 and a repeated root k,

□ $\mathrm{f}(x)=-2(x+2)(x-1)(x-k)^2$ [M1] Attempt to form f(x) using given info

Given that $\mathrm{f}(x)$ has a remainder of 64 when divided by $x+1$,

□ $\mathrm{f}(-1)=64$ [M1] Attempt to establish relationship using given info

$64=-2(-1+2)(-1-1)(-1-k)^2$

$64=4(-1-k)^2$

$16=(-1)^2(1+k)^2$

$4=1+k$ and $-4=1+k$

$k=3$ and $k=-5$ [A1]

(a)(ii)

$\mathrm{f}(x)=-2(x+2)(x-1)(x-3)^2$

$\mathrm{f}(x)=(-2x^2-2x+4)(x^2-6x+9)$ [M1] Attempt to form and simplify f(x)

$\mathrm{f}(x)=-2x^4+12x^3-18x^2-2x^3+12x^2-18x+4x^2-24x+36$

$\mathrm{f}(x)=-2x^4+10x^3-2x^2-42x+36$ [A1]

(b)

$\mathrm{f}(x)=2x^3-5x^2-x+6$

$\mathrm{f}(-1)=2(-1)^3-5(-1)^2-(-1)+6=0$ □ $(x+1)$ is a factor of $\mathrm{f}(x)$ [M1]o.e. Attempt to find any factor

$2x^3-5x^2-x+6=0$

$(x+1)(2x^2-7x+6)=0$ [M1] Attempt to deduce quadratic expression

$(x+1)=0$ or $(2x^2-7x+6)=0$

$x=-1$ or $x=\dfrac{-(-7)\pm\sqrt{(-7)^2-4(2)(6)}}{2(2)}$ [M1] o.e. Attempt to solve f(x)

$x=-1$ or $x=2$ or $x=\dfrac{3}{2}$

Q10(i)

$x^2+y^2+4x-2y-15=0$

$(x+2)^2+(y-1)^2=20$

$C(-2, 1)$

$r=\sqrt{20}$

$r=4.4721$

$r=4.47$ units (3sf)

[B1] o.e. Attempt to express as standard equation form or using $C(-g, -f)$ in

[M1A1] o.e. Attempt to find

(ii)

Length between the point $(1, 3)$ and radius $(-2, 1)$

$=\sqrt{(-2-1)^2+(1-3)^2}$

[M1] Attempt to find dist between point and radius

$=\sqrt{13}=3.6055$

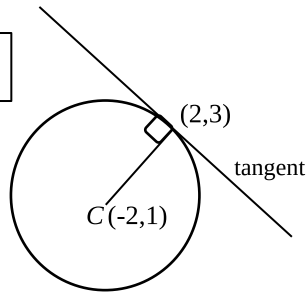
Since 3.60 < 4.47 (radius), point $(1, 3)$ lies inside the circle. [A1]

(iii)

$m_{\text{normal}}=-1\div\frac{3-1}{2+2}$

$=-2$

[M1] Attempt to find gradient tangent/ normal



Equation of the tangent to the circle A at the point $(2, 3)$:

$y-3=-2(x-2)$ [M1] Attempt to find eqn of normal

$y=-2x+7$ [A1]

(iv)

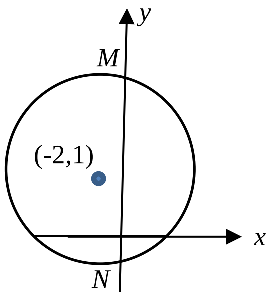
sub $x=0$ into $x^2+y^2+4x-2y-15=0$,

$y^2-2y-15=0$

$(y-5)(y+3)=0$

$y=5,-3$

[M1] Attempt to find y-coordinates of point M & N



Hence, $M(0,5)$ and $N(0,-3)$.

Therefore, Length of $MN=5+3$

$=8$ units [A1]

Q11(i)

$v = 5(3t-2)^2 - 45$

[M1] Attempt to differentiate "*v*"

$a = 30(3t-2)$

For initial acceleration, $t = 0$.

$a = -60$ m/s^2

[A1]

(ii)

For minimum velocity,

$a = 0$

[M1] Attempt to find *t* for min velocity

$30(3t-2) = 0$

$t = \frac{2}{3}$ s

Minimum velocity, $v = 5\left[3\left(\frac{2}{3}\right) - 2\right]^2 - 45$

[M1]

$v = -45$ m/s

[A1]

(iii)

At instantaneous rest, $v = 0$

$5(3t-2)^2 - 45 = 0$

[M1]

$(3t-2)^2 = 9$

$t = \frac{5}{3}$ s or $t = 1\frac{2}{3}$ s

[A1]

(iv)

$s = \int 5(3t-2)^2 - 45\, dt$

$s = \frac{5}{9}(3t-2)^3 - 45t + C$

[M1] Attempt to find displacement

When $t = 0$, $s = 0$, $s = \frac{5}{9}[-2]^3 + C \Rightarrow C = \frac{40}{9}$

[M1] Attempt to find "*C*"

$s = \frac{5}{9}(3t-2)^3 - 45t + \frac{40}{9}$

When $t = \frac{5}{3}$ s, $s = \frac{5}{9}\left[3\left(\frac{5}{3}\right) - 2\right]^3 - 45\left(\frac{5}{3}\right) + \frac{40}{9} = -\frac{500}{9}$ m

[M1] Attempt to find "*s*" at either *s* or *s*

When $t = 2$ s, $s = \frac{5}{9}[3(2) - 2]^3 - 45(2) + \frac{40}{9} = -50$ m

Total distance $= \frac{500}{9} + \left(\frac{500}{9} - 50\right)$ or $2\left(\frac{500}{9}\right) - 50$

$= 61\frac{1}{9}$ m

[A1] also accept m or 61.1 m (3sf)

Q12(i)

For $y = 0$, $0 = \frac{1}{2}\ln(4-x)$

$0 = \ln(4-x)$

$e^0 = 4-x$

$x = 3$

Hence, A (3, 0) [B1]

(ii)

Given B are $(0, \ln a)$, $\ln a = \frac{1}{2}\ln(4-0)$

$\ln a^2 = \ln 4$

$a^2 = 4$

$a = 2$ or $a = -2$ (reject) [B1]

(iii)

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{4-x}(-1)$ [M1] Attempt to find gradient tangent using differentiation

$\frac{dy}{dx} = -\frac{1}{2(4-x)}$

$m_{\text{tangent at }(3,0)} = -\frac{1}{2}$

$m_{\text{normal at }(3,0)} = 2$ [M1] Attempt to find gradient normal

Equation of AC: $y - 0 = 2(x-3)$ [M1] Attempt to find eqn of AC

$y = 2x - 6$

At $x = 0$, $y = -6$

Hence, C (0, -6) [A1]

(iv)

$y = \ln(4-x)^{\frac{1}{2}}$

$e^{2y} = 4-x$

$x = 4 - e^{2y}$

$k = 2$ [B1]

(v)

Area of the shaded region

$= \int_0^{\ln 2} 4 - e^{2y} \, dy + \left|\frac{1}{2} \times 3 \times 6\right|$ [M1M1] Attempt to find area bounded by y-axis and curve and area of triangle

$= \left[4y - \frac{e^{2y}}{2}\right]_0^{\ln 2} + 9$

$= \left[4(\ln 2) - \frac{e^{2(\ln 2)}}{2}\right] - \left[-\frac{1}{2}\right] + 9$ [M1]

$= (4\ln 2 + 7.5)$ units^2 (shown) [A1]