Anglo-Chinese School
(Independent)

FINAL EXAMINATION 2017

YEAR THREE EXPRESS
ADDITIONAL MATHEMATICS
PAPER 2

Wednesday **11 October 2017** **1 hour 30 minutes**

Additional Materials:
Answer Paper (8 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

Mathematical Formulae

ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae* for *ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1. Solve the equation $\frac{4^x}{4} = \sqrt{2}\left(2^x\right)$. [3]

2. The roots of the equation $x^2 = (k-3)x - 2$, are $\alpha + 1$ and $\alpha + 4$ where $k > 0$. Find the value of k in the form $a + \sqrt{b}$. [4]

3. Express $\frac{-x^2 - x + 7}{(2x+3)(x-1)^2}$ in partial fractions. [4]

4. A metallic sphere is heated to a temperature of 287°C and left to cool in an air conditioned room. As the sphere cools, its temperature, T°*C*, *t* mins after it is left in the room is given by $T = 22 + 265e^{-kt}$, where *k* is a constant. When *t* = 5, the temperature of the sphere reaches 200°C.

(i) Find the value of *k* correct to 3 significant figures. [2]

(ii) Find the temperature of the sphere after 20 mins. [1]

(iii) Explain what happens to T as *t* becomes very large. [1]

5. It is given that $2x^5 - x^4 + 3x^3 - 4x^2 + 2x + 3 = (2x+1)(x-1)Q(x) + ax + b$, where $Q(x)$ is a polynomial and *a* and *b* are constants.

(i) State the degree of $Q(x)$. [1]

(ii) Find the value of *a* and *b*. [3]

(iii) Hence, state the remainder when $2x^5 - x^4 + 3x^3 - 4x^2 + 2x + 3$ is divided by $2x^2 - x - 1$. [1]

6. **(i)** Solve the simultaneous equations:

$$y = |3 - x| - 1$$

$$y = \frac{1}{4}x + 2$$

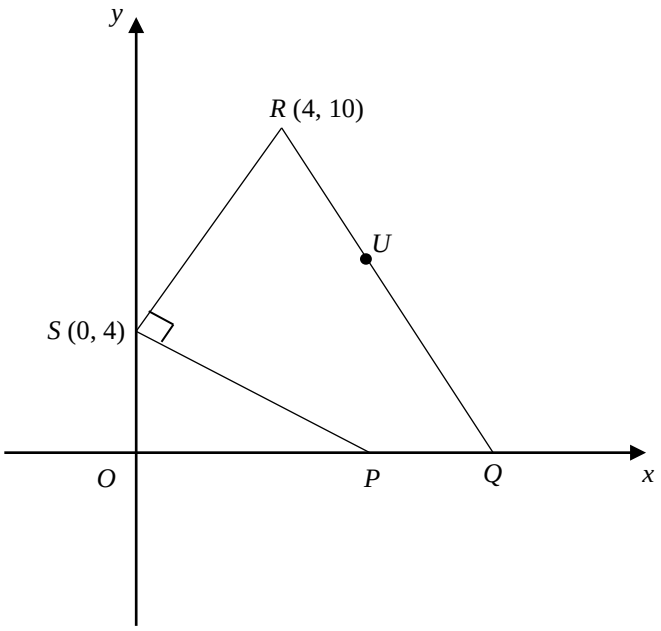
[3]

(ii) Sketch on the same axes, the graph of $y = |3 - x| - 1$ and $y = \frac{1}{4}x + 2$ for $0 \le x \le 10$.

[3]

(iii) Hence, state the range of values of *x* for which $4|3 - x| \le x + 12$. [2]

7. The diagram shows the quadrilateral *PQRS*. Points *P* and *Q* are on the *x*-axis. Point *U* lies on *RQ*.



(i) Find the equation of *SR*. [2]

(ii) Given that *SP* is perpendicular to *SR*, find the coordinates of *P*. [2]

(iii) Given that the area of *PQRS* is 41 units, find the coordinates of *Q*. [3]

(iv) If *RU* : *UQ* = 2 : 3, find the coordinates of *U*. [2]

(v) Explain with working, if the point *U* lies on the perpendicular bisector of *SP*. [3]

8 **(a)** Solve, for $0^o \le x \le 360^o$, $\tan^2 x = \dfrac{3}{\cos x} - 3$. [4]

(b) **(i)** Prove that $\dfrac{\sin x}{\sec x - 1} + \dfrac{\sin x}{\sec x + 1} = 2\cot x$. [3]

(ii) Hence, or otherwise, solve $\dfrac{\sin 2x(\sec 2x + 1) + \sin 2x(\sec 2x - 1)}{2\sqrt{3}} = -1$ for $-\pi < x < \pi$, leaving your answer in terms of π. [4]

[Please turn over

9 Answer the whole of this question on a single sheet of graph paper.

The table shows some experimental values of two variables, x and y.

x	0.8	1.0	1.2	1.4	1.6	1.8
y	21.0	14.9	10.6	9.2	4.5	2.1

It is believed that x and y are related by the equation $y = ax + \frac{b}{x}$, where a and b are constants.

(i) Using 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 2 units on the vertical axis, plot the graph of xy against x^2 [3]

(ii) Use your graph to

(a) estimate the value of a and of b, [2]

(b) identify the abnormal reading and estimate its correct value, [2]

(c) find the value of y when $x = 1.5$. [2]

----------The End ----------

ANSWERS:

1. $x = 2\frac{1}{2}$
2. $k = 3 + \sqrt{17}$
3. $\frac{-x^2 - x + 7}{(2x+3)(x-1)^2} = \frac{1}{2x+3} - \frac{1}{x-1} + \frac{1}{(x-1)^2}$

4. (i) $k = 0.0796$ (ii) 75.9°C (iii) $T = 22$

5. (i) 3 (ii) a = 3 and b = 2 (iii) $3x + 2$

6. (i) 0, 8 (iii) $0 \le x \le 8$

7. (i) $y = \frac{3}{2}x + 4$ (ii) $(6, 0)$ (iii) (9, 0) (iv) $(6, 6)$ (v) U does not lie on the $\perp$ bisector

8. (a) $x = 0^o, 60^o, 300^o \; and \; 360^o$ (c) $-\frac{\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{3}$ and $\frac{5\pi}{6}$

9. (iia) $a = -4.95$, $b = 20.00$ (iib) $y = 7.14$ (iic) $y = 5.73$