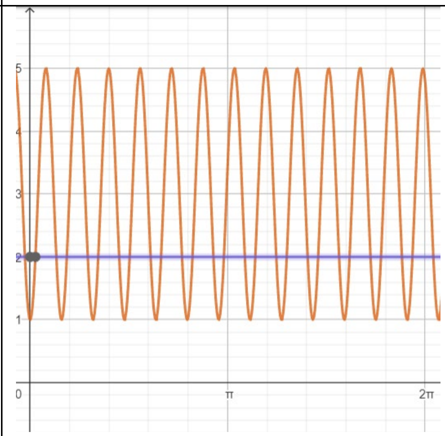
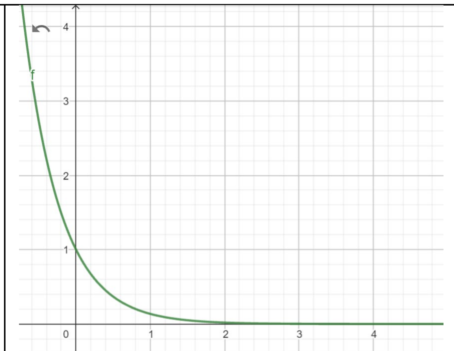
Marking Scheme BSSS AM4 4049 P2 2023

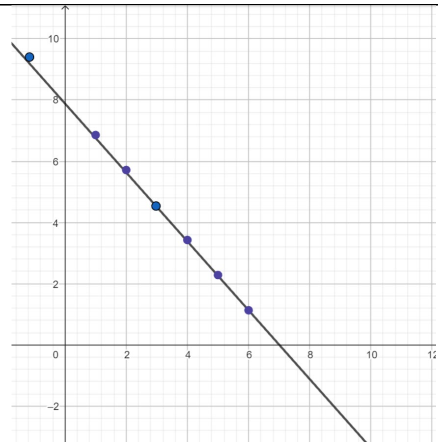
1	By Factor Theorem, $f(x)=(x-2)(2x+1)(x-k)$ By Remainder Theorem, $f(1)\ =-6$ $\Rightarrow (1-2)(2+1)(1-k)=-6$ $\Rightarrow \quad (-1)(3)(1-k)=-6$ $\Rightarrow \quad 1-k=2$ $\Rightarrow \quad k=-1$	M1 M1 A1
2a	$d=3-2\cos kt$ Period = 5min/10 = $\frac{2\pi}{k}$ = 0.5min $k=4\pi$ rad/min	M1 A1
2b	 d>2, $3-2\cos 4\pi t>2$ $\frac{1}{2}>\cos 4\pi t$ Let $\cos 4\pi t=\frac{1}{2}$, $\alpha=\frac{\pi}{3}$, $4\pi t$ lie in quad 1&4 $4\pi t=\alpha, 2\pi-\alpha, 2\pi+\alpha, 4\pi-\alpha, 4\pi+\alpha,\ldots$ $t=\frac{\pi}{3(4\pi)},\frac{5\pi}{3(4\pi)},\frac{7\pi}{3(4\pi)},\frac{15\pi}{3(4\pi)},\frac{17\pi}{3(4\pi)},\ldots$ $t=\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{15}{12},\ldots$ from the sketch, for d>2, $t=\frac{5}{12}-\frac{1}{12}$ $=\frac{1}{3}$ In 5 min, for 10 cycles, time = $\frac{10}{3}$ min.	M1 M1 M1 A1

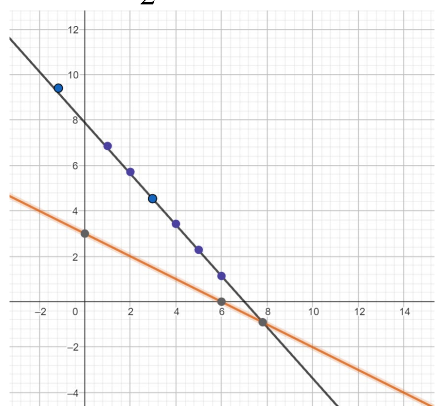
3a	$y=2\ln(x-1)$ when $y=2, x=a$ $2=2\ln(a-1)$ $a=e+1$	M1 A1
3b	$y=2\ln(x-1)$ $x=1+e^{\frac{y}{2}}$ $\int_0^2 x\ dy=\int_0^2 1+e^{\frac{y}{2}}\ dy$ $=\left[y+2e^{\frac{y}{2}}\right]_0^2$ $=2+2e-2$ $=2e$ $\int_2^{1+e} 2\ln(x-1)\ dx=$  $=2(e+1)-\int_0^2 x\ dy$ $=2(e+1)-2e$ $=2$ (Shown)	M1 - x M1 - intg M1-intg A1 - ans M1 A1
4a	$\frac{\sec^2 x}{1+\sec^2 x}=\frac{1}{\cos^2 x+1}$ $=\frac{2}{2\cos^2 x+2}$ $=\frac{2}{\cos 2x+1+2}$ $=\frac{2}{\cos 2x+3}$ (Shown)	M1 M1 A1
4b	$\frac{\sec^2 2x}{1+\sec^2 2x}=\frac{4}{7}$ $\Rightarrow \frac{2}{\cos 4x+3}=\frac{4}{7}$ $\Rightarrow \frac{14}{4}=\cos 4x+3$ $\Rightarrow \cos 4x=\frac{2}{4}$ $\Rightarrow 4x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3},\ldots$ $\Rightarrow x=\frac{\pi}{12},\frac{5\pi}{12},\frac{7\pi}{12},\frac{11\pi}{12},\ldots$ for $-2\pi\le x\le 2\pi$, there will be 16 solutions.	M1 M1 A1 A1

5a		G1- Shape of Graph + y-Intercept G1-Labels
5b	$\frac{dy}{dx}=e^{2x}+e^{-2x}$ Since $e^{2x}>0$ and $e^{-2x}>0$, $\frac{dy}{dx}=e^{2x}+e^{-2x}>0$, for all values of x. $\therefore$ the function is an increasing function.	M1 A1
5c	For equation of function, $y=\int e^{2x}+e^{-2x}$ $y=\frac{1}{2}e^{2x}-\frac{1}{2}e^{-2x}+c$ when $x=0, y=3$, $c=3$ $\therefore y=\frac{1}{2}e^{2x}-\frac{1}{2}e^{-2x}+3$	M1 M1 A1
6a	$\angle DAG=\angle DCA$ (Angle in alternate segment) $=\angle ECF$ $=\angle ECB$ (Given)	M1 A1
6b	E is midpoint of AC (Given) F is midpoint of DG (Given) $\therefore$ $\triangle$ACD is similar to $\triangle$ECF. (Midpoint Theorem) $\angle$BCA=$\angle$BDA (Angle in the same segment ADCB) $\angle$BEC=$\angle$AED (Angle in the same segment AB) $\therefore$ $\triangle$BCE is similar to $\triangle$DCA. (AA) $\therefore$$\triangle$BCE is similar to $\triangle$FCE.	M1 M1 M1 A1
6c	Since $\triangle$ACD is similar to $\triangle$ECF. $\frac{AC}{EC}=\frac{CD}{CF}=\frac{DA}{FE}=\frac{1}{2}$ Since $\triangle$BCE is similar to $\triangle$ECF $\frac{BC}{EC}=\frac{CE}{CF}=\frac{EB}{FE}$ $\therefore EC \times BE=BC \times FE$ $=BC \times \frac{1}{2}DA$ $=\frac{1}{2}AD\times BC$ (Shown)	 M1 A1
	Alternatively Since $\triangle$BCE is similar to $\triangle$ECF $\frac{BC}{EC}=\frac{CE}{CF}=\frac{EB}{FE}$ $\therefore EC \times BE=BC \times FE$ $=BC \times \frac{1}{2}DA$ (Midpoint Theorem) $=\frac{1}{2}AD\times BC$ (Shown)	M1 A1
7	$2^{1+3x}+2^2=7\left(4^x\right)+5\left(2^x\right)$ $2(2^x)^3+4=7(2^x)^2+5(2^x)$ Let $y=2^x$, $2y^3+4=7y^2+5y$ $2y^3-7y^2-5y+4=0$ Let $f(y)=2y^3-7y^2-5y+4$, $f(-1)=2(-1)^3-7(-1)^2-5(-1)+4$ $=-2-7+5+4$ $=0$ By Factor Theorem, $(y+1)$ is a factor of $f(y)$. $2y^3-7y^2-5y+4=(y+1)(2y^2+by+4)$ When y =1, $2-7-5+4=(2)(2+b+4)$ $-6=(2)(b+6)$ $-3-6=b$ $b=-9$ $f(y)=(y+1)(2y^2-9y+4)$ $=(y+1)(2y-1)(y-4)$ $f(y)=0$ $(y+1)(2y-1)(y-4)=0$ $y=-1$ or $y=\frac{1}{2}$ or $y=4$ $2^x=-1$ (Not admissible) or $2^x=2^{-1}$ or $2^x=2^2$ $x=-1$ or $x=2$	M1 A1 M1 M1 M1-factorise M1 A1 – rej A1
8a	$V_A=t^2-4t+7$ $a_B=6t-16$ $V_B=\int 6t-16\ dt$ $V_B=3t^2-16t+C$ At O, $V_B=23,\ t=0$, $C=23$ $V_B=3t^2-16t+23$ For t such that $V_A>V_B$, $t^2-4t+7>3t^2-16t+23$ $0>2t^2-12t+16$ $0>t^2-6t+8$ $0>(t-2)(t-4)$ $2<t<4$	M1- - intg M1- C M1 M1 M1 A1

8b	$s_A = \int t^2 - 4t + 7 \, dt$ $s_A = \frac{1}{3}t^3 - 2t^2 + 7t + C_A$ At O, $t = 0$, $s_A = 0, C_A = 0$ $s_A = \frac{1}{3}t^3 - 2t^2 + 7t$ $s_B = \int 3t^2 - 16t + 23 \, dt$ $s_B = t^3 - 8t^2 + 23t + C_B$ At O, $t = 0$, $s_B = 0, C_B = 0$ $s_B = t^3 - 8t^2 + 23t$ For overtaking, $s_A = s_B$ $\frac{1}{3}t^3 - 2t^2 + 7t = t^3 - 8t^2 + 23t$ $\frac{2}{3}t^3 - 6t^2 + 16t = 0$ $2t(t^2 - 9t^2 + 24) = 0$ $t = 0$ or $t^2 - 9t^2 + 24 = 0$ Consider $t^2 - 9t^2 + 24 = 0$ Discriminant $= (-9)^2 - 4(1)(24) < 0$ $\therefore$ the only solution to $s_A = s_B$ is $t = 0$. Hence, there will not be any overtaking after O.	M1- s_A M1 - s_B M1 – Solve M1 – D<0 A1
9a	$\frac{d}{dx}\left[x^2 \ln x - x\right] = 2x \ln x + x - 1$	B1
9bi	At a, $y = 0$. $2x \ln x = 0$ $x = 0$ or $\ln x = 0$ $x = 0$ or $x = 1$ $\therefore a = 1$	M1 M1 A1
9bii	For equation of line l_1, . gradient of $l_1 = \frac{0-1}{1-0} = -1$ l_1 is $y = -x + 1$	M1 A1
9biii	Area needed = $= \int_1^2 2x \ln x dx + \frac{1}{2}(1)(1)$ $= \int_1^2 \frac{d}{dx}\left[x^2 \ln x - x\right] - x + 1 \, dx + \frac{1}{2}$ $= \left[\left(x^2 \ln x - x\right) - \frac{1}{2}x^2 + x\right]_1^2 + \frac{1}{2}$ $= \left[\left(2^2 \ln 2 - \frac{1}{2}2^2\right) - \left(1 \ln 1 - \frac{1}{2}1^2\right)\right] + \frac{1}{2}$ $= \left[4 \ln 2 - 2 + \frac{1}{2}\right] + \frac{1}{2}$ $= 4 \ln 2 - 1$	M1 M1 M1 A1

10a	$\frac{2}{r^2 + r} = \frac{A}{r} + \frac{B}{r+1}$ Comparing numerators, $2 = A(r+1) + B(r)$ when $r = 0$, $2 = A$ when $r = -1$, $2 = -B$ $B = -2$ $\frac{2}{r^2 + r} = \frac{2}{r} - \frac{2}{r+1}$	M1 M1 A1
10b	$\frac{2}{1 \times 2} + \frac{2}{2 \times 3} + \cdots\cdots + \frac{2}{98 \times 99} + \frac{2}{99 \times 100}$ $= \left(\frac{2}{1} - \frac{2}{2}\right) + \left(\frac{2}{2} - \frac{2}{3}\right) + \ldots + \left(\frac{2}{98} - \frac{2}{99}\right) + \left(\frac{2}{99} - \frac{2}{100}\right)$ $= \frac{2}{1} - \frac{2}{100}$ $= 1.98$	M1 A1
10c	$\int_1^3 \frac{3}{r^2 + r} \, dr$ $= \frac{3}{2}\int_1^3 \frac{2}{r^2 + r} \, dr$ $= \frac{3}{2}\int_1^3 \frac{2}{r} - \frac{2}{r+1} \, dr$ $= 3\left[\ln r - \ln(r+1)\right]_1^3$ $= 3\left[\ln 3 - \ln 4 + \ln 2\right]$ $= 3\ln\left(\frac{3 \times 2}{4}\right)$ $= 3\ln\left(\frac{3}{2}\right)$ $= \ln\left(\frac{3}{2}\right)^3$ $= \ln\left(\frac{27}{8}\right)$	M1 M1 M1 M1 A1
11a	$y = ae^{bt}$ $\ln y = \ln a + bt$	G1-Eqn G1- Table of Values G1- Accurate Graph and Plotting G1- Labels

		
11b	From the graph, $\ln a = 8$ $a = e^8$ $= 2980.95$ $= 2980$ (to 3 sig. fig.) $b = \dfrac{8-0}{0-7}$ $= -1.1428$ $= -1.14$ (to 3 sig. fig.)	B1 B1

11c	From the graph, When $t = 3$, $\ln y = 4.4$ $y = 81.450$ $y = 81.5$ (to 3 sig. fig.)	M1 A1
11d	$y = \dfrac{e^3}{\sqrt{e^t}}$ $\ln y = 3 - \dfrac{1}{2}t$  From the graph, M contains a higher level of active ingredients after $t = 8$ years.	M1 G1 – Table of Values G1- Accurate Graph + Plotting A1