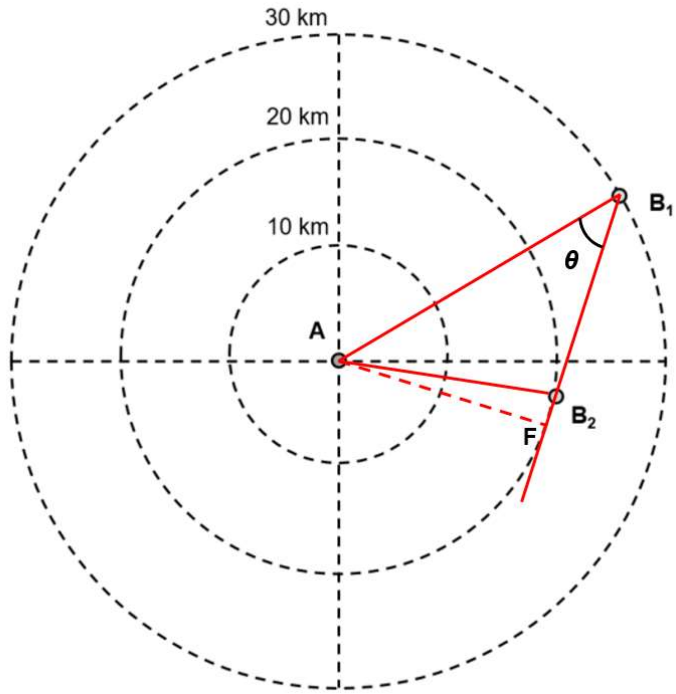
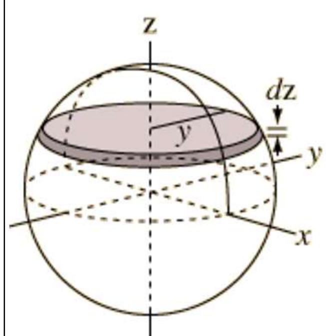
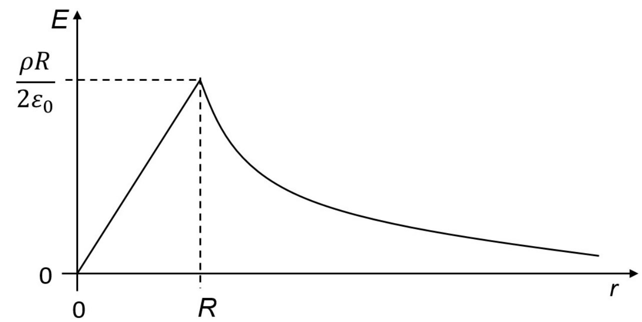
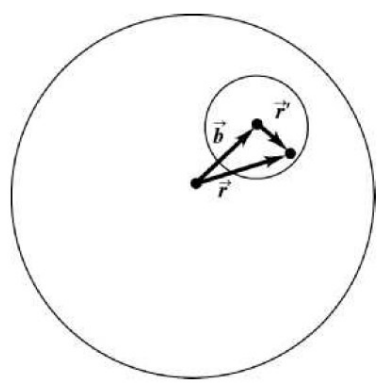
EJC H3 Physics 2020 Preliminary Examination Marks Scheme and Solution

Question		Solution	Marks
1	**(a)**	Change in displacement of B relative to A $= \left\|\begin{pmatrix} 30000\sin 30° + 20000\sin 10° \\ 30000\cos 30° - 20000\cos 10° \end{pmatrix}\right\|$ $= 19512\ m$ OR Using cosine rule, $B_1B_2 = \sqrt{30^2 + 20^2 - 2(20)(30)\cos(100° - 60°)} = 19.512\ km$ $speed = \frac{s}{t} = \frac{19512}{3600} = 5.42\ m\ s^{-1}$	**M1** **(M1)** **A1**
	(b)	 Let F be the point of closest approach for A and B. Using sine rule: $\frac{\sin\theta}{20} = \frac{\sin 40°}{19.51}$ $\theta = 41.2°$ $B_1F = 30\cos 41.2° = 22.5685\ km$ $Time\ taken = \frac{22568.5}{5.42} = 4163.9s = 69\min 24\ s\ \ from\ 12\ pm$ Time they are the closest is 1309HR	 **C1** **C1** **A1**

	(c)	$\angle B_1AF = 180° - 90° - 41.2° = 48.8°$ $Bearing = 60° + 48.8° = 108.8°$ $speed = \frac{AF}{time\ taken} = \frac{30000 \sin 41.2°}{9 \times 60} = 36.6\ m\ s^{-1}$ Also accept if student use time taken as $9 \times 60 + 24 = 564\ s$ and gets speed as 35.0 m s^{-1}.	**A1** **A1**

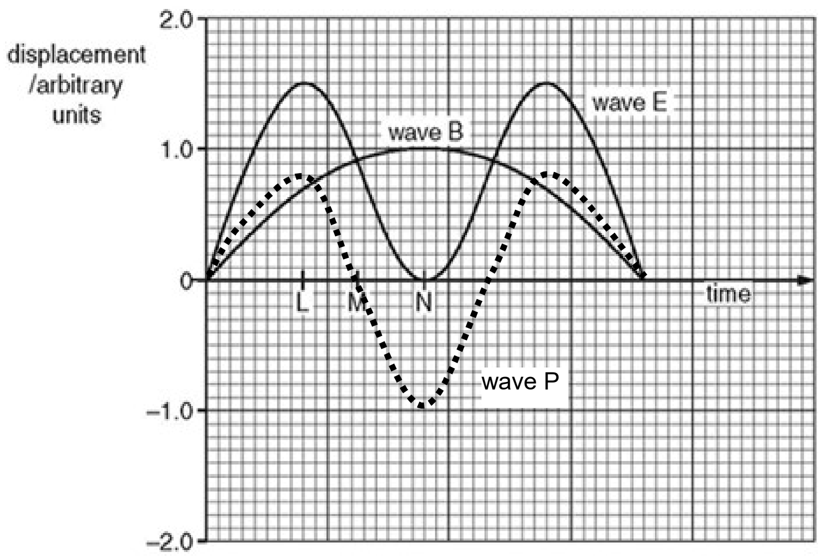
Question			**Solution**	**Marks**
2	**(a)**	**(i)**	The expression for the moment of inertia of a sphere can be developed by summing the moment of infinitely thin disks about the z axis.  $dI = \frac{1}{2}y^2dm = \frac{1}{2}y^2\rho dV = \frac{1}{2}y^2\rho\pi y^2dz$ $I_{CM} = \frac{1}{2}\rho\pi\int_{-R}^{R} y^4dz$ $= \frac{1}{2}\rho\pi\int_{-R}^{R}(R^2 - z^2)^2dz$ $= \frac{8}{15}\rho\pi R^5$ $= \frac{2}{5}MR^2$ Accept summing up spherical hollow shells. Clearly defined variables Correct definition of dI Correct algebra leading to final expression	 **B1** **B1** **B1**
		(ii)	Using parallel axis theorem $I_P = I_{CM} + Md^2$ $I_P = \frac{2}{5}MR^2 + M(L+R)^2$	**M1** **A1**
	(b)	**(i)**	$\tau = -Mg(L+R)\sin\theta$ (no marks if negative sign is omitted.)	**A1**
		(ii)	Using Newton's 2nd law for rotation: $\tau = I\alpha$ $-Mg(L+R)\sin\theta = \left[\frac{2}{5}MR^2 + M(L^2 + 2LR + R^2\right]\alpha$ $\alpha = \frac{Mg(L+R)\sin\theta}{M\left(\frac{2}{5}R^2 + R^2 + 2LR + L^2\right)}$ $= -\frac{g(L+R)\sin\theta}{\left(\frac{7}{5}R^2 + L^2 + 2LR\right)}$	**B1** **A1**

		(iii)	θ must be small such at $\sin\theta \approx \theta$ $\alpha = -\dfrac{g(L+R)}{\left(\frac{7}{5}R^2 + L^2 + 2LR\right)}\sin\theta$ $\alpha = -\dfrac{g(L+R)}{\left(\frac{7}{5}R^2 + L^2 + 2LR\right)}\theta$ $\omega^2 = \dfrac{g(L+R)}{\left(\frac{7}{5}R^2 + L^2 + 2LR\right)}$ $T = 2\pi\sqrt{\dfrac{\frac{7}{5}R^2 + L^2 + 2LR}{g(L+R)}}$ $= 2\pi\sqrt{\dfrac{\frac{7}{5}(0.30)^2 + 1^2 + 2(0.30)(1)}{9.81(1+0.30)}}$ $= 2.31\ s$	**M1** **M1** **A1**

Question		Solution	Marks
3	(a)	Using Gauss Law, $$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$ Using a cylindrical Gaussian surface, with $r < R$ $$E(2\pi rL) = \frac{\rho\pi r^2 L}{\varepsilon_0}$$ $$E = \frac{\rho r}{2\varepsilon_0}$$	**M0** **A1**
	(b)	Using Gauss Law, $$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$ Using a cylindrical Gaussian surface, with $R < r$ $$E(2\pi rL) = \frac{\rho\pi R^2 L}{\varepsilon_0}$$ $$E = \frac{\rho R^2}{2\varepsilon_0 r}$$	**M1** **A1**
	(c)	 Linear graph from 0 < r < R and 1/r graph for R < r Labelling of coordinates of max point	**B1** **B1**
	(d)	Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder.  As shown in diagram, $\vec{r'} = \vec{r} - \vec{b}$ $$\vec{E}_{off-axis} = \frac{\rho\,\vec{r'}}{2\varepsilon_0} = \frac{\rho(\vec{r}-\vec{b})}{2\varepsilon_0}$$ $$\vec{E}_{hole} = \vec{E}_{cylinder} - \vec{E}_{off-axis}$$ $$= \frac{\rho\vec{r}}{2\varepsilon_0} - \frac{\rho(\vec{r}-\vec{b})}{2\varepsilon_0} = \frac{\rho\vec{b}}{2\varepsilon_0}$$ Direction: Along the direction from centre of cylinder to centre of hole.	 **B1** **B1**
	(e)	Cylindrical Gaussian surface about central axis encloses no charge (based on equation in (d), as $\vec{b} = \vec{0}$) Electric field strength is zero.	**M1** **(M1)** **A1**

Question	Solution	Marks
4	Using $v = L\frac{di}{dt}$	B1
	Instantaneous power supplied to initiate the current in the inductor is $$P = iv = Li\frac{di}{dt}$$	M1
	$Total\ energy\ stored = \int_0^{t_f} P\ dt$ $= \int_0^{t_f} Li\frac{di}{dt}\ dt$ $= \int_0^{I_f} Li\ di$ $= \left[\frac{1}{2}Li^2\right]_0^{I_f}$	C1
	$= \frac{1}{2}LI_f^2$	A1

Question			Solution	Marks
5	**(a)**		The total energy of a moving satellite *m* under the influence of the gravitational field due to the Earth of mass *M* is given by: $TE = KE + GPE$ $= \frac{1}{2}mv^2 - \frac{GMm}{r}$ Since the satellite have an ellipsoidal orbit, $v^2 = v_{tangent}^2 + v_{radial}^2$ Since satellite is in a central force field $\vec{\tau} = \vec{r} \times \vec{F} = \vec{0}$ and resulted in $L = mrv_t$ Therefore, sub in the about equations and simplifying, $TE = \frac{1}{2}mv_t^2 + \frac{1}{2}mv_r^2 - \frac{GMm}{r}$ $= \frac{1}{2}mv_r^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$	**B1** **B1** **A0**
	(b)	**(i)**	Kepler's first law states that all planets move in **elliptical orbits** with the Sun at one focus of the ellipse.	**B1**
		(ii)	At turning points, $v_r = 0$ $E = \frac{L^2}{2mr^2} - \frac{GMm}{r}$ $Er^2 + GMmr - \frac{L^2}{2m} = 0$ $r^2 + \frac{GMm}{E}r - \frac{L^2}{2mE} = 0$ By comparing coefficients, $A = \frac{GMm}{E}$ $B = -\frac{L^2}{2mE}$	**M1** **A1** **A1**
		(iii)	Since the roots of the equation in (b)(ii) are r_a and r_p, $(r - r_p)(r - r_a) = 0 \Rightarrow r^2 - (r_p + r_a)r + r_a r_p = 0$ Comparing coefficients, $-\frac{GMm}{E} = (r_a + r_p)$ $E = -\frac{GMm}{(r_a + r_p)}$ $= -\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})(10.0)}{(25 + 35) \times 10^6}$ $= -6.64 \times 10^7 J$ (accept if algebraic method gives correct final answer)	**C1** **C1** **C1** **A1**

Question			Solution	Marks
6	(a)		$v = f\lambda$ $340 = (20 \times 10^3)\lambda$ or $340 = (80 \times 10^3)\lambda$ λ = 17 mm or λ = 4.3 mm 4.3 mm to 17 mm. (also accept 12.8 mm for the range in between)	**M1** **A1**
	(b)	(i)	Displacement at L = 1.5 – 0.7= 0.8 (units) Displacement at M = 0 (units) since the two curves intersect Displacement at N = 0 – 1.0 = –1.0 (units) Two answers correct All three answers correct	 **B1** **B1**
		(ii)	 Correct line through points L, M and N Complete waveform drawn (symmetrical about the vertical line through N)	**B1** **B1**
	(c)	(i)	Value of Δf = 51.25 - 50.80 = 0.45 kHz and f = 50.80 kHz $\frac{\Delta f}{f} = \frac{2v}{c}$ $\frac{0.45}{50.80} = \frac{2v}{340}$ Insect's speed, v = 1.51 m s^{-1}	**C1** **A1**
		(ii)	Increase in frequency due to more frequent reflection of wave. (sound wave that is being reflected off the insect is being compressed) Towards bat.	**B1** **(B1)** **B1**
		(iii)	Any reasonable factor that interfere with the reflected waves - Echo from surrounding objects - Presence of multiple bats using the same frequency sound waves	**B1**

			- Evolved insects that produces sonic clicks to jam echolocation.	
		(iv)	Higher frequency wavelength of wave is smaller. More sensitive to small change in frequency due to Doppler's effect.	**B1** **B1**
	(d)	**(i)**	From Fig 6.4: Graph of ln I against x is a straight line with negative gradient, equation of the straight line is : ln I = mx + ln I_o where m is the gradient and ln I_o is the y-intercept $\ln\left(\frac{I}{I_o}\right) = \mathrm{m}x$ $\therefore\ \frac{I}{I_o} = \mathrm{e}^{\mathrm{m}x}$ or $I = I_{\mathrm{o}}\mathrm{e}^{\mathrm{m}x}$ This equation is of the form of $I = I_{\mathrm{o}}\mathrm{e}^{-\alpha \mathrm{x}}$ where m = - α	 **B1** **B1** **B1**
		(ii)	Using Fig. 6.4, gradient = $\frac{1.00 - 2.15}{2.75 - 1.00} = -0.657 = -0.7$ Hence α = 0.7 m^{-1}	 **B1**
		(iii)	Density of air. Denser air leads to smaller attenuation coefficient.	**B1** **B1**

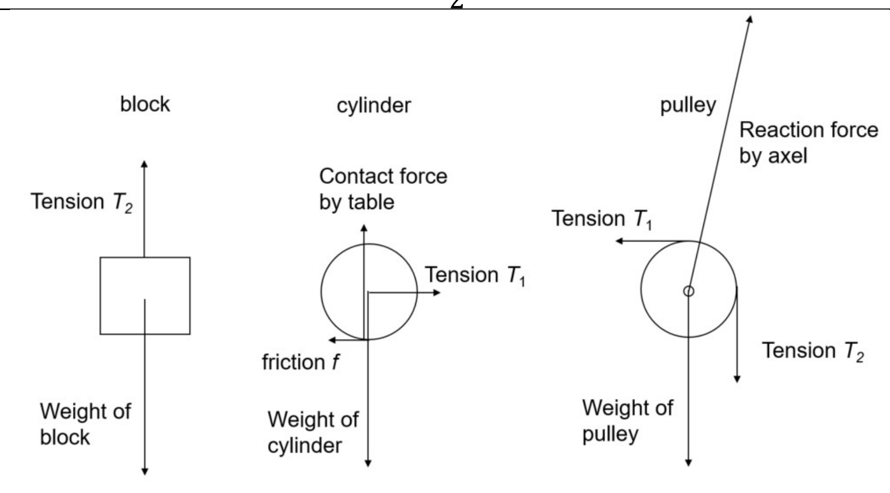
<table>
<tr><th colspan="3">Question</th><th>Solution</th><th>Marks</th></tr>
<tr><td>7</td><td>(a)</td><td>(i)</td><td>Consider a hollow cylindrical shell with radius r, and height L

$$dm = 2\pi\rho L r$$

$$I = \int r^2 \ dm$$
$$= \int_{R_{in}}^{R_{out}} 2\pi\rho L r^3 dr$$
$$= \left[\frac{\pi\rho L}{2} r^4\right]_{R_{in}}^{R_{out}}$$
$$= \frac{\pi L}{2}\left(\frac{M}{\pi\left(R_{out}^2 - R_{in}^2\right)L}\right)\left(R_{out}^4 - R_{in}^4\right)$$
$$= \frac{\pi L}{2}\left(\frac{M}{\pi L}\right)\left(R_{out}^2 + R_{in}^2\right)$$
$$= \frac{1}{2}M(R_{out}^2 + R_{in}^2)$$</td><td>M1

M1

A1</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>For a solid cylinder, R_{in} = 0

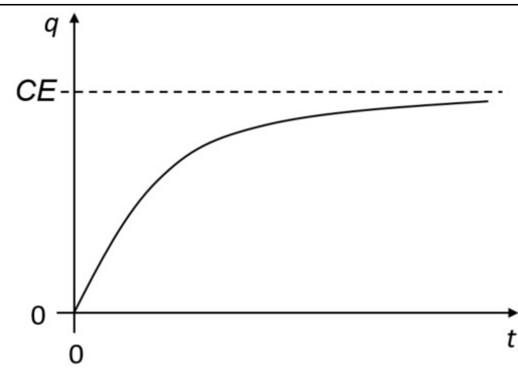
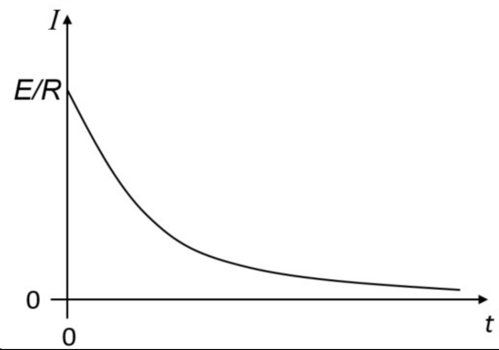
$$I_{solid} = \frac{1}{2}MR_{out}^2$$</td><td>A1</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td>

All force correctly labelled
Relative length of vector arrows are correct
(W all same and W > T_2)
$T_2 > T_1$
Horizontal component of reaction force by axel equals T_1.</td><td>B1
B1

B1
B1</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>$T_2 > T_1$ so that there is resultant clockwise torque acting on the pulley
This ensures that the string does not slip on the pulley.</td><td>B1
B1</td></tr>
<tr><td></td><td></td><td>(iii)</td><td>From the block:
$Mg - T_2 = Ma$ --- (1)

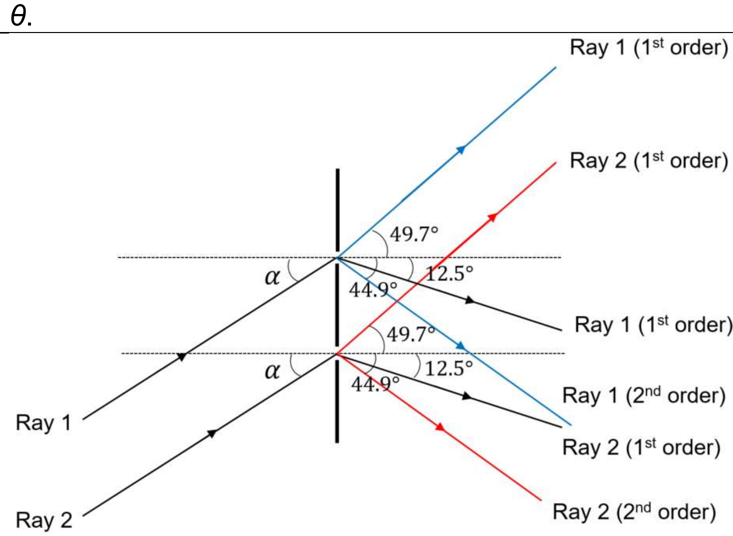
From the cylinder:
$T_2 R - T_1 R = \frac{1}{2}MR^2\alpha$ --- (2)</td><td></td></tr>
</table>

			From the pulley: $T_1 - f = Ma$ --- (3) Sub (1) and (3) into (2) $$Mg - Ma - (Ma + f) = \frac{1}{2}Ma$$ $$Mg - f = \frac{5}{2}Ma \text{ --- (4)}$$ Torque acting on cylinder: $$2fR = \frac{1}{2}M(2R)^2\alpha$$ $$f = MR\alpha \text{ --- (5)}$$ Since cylinder is rolling without slipping, $a = 2R\alpha$ (5) becomes $$f = MR\left(\frac{a}{2R}\right) = \frac{1}{2}Ma$$ Sub into (4), $$Mg - \frac{1}{2}Ma = \frac{5}{2}Ma$$ $$a = \frac{g}{3}$$	**C1** **M1** **M1** **A1**
		(iv)	Using conservation of energy, GPE loss by block = gain in translational KE of cylinder and block + rotational KE of pulley and cylinder $$Mgh = \frac{1}{2}Mv_{cylinder}^2 + \frac{1}{2}Mv_{block}^2 + \frac{1}{2}I_{pulley}\omega_{pulley}^2 + \frac{1}{2}I_{cylinder}\omega_{cylinder}^2$$ we also know that $v_{cylinder} = v_{block} = v$ Since cylinder and pulley are rolling, $$v_{cylinder} = 2R\omega_{cylinder}$$ $$v_{pulley} = R\omega_{pulley}$$ We will arrive at $$Mgh = Mv^2 + \frac{1}{2}\left(\frac{1}{2}M(2R)^2\right)\left(\frac{v}{2R}\right)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$ $$gh = \frac{3}{2}v^2 \text{ --- (1)}$$ Differentiating (1) w.r.t time, $$g\frac{dh}{dt} = \frac{3}{2}2(v)\frac{dv}{dt}$$ $$gv = 3va$$ $$a = \frac{g}{3}$$	**B1** **B1** **B1** **B1**
	(c)		Acceleration of the block is larger. All GPE lost by block converted to translational KE of the cylinder.	**B1** **B1**

Question			**Solution**	**Marks**
8	**(a)**	**(i)**	CE	**A1**
		(ii)	  	**A1** **A1**
	(b)	**(i)**	$\frac{1}{2}CE^2$	**A1**
		(ii)	CE^2	**A1**
		(iii)	By Kirchhoff's voltage law, and noting $i = \frac{dq}{dt}$ $E - iR - \frac{q}{C} = 0$ $Ri + \frac{q}{C} = E$ $R\frac{dq}{dt} + \frac{q}{C} = E \quad (\text{shown})$	**M1** **A0**
		(iv)	$$\frac{dq}{dt} = -\frac{B}{\tau}e^{-\frac{t}{\tau}}$$ Using the equation from (b)(iii), $$R\left(-\frac{B}{\tau}e^{-\frac{t}{\tau}}\right) + \frac{1}{C}\left(A + Be^{-\frac{t}{\tau}}\right) = E$$ when $t \longrightarrow \infty$, $R(0) + \frac{1}{C}(A + 0) = E$ $\Rightarrow A = CE$ when $t = 0$, $q = 0$	**B1** **B1**

			$-\frac{RB}{\tau}+\frac{1}{C}(A+B)=E$ $-\frac{RB}{\tau}+\frac{B}{C}=0$ $\Rightarrow \tau = RC$ when $t=0$, $q=0$ Using $q=A+Be^{-\frac{t}{\tau}}$ $0=A+B$ $\Rightarrow B=-A=-CE$ Correct expression for A and B Correct expression for τ **OR** $R\frac{dq}{dt}+\frac{q}{C}=E$ $\frac{dq}{dt}=\frac{CE-q}{RC}$ $\int_0^q \frac{1}{CE-q}dq=\frac{1}{RC}\int_0^t dt$ (correct integration) $-\left[\ln(CE-q)\right]_0^q=\frac{1}{RC}[t]_0^t$ $\ln\left(\frac{CE-q}{CE}\right)=-\frac{t}{RC}$ $\frac{CE-q}{CE}=e^{-\frac{t}{RC}}$ $q=CE-CEe^{-\frac{t}{RC}}$ Hence, $A=CE$, $B=$ **-**CE and $\tau=RC$	 **A1** **A1** **(B1)** **(B1)**
		(v)	$q=CE-CEe^{-\frac{t}{RC}}$ $\frac{dq}{dt}=\frac{CE}{RC}e^{-\frac{t}{RC}}$ $I=\frac{E}{R}e^{-\frac{t}{RC}}$	 **M0** **A1**

		(vi)	$P = I^2R$ $= \left(\frac{E}{R}e^{-\frac{t}{RC}}\right)^2 R$ $= \frac{E^2}{R}e^{-\frac{2t}{RC}}$ $\frac{dE_R}{dt} = \frac{E^2}{R}e^{-\frac{2t}{RC}}$ $E_R = \frac{E^2}{R}\int_0^\infty e^{-\frac{2t}{RC}}dt$ $= -\frac{1}{2}CE^2\left[e^{-\frac{2t}{RC}}\right]_0^\infty$ $= \frac{1}{2}CE^2$	**M1** **A1**
		(vii)	Yes. The result is in agreement as half of the energy supplied by the battery is used to **charge to capacitor** the other half is **dissipated in the resistor.**	**B1**
	(c)	**(i)**	Since the capacitors are connected in series, $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$ $\frac{1}{C_T} = \frac{1}{2.5\times10^{-6}} + \frac{1}{1.1\times10^{-6}}$ $C_T = \frac{(2.5\times10^{-6})(1.1\times10^{-6})}{(2.5+1.1)\times10^{-6}} = 7.6\times10^{-7}F$ Since the capacitors are connected in series, charge on each capacitor are the same. Hence charge on each capacitor, $Q = CV$ = $(7.6 \times 10^{-7})(9.0)$ = 6.88 μC (both must be correct to be awarded)	**M1** **A1**
		(ii)	$V_1 = \frac{Q}{C_1} = \frac{6.8}{2.5} = 2.75\ V$ $V_2 = \frac{Q}{C_2} = \frac{6.88}{1.1} = 6.25\ V$ (both must be correct to be awarded) (ecf accepted if (c)(i) is wrong)	**A1**
		(iii)	Since no current in R_1, p.d. across R_2 = p.d. across C_2 = 6.25 V $current\ through\ R_2 = \frac{C_2}{I} = \frac{6.25}{58} = 0.108\ A$ Hence, R of variable resistor $= \frac{C_1}{I} = \frac{2.75}{0.108} = 25.5\ \Omega$	**M1** **C1** **A1**

Question				Solution	Marks
9	**(a)**	**(i)**		The *Principle of Superposition* states that when two or more waves of the same type meet, the resultant displacement at any point is the vector sum of the individual displacements that each wave would cause at that point.	**B1** **B1**
		(ii)		Phase difference is an **angular measure** of how much an oscillation is out of step with itself at 2 different instances in time (or how much 2 oscillations are out of step with each other at the same instant in time). Path difference between 2 waves is a measure of the difference in distance travelled from their respective coherent wave sources which are in phase with each other, and is usually expressed in terms of number of wavelengths.	**B1** **B1**
	(b)	**(i)**	**1**	$path\ difference = d\sin\alpha$	**A1**
			2	$Total\ path\ difference = d\sin\alpha + d\sin\beta$	**A2**
			3	For strong diffraction, Path difference should be an integer-multiple of a wavelength. $d\sin\alpha + d\sin\beta = m\lambda$ where *m* is an integer.	 **A1**
		(ii)	**1**	Bright lines produced by a 2 slit source is less sharp compared to a diffraction grating making it difficult to precisely measure θ.	**B1**
			2	Bright lines produced by a 2 slit source is less bright compared to a diffraction grating making it difficult to accurately measure θ.	**B1**
	(c)	**(i)**		 Upper 1st order deflected upwards and lower 1st order downwards.	**B1**
		(ii)		Using equation in (b)(i)3, $d(\sin\alpha + \sin 12.5°) = \lambda$ --- (1) $d(\sin 49.7° + \sin\alpha) = \lambda$ --- (2) Solving equation (1) and (2) $\alpha = 15.849° = 15.8°$	 **M1** **C1** **A1**

		(iii)	Using $d(\sin\alpha + \sin 12.5°) = \lambda$ --- (1) $d(\sin 15.8° + \sin 12.5°) = 633 \times 10^{-9}$ $d = 1.2931 \times 10^{-6} = 1.30 \times 10^{-6} m$	**A1**
		(iv)	In the upwards direction, the maximum order can be found from: $d(\sin 90° + \sin 15.849°) = m\lambda$ $(1.2931 \times 10^{-6})(1 - 0.27310) = m(633 \times 10^{-9})$ $m = 1.48 = 1\ (round\ down\ to\ nearest\ integer)$ **The first order is the maximum order.** In the downwards direction, the maximum order can be found from: $d(\sin 15.489° + \sin 90°) = m\lambda$ $(1.2931 \times 10^{-6})(0.27310 + 1) = m(633 \times 10^{-9})$ $m = 2.6007 = 2\ (round\ down\ to\ nearest\ integer)$ **The second order is the maximum order.** To find the angle for this second order, $d(\sin 15.489° + \sin\beta) = 2\lambda$ $(1.2931 \times 10^{-6})(0.27310 + \sin\beta) = 2(633 \times 10^{-9})$ $\beta = 44.906° = 44.9°$	**B1** **B1** **B1** **B1** **A1**