Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2)

Paper 1

9740/01

17 September 2014

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, **attach the question paper to the front of your answer script.**

The total number of marks for this paper is **100**.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	4	6	7	8	8	8	9	12	11	13	14	100

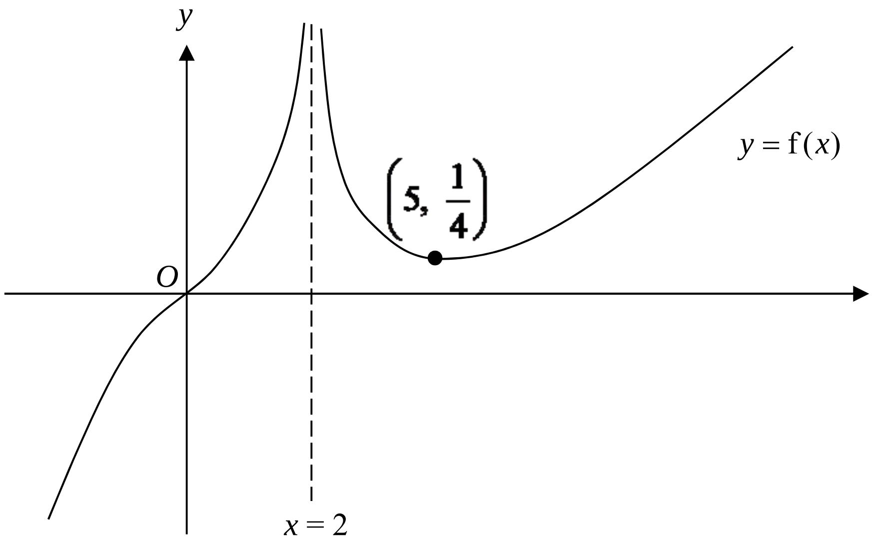
1 **(i)** A fisherman has 800kg of fish, consisting of mackerel, salmon and tuna. He may choose to sell all his fish at either Market A or Market B. The rates offered by the respective markets and his total returns are as follows:

Market	Price (in dollars) per kg			Total Returns (in dollars)
	Mackerel	Salmon	Tuna	
A	7	21	39	20 300
B	5	23	49	23 900

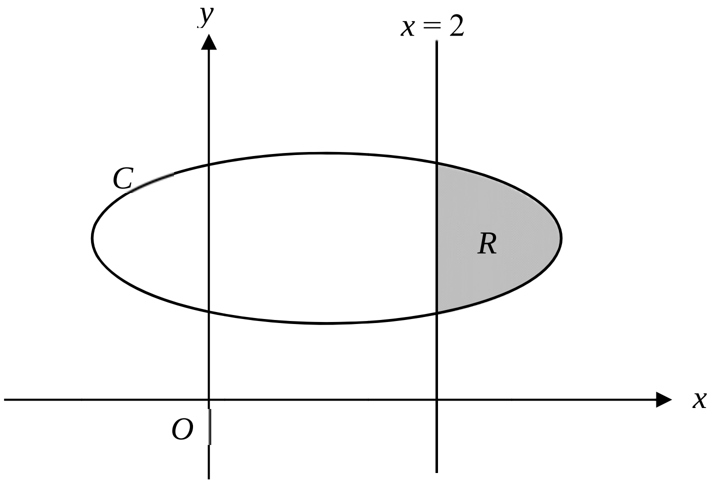
Determine the weight of the salmon that the fisherman has. [3]

(ii) Another fisherman has 600kg of the same types of fish and he claims that he can obtain the exact same total returns as the fisherman in part **(i)** at the respective markets. Determine whether his claim is possible. [1]

2 **(a)** The diagram below shows the curve with equation $y = \mathrm{f}(x)$ which passes through the origin O, has a vertical asymptote $x = 2$ and a stationary point at $\left(5, \frac{1}{4}\right)$. Sketch the graph of $y^2 = \mathrm{f}(x)$, indicating the coordinates of the points where the graph crosses the axes, stationary point(s) and the equations of any asymptotes. [2]



(b) The diagram shows the region R bounded by the curve C with equation $\frac{(x-1)^2}{4}+(y-2)^2=1$ and the line $x=2$. Find the numerical value of the volume of the solid formed when R is rotated through 2π radians about the x-axis. [4]



3 Given that $e^y = 1+3x+2x^2$,

(i) Show that $\left(\frac{dy}{dx}\right)^2+\frac{d^2y}{dx^2}=4e^{-y}$. [2]

(ii) By further differentiation of the result in part **(i)**, find the first three non–zero terms in Maclaurin's series for y. [3]

(iii) By solving $1+3x+2x^2=1.0302$, use a suitable value of x and the result obtained in **(ii)** to find an approximate value for $\ln(1.0302)$. Give your answer correct to 4 decimal places. [2]

4 **Do not use a calculator in answering this question.**

Relative to the origin O, two points A and B have position vectors given by $\mathbf{a}=p\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $\mathbf{b}=\mathbf{i}$ respectively.

(i) The point C is on AB such that $AC:CB=2:1$. Find the position vector of C in terms of p. Hence find the exact area of triangle OAC. [3]

(ii) The point D is on OC produced such that $OD=2CD$. The point E is such that $\overrightarrow{AE}=\overrightarrow{OC}$. Find the area of trapezium $OAED$. [2]

(iii) Given that the angle between **a** and **b** is $135°$, find the value of p. [3]

5 The complex number z is given by $z = r\,\mathrm{e}^{\mathrm{i}\theta}$, where $0 < r \le 2$ and $\frac{\pi}{6} < \theta \le \frac{\pi}{2}$.

(i) Given that $w = 2\left(\sqrt{3} - \mathrm{i}\right)z$, find $|w|$ in terms of r and arg w in terms of θ. [2]

(ii) Given that θ has a fixed value, draw an Argand diagram to show the locus of z as r varies. On the same diagram, show the corresponding locus of w. You should identify the modulus and argument of the end-points of each locus. [4]

(iii) Find the range of values that $\left|\frac{w^2}{2z^*}\right|$ can take. [2]

6 Show that the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + y - 3(xy)^2 = 0$$

may be reduced by using the substitution $u = xy$ to $\frac{\mathrm{d}u}{\mathrm{d}x} - 3u^2 = 0$.

Hence find the general solution of y in terms of x. [4]

(i) Sketch the solution curve that passes through $\left(1, \frac{1}{3}\right)$, indicating any stationary points and asymptotes clearly. [3]

(ii) State the particular solution for which y has no turning point. [1]

7 The curve C has equation

$$y = e^{x} - \lambda ,$$

where λ is a constant, $\lambda > 1$.

(i) Sketch C. [2]

Hence, on separate diagrams, sketch the graphs of

(ii) $y = e^{|x|} - \lambda$, [2]

(iii) $y = \dfrac{1}{e^{x} - \lambda}$, $x \neq \ln \lambda$, [3]

including the coordinates of the points where the graphs crosses the axes and the equations of any asymptotes.

For $x \neq 1$, use graphs in parts **(ii)** and **(iii)** to deduce the number of real solutions for the equation

$$(e^{x} - e)(e^{|x|} - e) = 1 .$$ [2]

8 **(a)** Find $\int x \sec^{2}(x + a)\,dx$, where a is a constant. [3]

(b) Find $\int \dfrac{x - 1}{x^{2} - 2x + 2}\,dx$. [2]

Hence find

(i) the exact value of $\int_{1}^{2} \dfrac{x - 4}{x^{2} - 2x + 2}\,dx$, [4]

(ii) $\int_{2-p}^{p} \left| \dfrac{x - 1}{x^{2} - 2x + 2} \right| dx$ where p is a constant, $p > 1$. Leave your answer in terms of p. [3]

9 **(a)** A sequence of positive numbers $x_1, x_2, x_3, ..., x_n$ is a geometric progression. Show that the sequence of numbers $y_1, y_2, y_3, ..., y_n$ given by the relation,

$$y_n = \log_k x_n + k, \quad \text{where } k \text{ is a positive constant,}$$

is an arithmetic progression. [4]

(b) Thomas takes an education loan of $\$20\,000$ from a bank for his undergraduate studies. Starting from the month he obtained the loan, Thomas makes a monthly payment of $\$x$ to the bank in the middle of every month. At the end of each month, the bank charges him an interest of 1% for the remaining amount owed. The repayment plan continues until he has fully paid for his loan.

(i) During his studies, Thomas is only able to make a monthly payment such that the amount owed remains at \$20 000 at the end of every month after the interest is charged. Find the value of x for this payment plan. [2]

After completing his studies, Thomas still owes the bank an outstanding amount of \$20 000. He adjusts the repayment amount in the month after completing his studies so that he can repay his loan in full after a certain time.

(ii) Show that starting from the month the repayment amount is adjusted, the monthly payment of $\$x$ such that the loan is fully paid after the nth payment is given by

$$x = \frac{200(1.01^{n-1})}{1.01^n - 1}.$$ [3]

(iii) Hence determine the monthly amount that Thomas should make to fully pay up the loan in exactly 3 years' time. [2]

10 The function f is defined by

$$\mathrm{f} : x \mapsto 2 + \frac{3}{x^2 + 2x + 2}, \quad x \in \mathbb{R}.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. Show that f does not have an inverse. [2]

(ii) The function f has an inverse if its domain is restricted to $x \geq k$. State the smallest possible value of k. [1]

For the rest of the question, use the new domain found in part **(ii)**.

(iii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iv) Show that f^2 exists and find the range of f^2. [3]

(v) Describe a sequence of two geometrical transformations by which the graph of $y = 2 + \dfrac{3}{x^2 + 2x + 2}$ can be obtained from the graph of $y = \dfrac{3}{4x^2 + 4x + 2}$. [2]

(vi) By adding a suitable graph to $y = \mathrm{f}(x)$ in part **(i)**, solve the inequality

$$\frac{3}{9x^2 + 6x + 2} < 3x - 2.$$ [2]

11 A curve has parametric equations

$$x = \tan t, \quad y = 2\cos t, \quad \text{for } 0 \leq t < \frac{\pi}{2}.$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. What can be said about the tangent to the curve as $t \to 0$?

Hence sketch the curve. [4]

(ii) The point P on the curve has a non-zero parameter p. Given that the normal to the curve at P passes through the origin, find the value of p.

Hence show that equation of the normal to the curve which passes through the origin is given by $y = x\sqrt{2}$. [5]

(iii) Find the exact area of the region bounded by the curve, the line $y = x\sqrt{2}$ and the y-axis. [5]

END OF PAPER