| Full Name | Class Index No | Class |
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# Anglo-Chinese School (Barker Road)

**PRELIMINARY EXAMINATION 2023**
**SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)**

**ADDITIONAL MATHEMATICS**
**4049**
**PAPER 2**

**2 HOURS 15 MINUTES**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

| For Examiner's Use |
|---|
| |

This question paper consists of **20** printed pages and **2** blank pages.

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** Mr Tan invested some money in stock in May.
The value of a stock, $\$y$, varies with time, $x$, can be modelled by the equation
$y=-\frac{1}{3}x^2+2x+1$, where $y$ is in thousands and $x$ is in months.

**(a)** State the initial value of the stock. [1]

**(b)** Express $y$ in the form $a(x-b)^2+c$. [2]

**(c)** Determine the best time to sell off his stocks to make a maximum profit.
Find the value of the stock at this time. [2]

**2** It is given that $\mathrm{f}(x) = ax^3 + 2bx^2 - 34x + 12$, where $a$ and $b$ are constants, has a factor of $x - 3$ and leaves a remainder of 32 when divided by $x + 1$.

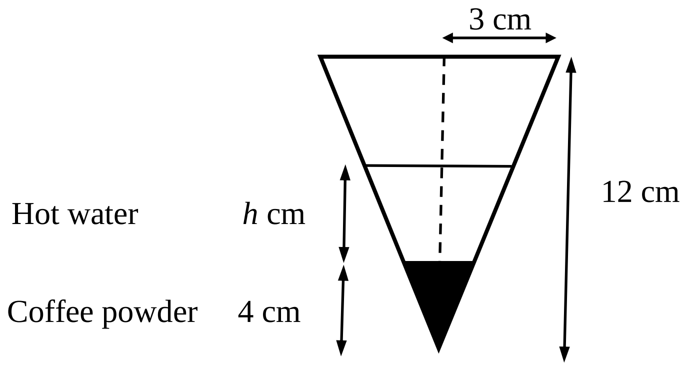
**(a)** Find the value of $a$ and of $b$. [4]

**(b)** Find the range of values of $k$ for which the line $y + 9 = kx$ intersects the curve $x = \sqrt{y - 3x}$ at two distinct points. [4]

**3** **(a)** Show that $\dfrac{2\cos^2\theta - \sin\theta\cos\theta + 1}{\sin^2\theta} = 3\cot^2\theta - \cot\theta + 1$. [3]

**(b)** Hence solve the equation $2\cos^2\theta - \sin\theta\cos\theta + 1 = \sin^2\theta$ for $-\pi \le \theta \le \pi$. [4]

**4** A coffee filter is in the shape of a right circular cone. The cone has a base radius of 3 cm and a height of 12 cm. The coffee powder, as seen in the cross-section of the filter below, has a depth of 4 cm. Let $h$ be the height of the hot water in the filter at time $t$ seconds. Assume that the amount of water in the coffee powder is negligible.



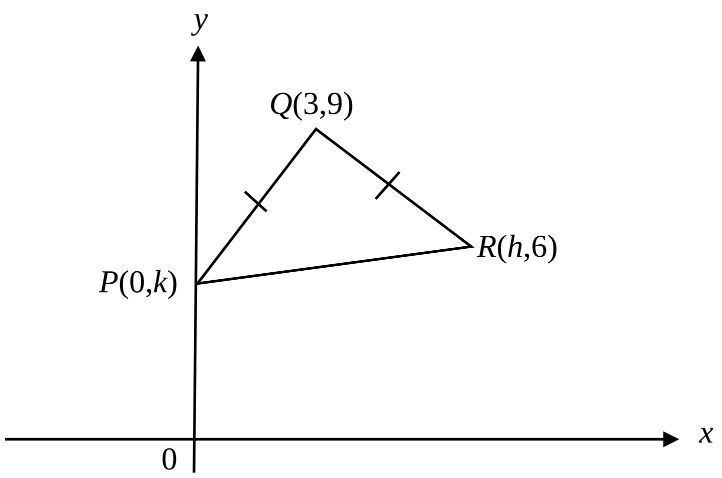

**(a)** Show that the volume of hot water, $V$, in the filter at the time $t$ seconds is given by $V = \frac{\pi}{48}(h+4)^3 - \frac{4\pi}{3}$. [3]

[The volume of a cone of height $h$ and base radius $r$ is $\frac{1}{3}\pi r^2 h$ ]

Initially, the filter contains only coffee powder. A machine dispenses hot water into the filter at a constant rate of 5 $cm^3$ per second and hot water drips out of the filter at a constant rate of 2 $cm^3$ per second.

**(b)** Find the exact rate of change of the depth of water when $h = 8$. [4]

**5**



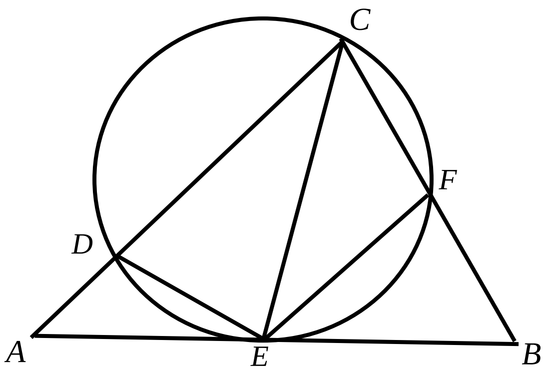

The diagram shows an isosceles triangle $PQR$ with vertices at $P(0, k)$, $Q(3, 9)$ and $R(h, 6)$, where $h$ and $k$ are positive constants and $h > k$.

**(a)** Show that $h + k = 12$. [3]

**(b)** It is now given that $h = 7$.
The point $S$ is such that $PQRS$ is a kite. Given that $S$ lies on the line $21y = 7x - 8$, find the coordinates of $S$. [4]

**(c)** Find the area of the kite. [2]

**6**




In the diagram above, *CDEF* is a circle and the line *AB* is a tangent to the circle at *E*. *ADC* and *CFB* are straight lines and *AE* = *EB* and *CF* = *FB*.

**(a)** Prove that *AC* is parallel to *EF*. [1]

**(b)** Prove that angle *AED* = angle *CEF* . [2]

**(c)** Prove that triangle *AED* is similar to triangle *CEF* . [2]

**7** Theresa bought a house on 2 January 1970. The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuation price, $V$, of the house is related to $t$, the number of years since 1970. The variables $V$ and $t$ can be modelled by the equation $V = pq^t$, where $p$ and $q$ are constants.

The table below gives the values of $V$ and $t$ for some of the years 1970 to 2010.

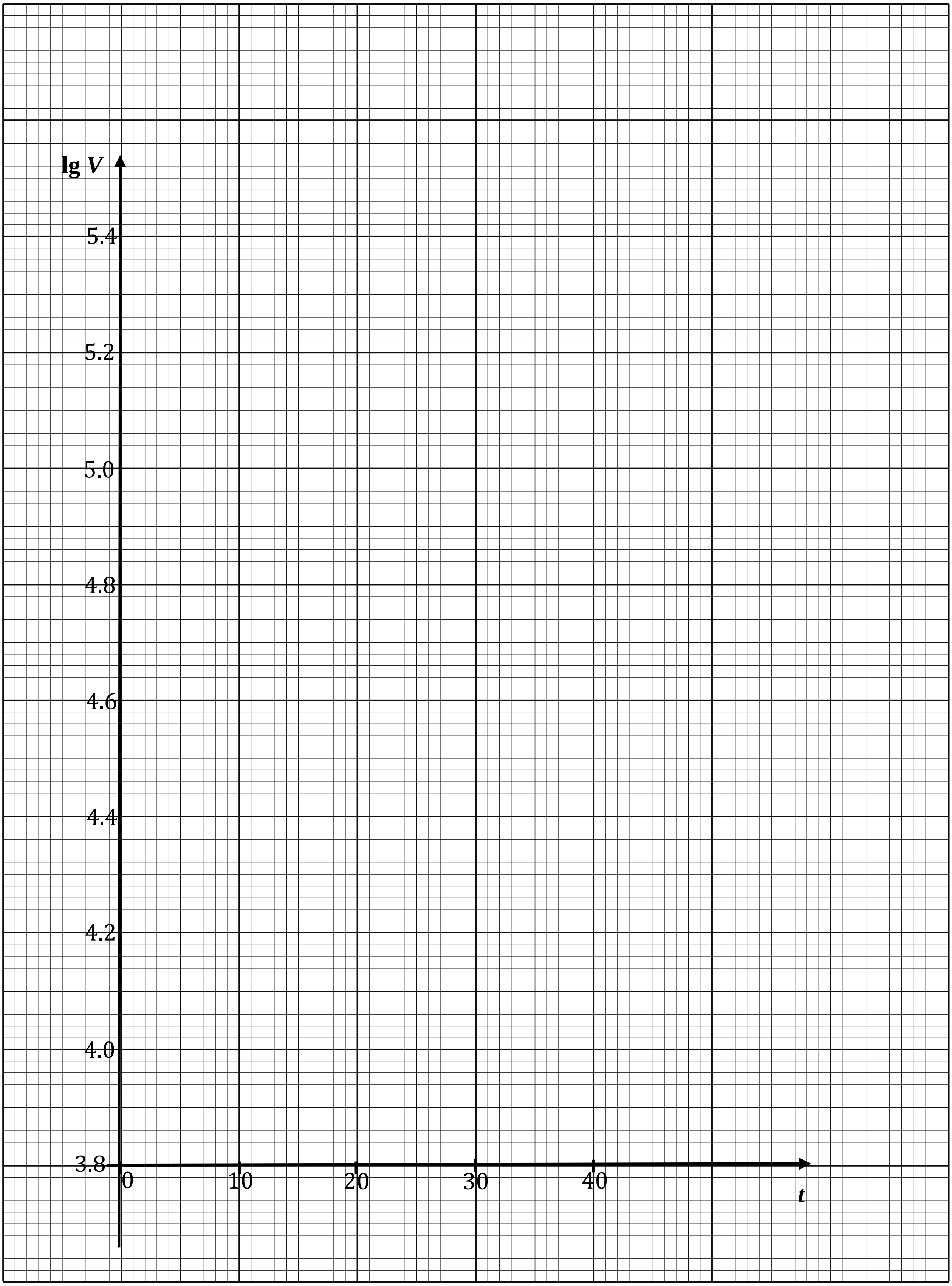
| Year | 1970 | 1980 | 1990 | 2000 | 2010 |
|---|---|---|---|---|---|
| $t$ | 0 | 10 | 20 | 30 | 40 |
| $V$ ($) | 8000 | 17500 | 38000 | 83000 | 190000 |

**(a)** Plot lg $V$ against $t$ and draw a straight line graph on the grid on the next page. [2]

**(b)** Use your graph to estimate the values of $p$ and $q$. [3]

**(c)** Determine the year in which Theresa's house will first be worth a million dollars. [3]

**(d)** Explain whether your answer to **part (c)** is likely to be reliable. [1]

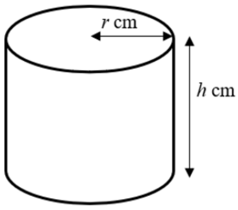

lg V
5.4
5.2
5.0
4.8
4.6
4.4
4.2
4.0
3.8
0
10
20
30
40
t

**8** It is given that $\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$.

**(a)** Explain clearly why $1<x<2$. [4]

**(b)** Hence solve $\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$ and show that it has only one solution. [5]

**9** A beverage company makes closed cylindrical cans, each with volume 400cm$^3$. The materials for the curved surface and base of the cans cost \$0.025 per cm$^2$ and \$0.03 per cm$^2$ respectively. Assume that the cost of wasted materials is negligble.

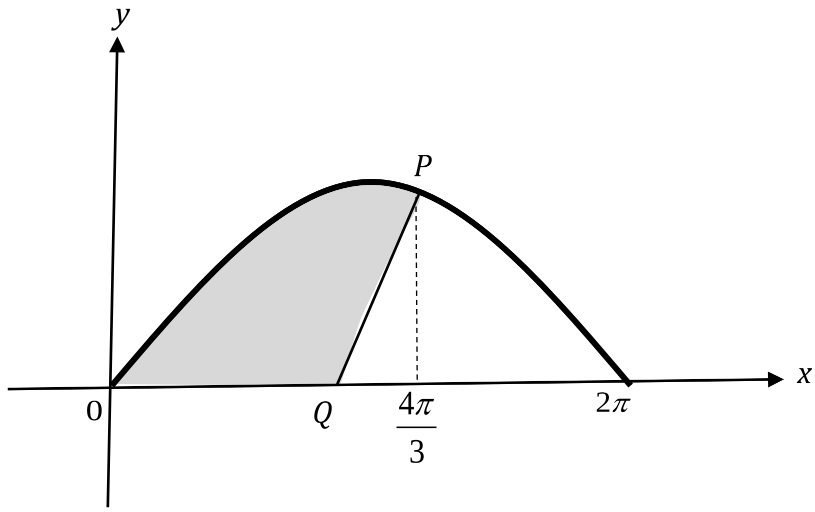



**(a)** Show that the cost in dollars, $C$, of the materal required to make the container is $C = \dfrac{3}{50}\pi r^2 + \dfrac{20}{r}$. [3]

**(b)** Given that $r$ can vary, find the value of $r$ for which the cost of material is stationary. [3]

**(c)** Justify, why the beverage company should choose to use the value of $r$ found in **part (b)** in producing the beverage can. [3]

**10**




The diagram shows part of the curve $y = 3\sin\frac{x}{2}$ that cuts the $x$ – axis at $x = 0$ and $x = 2\pi$ . The normal to the curve at $x = \frac{4\pi}{3}$ cuts the $x$-axis at $Q$.

**(a)** Find the coordinates of $Q$, leaving your answer in exact form. [6]

**(b)** Find the area of the shaded region bounded by the curve, the normal *PQ* and the coordinate axes. [4]

**11** A particle moves in a straight line, so that, $t$ seconds after passing a fixed point $A$, its velocity, $v$ m/s, is given by $v = 10e^{-0.1t} - 5$. The particle comes to instantaneous rest at the point $B$.

**(a)** Find the value of $t$ when the particle reaches $B$. Express your answer in the form of $p \ln q$, where $p$ and $q$ are integers. [3]

**(b)** Calculate the distance $AB$. [4]

**(c)** Find the acceleration of the particle when $t = 3$. [2]

**(d)** Show that the particle is again at $A$ at some instant during the sixteenth second after first passing through $A$. [3]

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