PRESBYTERIAN HIGH SCHOOL

MATHEMATICS   **4045/01**
PAPER 1

31 July 2023   Monday   2 hours

2023 SECONDARY FOUR NORMAL
PRELIMINARY EXAMINATION

MARK
SCHEME

Answer all the questions.

1
65% $\dfrac{\pi}{5}$ 0.6 $\sqrt{0.4}$ $\dfrac{5}{8}$

Write these numbers in order of size, starting with the smallest. [2]

0.6, $\dfrac{5}{8}$, $\dfrac{\pi}{5}$, $\sqrt{0.4}$, 65%	B1 for any 2 correct

2 Find the smallest integer satisfying $4x > -75$. [2]

$4x > -75$ $x > -18\dfrac{3}{4}$ smallest integer $= -18$	B1 B1

3 In a class of 40 pupils, 26 are boys.

10 of the boys wear spectacles and $\dfrac{3}{7}$ of the girls wear spectacles.

(a) Write the ratio boys : girls in its simplest form. [1]

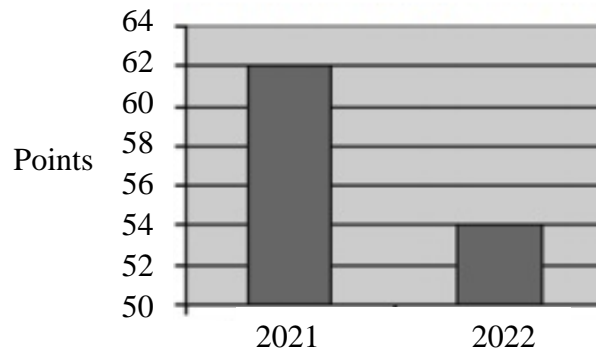|	
boys : girls 13 : 7	B1

(b) Write the ratio boys who wear spectacles : girls who wear spectacles in its simplest form.

[1]

boys wear spec: girls wear spec
     10 : 6
      5 : 3
	B1

4   The graph shows the number of points collected by a football team for the football league seasons in 2021 and 2022 respectively.

State one feature of the graph that may be misleading and explain why.   [2]

Answer

| The vertical axis does not start from zero. | B1 |
| It makes the difference in values on the vertical axis exaggerated./ It looks like the points collected in 2021 is triple of those collected in 2022. | B1 |

5   James bought 24 Snickers, 60 Mars and 72 M&M chocolates. If each type of chocolate is distributed equally into a number of party goodie bags, what is the greatest number of goodie bags that can be prepared?   [2]

| $24 = 2^3 \times 3$
 $60 = 2^2 \times 3 \times 5$
 $72 = 2^3 \times 3^2$
 $\text{HCF} = 2^2 \times 3$
 $\quad = 12 \text{ bags}$ | B1 for any 2 prime factorisation

 B1 |

6 Draw an enlargement of the figure using a scale factor of $\frac{3}{2}$.
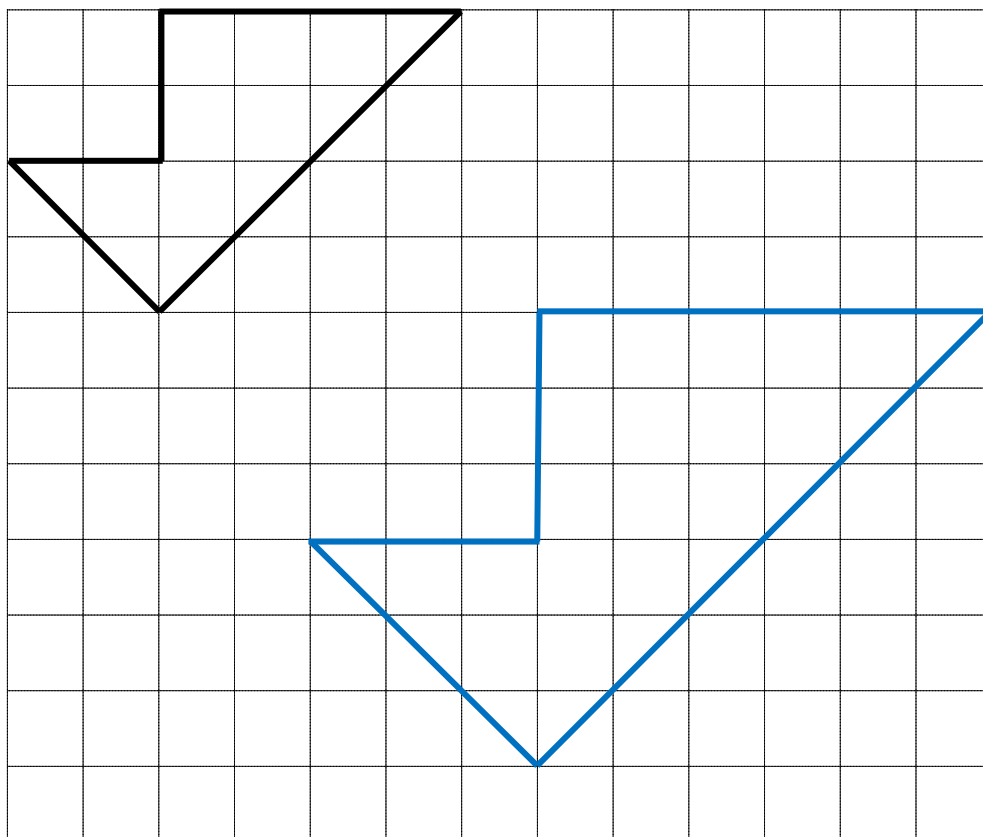
[2]

7
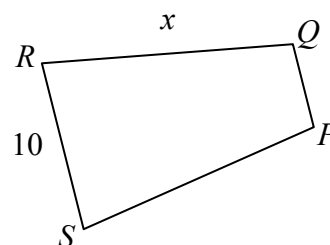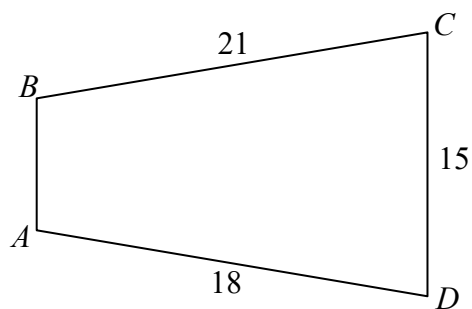

Figure *PQRS* is similar to figure *ABCD*.
All the lengths are in centimetres.

Find the value of *x*. [2]

$\dfrac{BC}{QR} = \dfrac{CD}{RS}$	
$\dfrac{21}{x} = \dfrac{15}{10}$	M1
$15x = 210$	
$x = 14$	A1

8   Given that sin $y°$ = 0.65, find the two possible values of y.   [2]

$y = 40.5416°$ or $180° - 40.5416°$	
$= 40.5°$ or $139.5°$	B1, B1

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9   The first 5 terms of a sequence are 16, 13, 10, 7, 4.

   (a)   Write down the next two terms.   [1]

1, –2	B1

   (b)   Find an expression for the nth term of this sequence.   [1]

$19 - 3n$	B1

10 A group of students were asked to vote for their favourite T-shirt colour.
The results are shown in the accurate pie chart below.
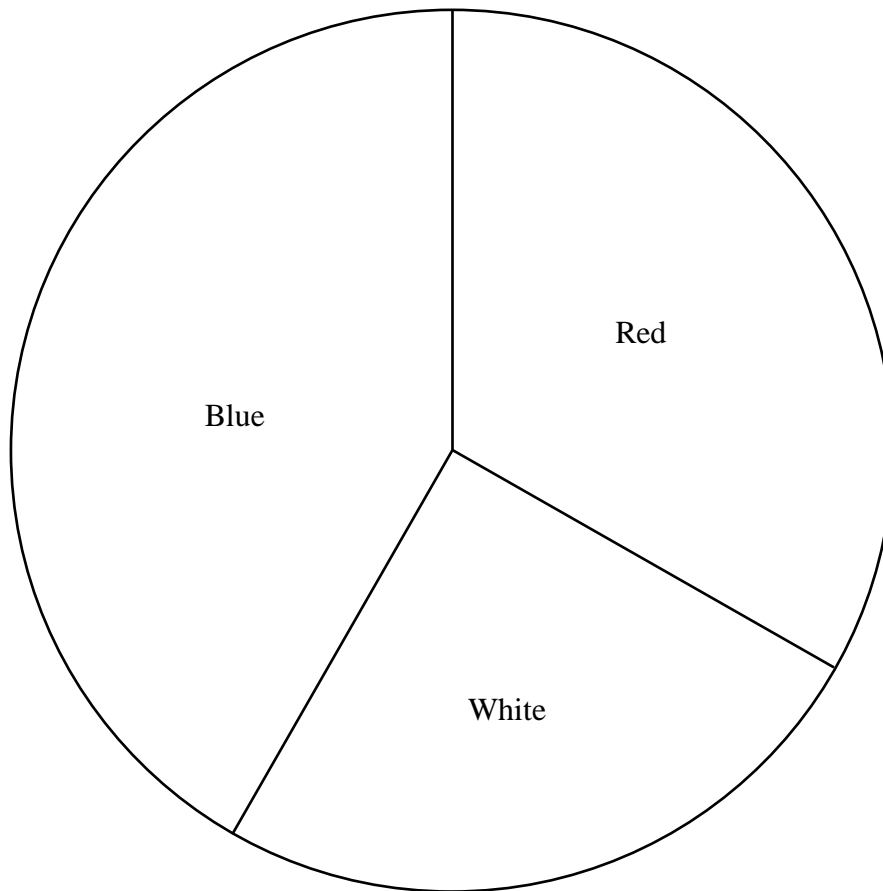
The number of students who chose blue is 12 more than the number of students who chose red.
Find the total number of students in the group. [3]

blue $=150°$, red $=120°$ $30°$ rep 12 students $360°$rep $12 \times \dfrac{360°}{30°}$ $= 144$ students	B1 for difference in angles M1 A1

11 Mark's monthly salary is \$3100.

In a particular month, he spent $\dfrac{3}{10}$ of his salary on food, gave \$150 to his parents and saved 20% of his salary. He spent the rest of his salary on outings with his friends.

Mark told his friends that he spent more than 48% of his salary on outings with his friends. Is his statement correct? Justify your answer with working. [3]

$\text{food} = \dfrac{3}{10} \times 3100 = \$930$ $\text{saving} = \dfrac{20}{100} \times 3100 = \$620$ $\text{outing} = 3100 - 620 - 930 - 150 = 1400$ $\%\text{ spent on outing} = \dfrac{1400}{3100} \times 100$ $\qquad\qquad\qquad = 45.16129$ $\qquad\qquad\qquad = 45.2\%$ His statement is <u>incorrect</u> since $45.2\% < 48\%$.	M1 for food/ saving M1 A1

12 Given $Q = 4m^2 - n$,

(a) find Q when $m = 3$ and $n = -5$, [1]

41	B1

(b) rearrange the formula to make m the subject. [2]

$Q = 4m^2 - n$ $4m^2 = Q + n$ $m^2 = \dfrac{Q+n}{4}$ $m = \pm\sqrt{\dfrac{Q+n}{4}}$	 M1 A1

13 A heptagon has 4 interior angles that are $80°$ each while the remaining interior angles are $(2x-5)°$, $(x+30)°$ and $(3x+60)°$.
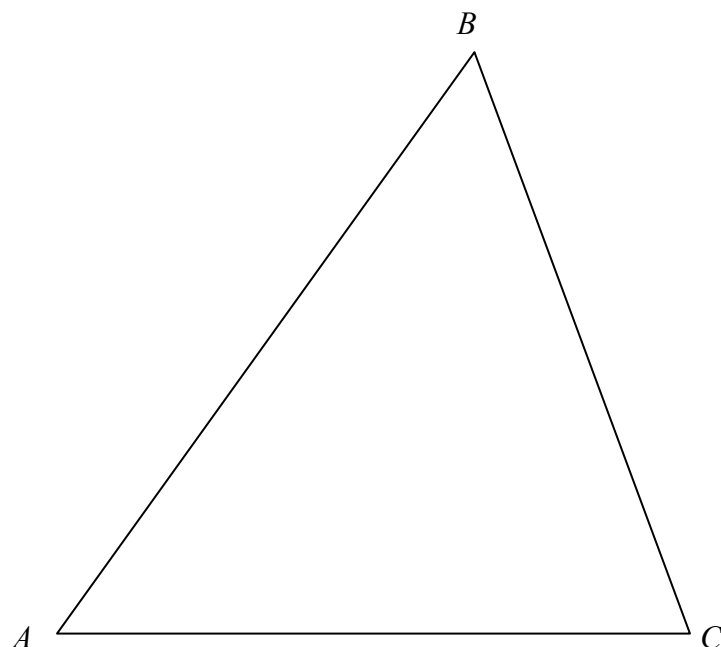
Find the value of x. [2]

$4(80)+(2x-5)+(x+30)+(3x+60)=(7-2)180$	B1 for sum of int angles
$6x+405=900$	
$x=82.5$	A1

14 Ben invested $5000 in an account earning compound interest of r% per year. At the end of 3 years, the total amount in the account is $5329.48.

Calculate r. [3]

$5000\left(1+\dfrac{r}{100}\right)^3=5329.48$	M1
$\left(1+\dfrac{r}{100}\right)^3=\dfrac{5329.48}{5000}$	
$1+\dfrac{r}{100}=\sqrt[3]{\dfrac{5329.48}{5000}}$	M1
$\dfrac{r}{100}=\sqrt[3]{\dfrac{5329.48}{5000}}-1$	
$r=\left(\sqrt[3]{\dfrac{5329.48}{5000}}-1\right)\times100$	
$=2.149978$	
$=2.15$	A1

M is the point of intersection of the bisector of angle BCA and the perpendicular bisector of AC.

(a) By using construction, find and label M. [2]

Correct perpendicular bisector/ correct angle bisector	B1
Correct location of M	B1

(b) Measure AM. [1]

5.1 cm (range: 5.0 to 5.2 cm)	B1

16 **(a)** Factorise $4y^2 - 1$. [1]

$(2y+1)(2y-1)$	B1

(b) Hence, or otherwise, express $\dfrac{1}{4y^2-1} + \dfrac{6}{2y+1}$ as a single fraction. [3]

$\dfrac{1}{(2y+1)(2y-1)} + \dfrac{6(2y-1)}{(2y+1)(2y-1)}$	M1 for LCM
$= \dfrac{1+12y-6}{(2y+1)(2y-1)}$	M1
$= \dfrac{12y-5}{(2y+1)(2y-1)}$	A1

17 Mary travels from Singapore to Hong Kong.
She exchanges S$2500 into Hong Kong dollars (HK$) when the exchange rate is S$1 = HK$5.85.

(a) While in Hong Kong, she spent HK$13200.
On her return, she exchanges the remaining Hong Kong dollars into Singapore dollars when the exchange rate is S$1 = HK$5.82.
How many Singapore dollars does she receive? [2]

$2500 \times 5.85 = HK\$14625$ $\text{balance} = 14625 - 13200$ $\quad\quad = HK\$1425$ $\quad\quad = S\$(\$1425 \div 5.82)$ $\quad\quad = S\$244.85$	B1 B1

(b) Mary wants to buy a business bag.
The business bag in Hong Kong costs HK$699. The same bag sold in Singapore costs S$139.
Considering the same exchange rate of S$1 = HK$5.85, would Mary pay less for the bag in Hong Kong or Singapore? Justify your answer with calculations. [2]

Answer

$HK\$699 = S\$\left(\dfrac{699}{5.85}\right)$ $\quad\quad\quad = S\$119.49$	B1
Mary should pay less for the bag in <u>Hong Kong</u> since S$119.49 < S$139.	B1

18 Two fair 4-sided dice, *A* and *B* are rolled once.
Die *A* has the numbers 1, 2, 3, 4 and Die *B* has the numbers 2, 4, 5, 7.
The product of the two numbers shown is recorded.
The table shows some of the possible products.

(a) Complete the table to show all the possible outcomes. [2]

Die *B*

	×	2	4	5	7
	1	2	4	5	7
Die *A*	2	4	8	**10**	**14**
	3	6	**12**	**15**	**21**
	4	8	**16**	**20**	**28**

(b) Find the probability that

(i) both dice show even numbers, [1]

$\dfrac{4}{16} = \dfrac{1}{4}$	B1

(ii) the product is a number less than 10, [1]

$\dfrac{8}{16} = \dfrac{1}{2}$	B1

(iii) the product is a factor of 16. [1]

$\dfrac{6}{16} = \dfrac{3}{8}$	B1

19 (a) Factorise completely $3ab - 5bc + 6a - 10c$. [2]

$3ab - 5bc + 6a - 10c$	
$= b(3a - 5c) + 2(3a - 5c)$	M1
$= (3a - 5c)(b + 2)$	A1

(b) $x^2 + 6x - 2 = (x + a)^2 + b$

    (i) Find a and b. [2]

$x^2 + 6x - 2 = x^2 + 6x + \left(\dfrac{6}{2}\right)^2 - \left(\dfrac{6}{2}\right)^2 - 2$	
$\qquad = (x + 3)^2 - \left(\dfrac{6}{2}\right)^2 - 2$	
$\qquad = (x + 3)^2 - 11$	
$a = 3, \quad b = -11$	B1, B1

    (ii) Hence solve $x^2 + 6x - 2 = 0$, giving your answers correct to 2 decimal places. [2]

$(x + 3)^2 - 11 = 0$	
$(x + 3)^2 = 11$	
$(x + 3) = \pm\sqrt{11}$	M1 (hence method)
$x = 0.3166, \quad -6.3166$	
$\qquad = 0.32, \qquad -6.32$	A1 for both answers

20 P is the point $(-8,\ 0)$ and Q is the point $(k,\ k+5)$ with $k>0$.

The length of PQ is 15 units.

(a) Form an equation in k and show that it simplifies to $k^2+13k-68=0$. [2]

Answer

$\sqrt{(k+8)^2+(k+5)^2}=15$	M1
$k^2+16k+64+k^2+10k+25=225$	
$2k^2+26k-136=0$	
$k^2+13k-68=0$	A1

(b) Using factorisation, solve $k^2+13k-68=0$ and find the coordinates of Q. [3]

$k^2+13k-68=0$	
$(k-4)(k+17)=0$	M1
$k=4,\quad -17$	A1
$Q(4,9)$	A1 for both answers

21 (a) The line $2x - 5y + 3 = 0$ has gradient m and y-intercept (a, b).
Find m and (a, b). [2]

$2x - 5y + 3 = 0$	
$y = \dfrac{2}{5}x + \dfrac{3}{5}$	
$m = \dfrac{2}{5}$	B1 for gradient
$(a, b) = \left(0, \dfrac{3}{5}\right)$	B1 for coordinates of y-intercept

(b) The diagram shows the graph of $y = x(x - 4)$ and it passes through the x-axis at $A(0, 0)$ and $B(k, 0)$.
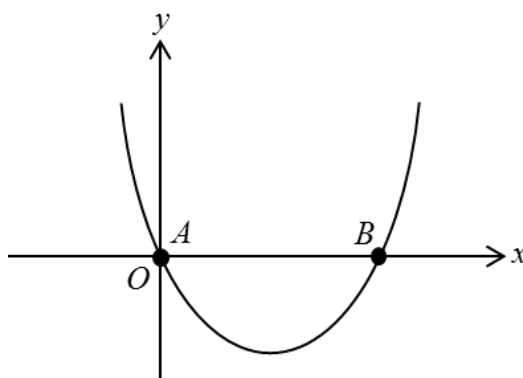
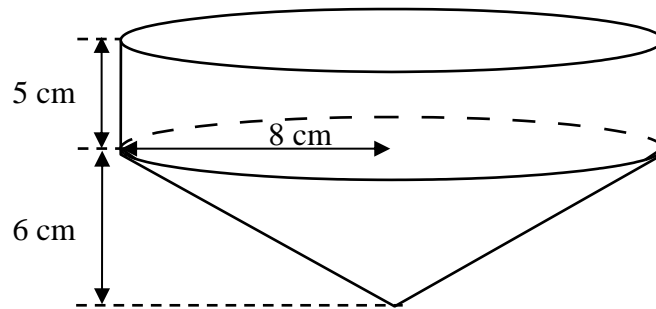**(i)** Show that the value of k is 4. [1]

Answer

$x(x - 4) = 0$	
$x = 0$ or 4	
$k = 4$	B1

(ii) Find the coordinates of the minimum point. [2]

$x = \dfrac{0 + 4}{2}$	B1 for eqn of line of symmetry
$x = 2$	
$y = 2(2 - 4) = -4$	
min point $(2, -4)$	B1

22 An open container is made up of a hollow cylinder and a hollow conical base of radius 8 cm. The height of the cylinder is 5 cm and the vertical height of the conical base is 6 cm.

Calculate the

(a) total volume of the container, [3]

vol of cone $= \dfrac{1}{3}\pi(8)^2(6) = 402.1238$	M1
vol of cylinder $= \pi(8)^2(5) = 1005.3096$	M1
total vol $= 1407.4335$	
$= 1410 \text{ cm}^3$	A1

(b) total surface area of the container. [3]

slant ht $= \sqrt{6^2 + 8^2} = 10$	B1
SA of cone $= \pi(8)(10) = 251.3274$	M1 for either SA of cone/ cylinder
SA of cylinder $= 2\pi(8)(5) = 251.3274$	
total SA $= 502.6548$	
$= 503 \text{ cm}^2$	A1