Solutions (Vectors)

1 (i)	$\frac{x}{2} = -1 - z\ ,\ y = 0 \Rightarrow \underset{\sim}{r} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ Vector parallel to $\pi_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\underset{\sim}{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ $\pi_1 : r \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \Rightarrow r \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = -2$
(ii)	Let foot of perpendicular from Q to π_1 be N. $\overrightarrow{ON} = \begin{pmatrix} 3 \\ 13 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 3+\lambda \\ 13+3\lambda \\ 6+2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 2$ $\Rightarrow \lambda = -4$ $\overrightarrow{ON} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$
(iii)	$\overrightarrow{ON} = \frac{\overrightarrow{OQ} + \overrightarrow{OQ'}}{2} \Rightarrow \overrightarrow{OQ'} = 2\begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 13 \\ 6 \end{pmatrix} = \begin{pmatrix} -5 \\ -11 \\ -10 \end{pmatrix}$ $\overrightarrow{PQ'} = \begin{pmatrix} -5 \\ -11 \\ -10 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -10 \\ -10 \end{pmatrix} = -2\begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix}$ $\therefore l_2 : \underset{\sim}{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix}$ OR $\therefore l_2 : \underset{\sim}{r} = \begin{pmatrix} -5 \\ -11 \\ -10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix}$

(iv)	$\pi_1 : r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = -2 \qquad \pi_2 : r \bullet \begin{pmatrix}a\\6\\4\end{pmatrix} = b$ $\begin{pmatrix}1\\3\\2\end{pmatrix} = k\begin{pmatrix}a\\6\\4\end{pmatrix} \Rightarrow a = 2$ **<u>Method 1:</u>** $\pi_1 : r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = -2 \qquad \pi_2 : r \bullet 2\begin{pmatrix}1\\3\\2\end{pmatrix} = b \;\Rightarrow\; \pi_2 : r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = \frac{b}{2}$ Distance between the 2 planes = $\left\lvert \dfrac{\frac{b}{2} - (-2)}{\sqrt{1+9+4}} \right\rvert = \sqrt{224} \;\Rightarrow b = 108$ or -116 **<u>Method 2:</u>** Distance QN = $\sqrt{(3-(-1))^2 + (13-1)^2 + (6-(-2))^2} = \sqrt{224}$ $b = \overrightarrow{OQ} \bullet \begin{pmatrix}2\\6\\4\end{pmatrix} = \begin{pmatrix}3\\13\\6\end{pmatrix} \bullet \begin{pmatrix}2\\6\\4\end{pmatrix} = 108$ OR $b = \overrightarrow{OQ}' \bullet \begin{pmatrix}2\\6\\4\end{pmatrix} = \begin{pmatrix}-5\\-11\\-10\end{pmatrix} \bullet \begin{pmatrix}2\\6\\4\end{pmatrix} = -116$ **<u>Method 3</u>** $\pi_1 : r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = -2 \Rightarrow r \bullet \frac{1}{\sqrt{14}}\begin{pmatrix}1\\3\\2\end{pmatrix} = -\frac{2}{\sqrt{14}}$ $\pi_2 : r \bullet \frac{1}{\sqrt{14}}\begin{pmatrix}1\\3\\2\end{pmatrix} = -\frac{2}{\sqrt{14}} \pm \sqrt{224}$ $\Rightarrow r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = -2 \pm 56 = 54 \text{ or } -58 \Rightarrow b = 108 \text{ or } -116$
(v)	$a \neq 2,\ b \in \Re$ or $a = 2,\ b = -4$

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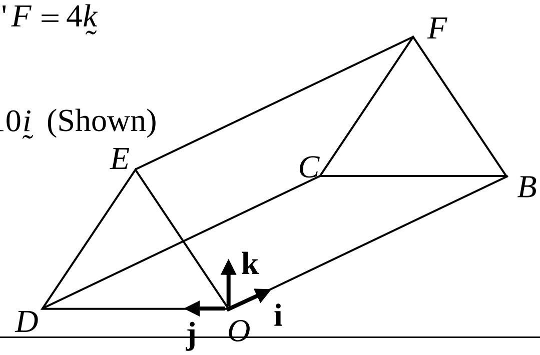
$\frac{FF'}{3} = \tan\theta \Rightarrow FF' = \frac{4}{3}\times 3 = 4 \quad \therefore \overrightarrow{F'F} = 4\underset{\sim}{k}$

$\overrightarrow{OF} = \overrightarrow{OB} + \overrightarrow{BF'} + \overrightarrow{F'F} = 3\underset{\sim}{j} + 4\underset{\sim}{k} + 10\underset{\sim}{i}$ (Shown)



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$\overrightarrow{DB} = \begin{pmatrix}10\\-6\\0\end{pmatrix} \qquad \overrightarrow{DE} = \begin{pmatrix}0\\-3\\4\end{pmatrix}$

$\underset{\sim}{n} = \begin{pmatrix}10\\-6\\0\end{pmatrix} \times \begin{pmatrix}0\\-3\\4\end{pmatrix} = \begin{pmatrix}-24\\-40\\-30\end{pmatrix} = -2\begin{pmatrix}12\\20\\15\end{pmatrix}$

$\sin\theta = \frac{\left|\begin{pmatrix}12\\20\\15\end{pmatrix} \bullet \begin{pmatrix}10\\3\\4\end{pmatrix}\right|}{\sqrt{769}\sqrt{125}}$

$\theta = 50.7228..^\circ = 50.7^\circ$

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(i)</td><td>

$\mathbf{a}\times\mathbf{b} = 4\mathbf{a}\times\mathbf{c}$

$(\mathbf{a}\times\mathbf{b}) - (4\mathbf{a}\times\mathbf{c}) = \mathbf{0}$

$(\mathbf{a}\times\mathbf{b}) - (\mathbf{a}\times 4\mathbf{c}) = \mathbf{0}$

$\mathbf{a}\times(\mathbf{b} - 4\mathbf{c}) = \mathbf{0}$

These are the three possible conclusion that can be drawn from the above equation.

1) $\mathbf{a} = \mathbf{0}$
2) $(\mathbf{b} - 4\mathbf{c}) = \mathbf{0}$
3) $\boldsymbol{a}$ is parallel to $\boldsymbol{b} - 4\boldsymbol{c}$, hence $\boldsymbol{b} - 4\boldsymbol{c} = \alpha\,\boldsymbol{a}$

We will present no. 3) as we are expected to show the given expression in (i).

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(ii)	$\frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert=\sqrt{126}$ $\frac{1}{2}\lvert 4\mathbf{a}\times\mathbf{c}\rvert=\sqrt{126}$ $\lvert\mathbf{a}\times\mathbf{c}\rvert=\frac{\sqrt{126}}{2}$ $\left\lvert\left(\frac{\mathbf{b}-4\mathbf{c}}{\sqrt{3}}\right)\times\mathbf{c}\right\rvert=\frac{\sqrt{126}}{2}$ $\lvert(\mathbf{b}\times\mathbf{c})-(4\mathbf{c}\times\mathbf{c})\rvert=\frac{\sqrt{3}\sqrt{126}}{2}$ $\lvert(\mathbf{b}\times\mathbf{c})\rvert=\frac{\sqrt{378}}{2}$
(iii)	Area of parallelogram with adjacent sides OB and OC.
(iv)	$(\boldsymbol{b}-4\boldsymbol{c})\cdot(\boldsymbol{b}-4\boldsymbol{c})=3\lvert\boldsymbol{a}\rvert^2$ $\lvert\boldsymbol{b}\rvert^2-8\,\boldsymbol{b}\cdot\boldsymbol{c}+16\lvert\boldsymbol{c}\rvert^2=3\lvert\boldsymbol{a}\rvert^2$ $\boldsymbol{b}\cdot\boldsymbol{c}=-\frac{10}{8}$ $\cos\theta=\frac{\boldsymbol{b}\cdot\mathrm{c}}{\lvert\boldsymbol{b}\rvert\lvert\boldsymbol{c}\rvert}=\frac{-\frac{10}{8}}{1(2)}$ $\theta=128.7^\circ$ and not 51.3^0

4	**<u>Question (i)</u>** $\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c}=\mathbf{0}$ $\Rightarrow$ $\mathbf{a},\mathbf{b},\mathbf{c}$ lie on the same plane $\Rightarrow\mathbf{a}\times(\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c})=\mathbf{a}\times\mathbf{0}$ $\Rightarrow\lambda(\mathbf{a}\times\mathbf{a})+\mu(\mathbf{a}\times\mathbf{b})+\gamma(\mathbf{a}\times\mathbf{c})=\mathbf{0}$ $\Rightarrow\mathbf{0}+\mu(\mathbf{a}\times\mathbf{b})=-\gamma(\mathbf{a}\times\mathbf{c})$ $\Rightarrow\mu(\mathbf{a}\times\mathbf{b})=\gamma(\mathbf{c}\times\mathbf{a})$ (shown) … (A) Similarly $\Rightarrow\mathbf{b}\times(\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c})=\mathbf{b}\times\mathbf{0}$ $\Rightarrow\lambda(\mathbf{b}\times\mathbf{a})+\mu(\mathbf{b}\times\mathbf{b})+\gamma(\mathbf{b}\times\mathbf{c})=\mathbf{0}$ $\Rightarrow\lambda(\mathbf{b}\times\mathbf{a})=-\gamma(\mathbf{b}\times\mathbf{c})$ $\Rightarrow\lambda(\mathbf{b}\times\mathbf{a})=\gamma(\mathbf{c}\times\mathbf{b})$ … (B) **<u>Question (ii)</u>** Consider,

	$\|\mathbf{b}\times\mathbf{c}\| = \|\mathbf{b}\|\|\mathbf{c}\|\sin\angle BOC$ ... (1) $\|\mathbf{c}\times\mathbf{a}\| = \|\mathbf{c}\|\|\mathbf{a}\|\sin\angle COA$ ... (2) $\|\mathbf{a}\times\mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\|\sin\angle AOB$ ... (3) Take $\frac{(2)}{(3)}: \frac{\|\mathbf{c}\times\mathbf{a}\|}{\|\mathbf{a}\times\mathbf{b}\|} = \frac{\|\mathbf{c}\|\|\mathbf{a}\|\sin\angle COA}{\|\mathbf{a}\|\|\mathbf{b}\|\sin\angle AOB}$ from (A): $\frac{\frac{\|\mu\|}{\|\gamma\|}\|\mathbf{a}\times\mathbf{b}\|}{\|\mathbf{a}\times\mathbf{b}\|} = \frac{\sin\angle COA}{\sin\angle AOB} \Rightarrow \frac{\|\mu\|}{\sin\angle COA} = \frac{\|\gamma\|}{\sin\angle AOB}$ Similarly, taking $\frac{(1)}{(3)}: \frac{\|\mathbf{b}\times\mathbf{c}\|}{\|\mathbf{a}\times\mathbf{b}\|} = \frac{\|\mathbf{b}\|\|\mathbf{c}\|\sin\angle BOC}{\|\mathbf{a}\|\|\mathbf{b}\|\sin\angle AOB}$ from (B): $\frac{\frac{\|\lambda\|}{\|\gamma\|}\|\mathbf{b}\times\mathbf{a}\|}{\|\mathbf{a}\times\mathbf{b}\|} = \frac{\sin\angle BOC}{\sin\angle AOB} \Rightarrow \frac{\|\lambda\|}{\sin\angle BOC} = \frac{\|\gamma\|}{\sin\angle AOB}$ Thus, $\frac{\|\mu\|}{\sin\angle COA} = \frac{\|\lambda\|}{\sin\angle BOC} = \frac{\|\gamma\|}{\sin\angle AOB}$ (proven)

Q5 solutions
$\overrightarrow{PQ} = (-2\underset{\sim}{a} - 3\underset{\sim}{b}) - (\underset{\sim}{a} - 2\underset{\sim}{b}) = -3\underset{\sim}{a} - \underset{\sim}{b}$ $\overrightarrow{PR} = (2\underset{\sim}{a} + \underset{\sim}{b}) - (\underset{\sim}{a} - 2\underset{\sim}{b}) = \underset{\sim}{a} + 3\underset{\sim}{b}$ Area of triangle PQR $= \frac{1}{2}\left\|\overrightarrow{PQ}\times\overrightarrow{PR}\right\|$ $= \frac{1}{2}\left\|(-3\underset{\sim}{a} - \underset{\sim}{b})\times(\underset{\sim}{a} + 3\underset{\sim}{b})\right\|$ $= \frac{1}{2}\left\|(-3\underset{\sim}{a}\times\underset{\sim}{a}) - (9\underset{\sim}{a}\times\underset{\sim}{b}) - (\underset{\sim}{b}\times\underset{\sim}{a}) - (3\underset{\sim}{b}\times\underset{\sim}{b})\right\|$ $= 4\left\|\underset{\sim}{a}\times\underset{\sim}{b}\right\|$ (Since $\underset{\sim}{a}\times\underset{\sim}{a} = \underset{\sim}{0}, \underset{\sim}{b}\times\underset{\sim}{b} = \underset{\sim}{0}$, and $\underset{\sim}{b}\times\underset{\sim}{a} = -\underset{\sim}{a}\times\underset{\sim}{b}$) Since area of triangle PQR = $\|\underset{\sim}{b}\|$, $4\|\underset{\sim}{a}\times\underset{\sim}{b}\| = \|\underset{\sim}{b}\|$. $4\|\underset{\sim}{a}\|\|\underset{\sim}{b}\|\sin\theta = \|\underset{\sim}{b}\|$ $\sin\theta = \frac{1}{4}$ (since $\underset{\sim}{a}\bullet\underset{\sim}{b} < 0, \theta$ is obtuse) $\theta = 165.5°$
$\overrightarrow{OM} = \frac{2\overrightarrow{OP} + \overrightarrow{OR}}{3}$ $= \frac{2(\underset{\sim}{a} - 2\underset{\sim}{b}) + (2\underset{\sim}{a} + \underset{\sim}{b})}{3}$ $= \frac{4\underset{\sim}{a} - 3\underset{\sim}{b}}{3}$

$$\left(\frac{4\underset{\sim}{a}-3\underset{\sim}{b}}{3}\right)\bullet\left(\underset{\sim}{a}+3\underset{\sim}{b}\right)=0$$

$$\left(4\underset{\sim}{a}-3\underset{\sim}{b}\right)\bullet\left(\underset{\sim}{a}+3\underset{\sim}{b}\right)=0$$

$$4\left|\underset{\sim}{a}\right|^2+12\underset{\sim}{a}\bullet\underset{\sim}{b}-3\underset{\sim}{a}\bullet\underset{\sim}{b}-9\left|\underset{\sim}{b}\right|^2=0$$

$$4+9\underset{\sim}{a}\bullet\underset{\sim}{b}-9\left|\underset{\sim}{b}\right|^2=0$$

$$9\left|\underset{\sim}{b}\right|^2-9\left|\underset{\sim}{b}\right|\cos\theta-4=0$$

Since $\sin\theta=\frac{1}{4},\cos\theta=-\frac{\sqrt{15}}{4}$

$$9\left|\underset{\sim}{b}\right|^2+\frac{9\sqrt{15}}{4}\left|\underset{\sim}{b}\right|-4=0$$

By GC,

$\left|\underset{\sim}{b}\right|=-1.31$ or $\left|\underset{\sim}{b}\right|=0.340$

Since $\left|\underset{\sim}{b}\right|>0$, $\left|\underset{\sim}{b}\right|=0.340$

6

$$\begin{pmatrix}1+\lambda\\ \lambda\\ 1-2\lambda\end{pmatrix}=\begin{pmatrix}1+7\mu\\ 2+5\mu\\ -1+\mu\end{pmatrix}$$

$\lambda-7\mu=0$ (1)

$\lambda-5\mu=2$ (2)

$2\lambda+\mu=2$ (3)

Solving (1) and (2): $\mu=1, \lambda=7$

(3): $2(7)+1=15\neq 2$

No unique solution and the lines are not parallel. Therefore, they are skew lines.

Let $\overrightarrow{OM}=\begin{pmatrix}1+\lambda\\ \lambda\\ 1-2\lambda\end{pmatrix}$ for some $\lambda\in\mathbb{R}$.

$$\overrightarrow{AM}\bullet\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow\begin{pmatrix}\lambda\\ \lambda-2\\ 2-2\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-2\end{pmatrix}=0$$

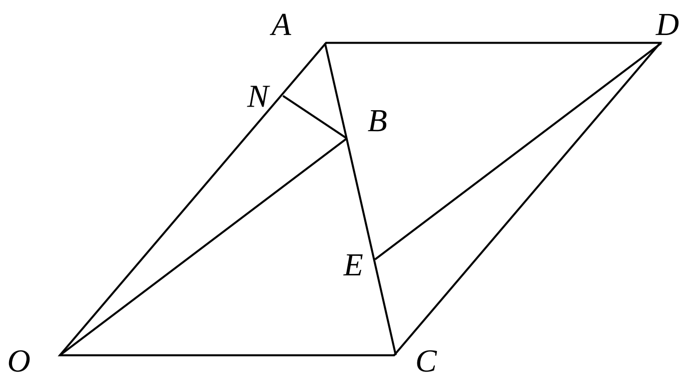
$$\lambda+\lambda-2-4+4\lambda=0\Rightarrow\lambda=1$$

$$|\overrightarrow{AM}|=\left\|\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\|=\sqrt{2}\text{ units}$$

	OR Let B = (1, 0, 1). $\vec{BA}=\begin{pmatrix}0\\2\\-2\end{pmatrix}$ Shortest distance = $\dfrac{1}{\sqrt{1+1+4}}\left\lvert\begin{pmatrix}0\\2\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\-2\end{pmatrix}\right\rvert=\dfrac{1}{\sqrt{6}}\left\lvert\begin{pmatrix}-2\\-2\\-2\end{pmatrix}\right\rvert=\sqrt{2}$

7(i)	$\lvert\mathrm{a}\rvert=2\lvert\mathrm{b}\rvert$ $\sqrt{16+36p^2+64}=2\sqrt{4+9+16p^2}$ $80+36p^2=4\left(13+16p^2\right)$ $28p^2=28$ $p=1$ or $p=-1$ $\left(\text{Reject } \because\ p>0\right)$ $\therefore\ p=1$
(ii)	Length of projection of **a** on **b**
(iii)	$\dfrac{1}{\lvert\mathbf{b}\rvert}\lvert\mathbf{b}\cdot\mathbf{a}\rvert=\dfrac{\left\lvert\begin{pmatrix}2\\-3\\4\end{pmatrix}\cdot\begin{pmatrix}4\\6\\-8\end{pmatrix}\right\rvert}{\sqrt{2^2+(-3)^2+4^2}}=\dfrac{42}{\sqrt{29}}$

8(i)	Since $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$ and A, B and C are collinear, $\therefore k=2$ $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}=\mathbf{b}+2\mathbf{b}-2\mathbf{a}=3\mathbf{b}-2\mathbf{a}$

(ii)	$\frac{3}{4}\lvert\mathbf{a}\rvert = \left\lvert \mathbf{b}.\frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}\right\rvert$ $\frac{3}{4}\lvert\mathbf{a}\rvert^2 = \lvert 4\rvert$ $\lvert\mathbf{a}\rvert^2 = \frac{16}{3}$ $\lvert\mathbf{a}\rvert = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$ Alternatively, $\frac{3}{4}\mathbf{a} = \left(\mathbf{b}\cdot\hat{\mathbf{a}}\right)\hat{\mathbf{a}} = \frac{(\mathbf{b}\cdot\mathbf{a})(\mathbf{a})}{\lvert\mathbf{a}\rvert^2}$ $\Rightarrow \frac{3}{4}\lvert\mathbf{a}\rvert = \frac{4\lvert\mathbf{a}\rvert}{\lvert\mathbf{a}\rvert^2}$ $\frac{3}{4}\lvert\mathbf{a}\rvert^2 = \lvert 4\rvert$ $\lvert\mathbf{a}\rvert^2 = \frac{16}{3}$ $\lvert\mathbf{a}\rvert = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$
(iii)	$\lvert \mathrm{b}\rvert = \sqrt{2^2+2^2+2^2} = 2\sqrt{3}$ $\cos\theta = \frac{\mathrm{a.b}}{\lvert\mathrm{a}\rvert\lvert\mathrm{b}\rvert} = \frac{4}{\left(\frac{4\sqrt{3}}{3}\right)\left(2\sqrt{3}\right)} = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ or 60^0
(iv)	

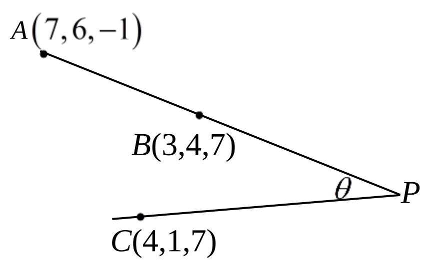
	$\overrightarrow{DE}=\dfrac{2\overrightarrow{DC}+\overrightarrow{DA}}{3}=\dfrac{2(-\mathbf{a})+(2\mathbf{a}-3\mathbf{b})}{3}=-\mathbf{b}=\begin{pmatrix}-2\\2\\-2\end{pmatrix}$ Alternatively, $\overrightarrow{AE}=\dfrac{2}{3}\overrightarrow{AE}=\dfrac{2}{3}(3\mathbf{b}-2\mathbf{a}-\mathbf{a})=2\mathbf{b}-2\mathbf{a}$ $\overrightarrow{DE}=\overrightarrow{DA}+\overrightarrow{AE}=\overrightarrow{CO}+\overrightarrow{AE}$ $=2\mathbf{a}-3\mathbf{b}+2\mathbf{b}-2\mathbf{a}$ $=-\mathbf{b}$ $=\begin{pmatrix}-2\\2\\-2\end{pmatrix}$

9(i)	$\overrightarrow{AB}=\begin{pmatrix}2\\-1\\4\end{pmatrix}-\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}$ $\overrightarrow{AC}=\begin{pmatrix}-2\\-1\\0\end{pmatrix}-\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}-2\\-2\\-1\end{pmatrix}$ $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}2\\-2\\3\end{pmatrix}\times\begin{pmatrix}-2\\-2\\-1\end{pmatrix}=\begin{pmatrix}8\\-4\\-8\end{pmatrix}=4\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ Choose normal vector $\underset{\sim}{n}_p$ for plane p = $\begin{pmatrix}2\\-1\\-2\end{pmatrix}$. p: $\mathbf{r}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=\begin{pmatrix}0\\1\\1\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=-3$ A cartesian equation of the plane p is $2x-y-2z=-3$
(ii)	Let the acute angle between l and p be θ. The angle between the normal vector $\underset{\sim}{n}_p$ (for plane p) and the direction vector $\underset{\sim}{m}_l$ (for line l),

	$\alpha=\cos^{-1}\dfrac{\begin{pmatrix}2\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-2\end{pmatrix}}{\left\|\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right\|\left\|\begin{pmatrix}1\\0\\-2\end{pmatrix}\right\|}=\cos^{-1}\dfrac{6}{3\sqrt{5}}=26.565°$ $\therefore\theta=90°-26.565°=63.4°(\text{to 1d.p.})$ or 1.11 rad <u>Alternative :</u> Let the acute angle between *l* and *p* be θ. $\sin\theta=\dfrac{\left\|\begin{pmatrix}2\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-2\end{pmatrix}\right\|}{\left\|\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right\|\left\|\begin{pmatrix}1\\0\\-2\end{pmatrix}\right\|}=\dfrac{6}{3\sqrt{5}}$ $\theta=\sin^{-1}\dfrac{6}{3\sqrt{5}}=63.4°(\text{to 1d.p.})$ or 1.11 rad (3 s.f.)
(iii)	Assume *Q* lies on the line *l*. $\begin{pmatrix}5\\-1\\-2\end{pmatrix}=\begin{pmatrix}2\\-1\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\-2\end{pmatrix}\Rightarrow\begin{cases}5=2+\lambda\\-1=-1\\-2=4-2\lambda\end{cases}\Rightarrow\begin{cases}\lambda=3\\ \\ \lambda=3\end{cases}$ Since $\lambda=3$ is consistent throughout, *Q* lies on the line *l*. --- <u>Alternative 1:</u> Since $\overrightarrow{OQ}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}=\begin{pmatrix}2\\-1\\4\end{pmatrix}+\begin{pmatrix}3\\0\\-6\end{pmatrix}$ $=\begin{pmatrix}2\\-1\\4\end{pmatrix}+3\begin{pmatrix}1\\0\\-2\end{pmatrix}$ *Q* lies on the line *l*. --- <u>Alternative 2:</u> $l:\ \dfrac{x-2}{1}=\dfrac{z-4}{-2},\ y=-1$

	$Q=(5,-1,-2)$, i.e. x = 5, y = -1, z = 4. $\frac{x-2}{1}=\frac{5-2}{1}=3$, $\frac{z-4}{-2}=\frac{-2-4}{-2}=3$. Hence, Q lies on the line l.
(iv)	Let F be the foot of perpendicular from the point Q to the plane p. l_{QF} : $\mathbf{r}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}, \mu\in\mathbb{R}$ Since F lies on l_{QF}, $\overrightarrow{OF}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}$, for some $\mu\in\mathbb{R}$. Since F also lies on plane p, $\overrightarrow{OF}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=-3$ $\left[\begin{pmatrix}5\\-1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right]\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=-3$ $2(5+2\mu)-(-1-\mu)-2(-2-2\mu)=-3$ $15+9\mu=-3\Rightarrow \mu=-2$ $\therefore\overrightarrow{OF}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}-2\begin{pmatrix}2\\-1\\-2\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ The foot of perpendicular from the point Q to the plane p is $(1,1,2)$. $\overrightarrow{QF}=\begin{pmatrix}1\\1\\2\end{pmatrix}-\begin{pmatrix}5\\-1\\-2\end{pmatrix}=\begin{pmatrix}-4\\2\\4\end{pmatrix}$ $\left\|\overrightarrow{QF}\right\|=\sqrt{(-4)^2+2^2+4^2}=6$ $RF=\sqrt{45-36}=3$ The locus of R is a circle that lies in plane p with <u>centre</u> $(1,1,2)$ and <u>radius</u> 3
(v)	Let Q' be the image of Q in plane p.

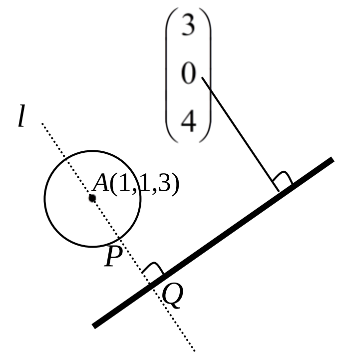
	$\overrightarrow{OF} = \frac{1}{2}(\overrightarrow{OQ} + \overrightarrow{OQ'})$ $\overrightarrow{OQ'} = 2\overrightarrow{OF} - \overrightarrow{OQ} = 2\begin{pmatrix}1\\1\\2\end{pmatrix} - \begin{pmatrix}5\\-1\\-2\end{pmatrix} = \begin{pmatrix}-3\\3\\6\end{pmatrix}$ <u>Alternative:</u> $\overrightarrow{BQ'} = \overrightarrow{BQ} + \overrightarrow{QQ'} = \left[\begin{pmatrix}5\\-1\\-2\end{pmatrix} - \begin{pmatrix}2\\-1\\4\end{pmatrix}\right] + 2\begin{pmatrix}-4\\2\\4\end{pmatrix} = \begin{pmatrix}-5\\4\\2\end{pmatrix}$ A vector equation of the line which is a reflection of the line l in plane p is $\mathbf{r} = \begin{pmatrix}2\\-1\\4\end{pmatrix} + \gamma\begin{pmatrix}-5\\4\\2\end{pmatrix}, \gamma \in \mathbb{R}$ or $\mathbf{r} = \begin{pmatrix}-3\\3\\6\end{pmatrix} + \gamma\begin{pmatrix}-5\\4\\2\end{pmatrix}, \gamma \in \mathbb{R}$

10 (i)	$2\mathbf{p} + \begin{pmatrix}7\\6\\-1\end{pmatrix} = 3\begin{pmatrix}3\\4\\5\end{pmatrix}$ $\mathbf{p} = \begin{pmatrix}1\\3\\8\end{pmatrix}$ 
(ii)	$\overrightarrow{AP}\bullet\overrightarrow{CP} = \left\lvert\overrightarrow{AP}\right\rvert\left\lvert\overrightarrow{CP}\right\rvert\cos\theta$ $\begin{pmatrix}-6\\-3\\9\end{pmatrix}\bullet\begin{pmatrix}-3\\2\\1\end{pmatrix} = \sqrt{126}\sqrt{14}\cos\theta$ $\cos\theta = \frac{21}{\sqrt{126}\sqrt{14}}$ $\theta = 60^0$
(iii)	Area = $\frac{1}{2}\left\lvert\overrightarrow{AP}\right\rvert\left\lvert\overrightarrow{CP}\right\rvert\sin\theta = \frac{1}{2}\sqrt{126}\sqrt{14}\frac{\sqrt{3}}{2} = \frac{21\sqrt{3}}{2}$ **OR**

	Area = $\frac{1}{2}\left\|\overrightarrow{AP}\times\overrightarrow{CP}\right\| = \frac{1}{2}\left\|\begin{pmatrix}-6\\-3\\9\end{pmatrix}\times\begin{pmatrix}-3\\2\\1\end{pmatrix}\right\| = \frac{21\sqrt{3}}{2}$

11

$l: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$



pt Q : $\begin{pmatrix} 1+3\lambda \\ 1 \\ 3+4\lambda \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = 40$

$\rightarrow \lambda = 1$

$\mathbf{q} = \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix}$

Let P be pt on sphere nearest to π

$AQ^2 = 9 + 0 + 16, AQ = 5 \rightarrow AP = 2 \;\&\; PQ = 3$

shortest dist between sphere & Plane = 3

$5\mathbf{p} = 3\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix}, \qquad \mathbf{p} = \frac{1}{5}\begin{pmatrix} 11 \\ 5 \\ 23 \end{pmatrix}$

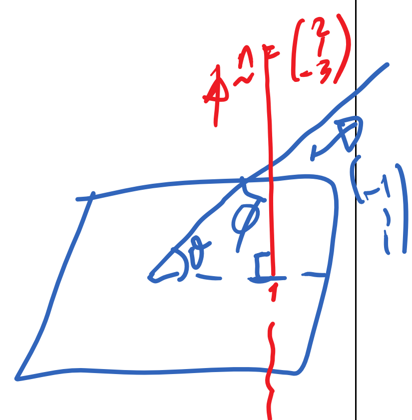
12 (i)

Normal of p_1 = $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{x} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

Direction of l_1 = $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

ACUTE angle between between l_1 and normal of p_1

$\cos^{-1}\left[\frac{\left| \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right|}{\sqrt{(4+1+9)(1+1+1)}} \right] = 51.9^\circ$ (ϕ)



Hence angle between l_1 and p_1 = $90^\circ - 51.9^\circ = 38.1^\circ$ (θ)

OR use the sine method directly

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<tr><td>(ii)</td><td>

$\begin{pmatrix}3.75\\0\\2.5\end{pmatrix}$ and $\begin{pmatrix}4\\-0.5\\2.5\end{pmatrix}$ are two points on the line $\dfrac{4x-15}{-2}=y; \quad z=\dfrac{5}{2}$

Substituting each point into equation of plane,
$3.75\alpha + 2.5\beta = 1$ …. (1)
$3.25\alpha + 2.5\beta = 0$ …. (2)
$\alpha = 2$, $\beta = -2.6$

Normal of $p_3 = \begin{pmatrix}2\\b\\1\end{pmatrix}$

Direction vector of $l_2 = \begin{pmatrix}-0.5\\1\\0\end{pmatrix}$

$\begin{pmatrix}2\\b\\1\end{pmatrix}.\begin{pmatrix}-0.5\\1\\0\end{pmatrix}=0; \quad b=1$

Eqn of plane p_4 containing line l_2 is

$\mathbf{r}.\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}3.75\\0\\2.5\end{pmatrix}.\begin{pmatrix}2\\1\\1\end{pmatrix}=10$

Distance between l_2 and $p_3 = \dfrac{10}{\sqrt{6}}-\dfrac{1}{\sqrt{6}}=\dfrac{3\sqrt{6}}{2}$

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<tr><td>13
(i)</td><td>

distance between point A and the plane π_1

$= \left| \dfrac{\begin{pmatrix}3\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix}}{\sqrt{2^2+1^2+2^2}} - \dfrac{6}{\sqrt{2^2+1^2+2^2}} \right| = \left|\dfrac{6-1-8}{3}-2\right| = 3 \text{ unit}$

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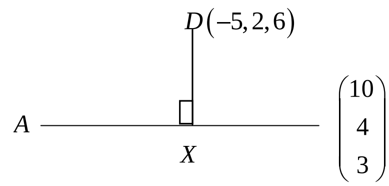
(ii)	$\overrightarrow{AB}=\begin{pmatrix}0\\-5\\-2\end{pmatrix}$, $\hat{\underset{\sim}{n}}=\dfrac{1}{\sqrt{2^2+1^2+(-2)^2}}\begin{pmatrix}2\\1\\-2\end{pmatrix}=\dfrac{1}{3}\begin{pmatrix}2\\1\\-6\end{pmatrix}$ length of projection $=\left\lvert\overrightarrow{AB}\times\hat{\underset{\sim}{n}}\right\rvert=\dfrac{1}{3}\left\lvert\begin{pmatrix}0\\-5\\-2\end{pmatrix}\times\begin{pmatrix}2\\1\\-2\end{pmatrix}\right\rvert$ $=\dfrac{1}{3}\left\lvert\begin{pmatrix}12\\-4\\10\end{pmatrix}\right\rvert=\dfrac{2}{3}\left\lvert\begin{pmatrix}6\\-2\\5\end{pmatrix}\right\rvert=\dfrac{2}{3}\sqrt{6^2+2^2+5^2}=\dfrac{2}{3}\sqrt{65}$ units Alternatively, use $\left\lvert\overrightarrow{AB}\bullet\hat{\underset{\sim}{n}}\right\rvert$, followed by Pythagoras thm
(iii)	Area of triangle $ABC=\dfrac{1}{2}(AC)(\dfrac{2}{3}\sqrt{65})=\dfrac{1}{2}(3\times 2)(\dfrac{2}{3}\sqrt{65})=2\sqrt{65}$ units^2
(iv)	$l: r=\begin{pmatrix}1\\-4\\5\end{pmatrix}+\lambda\begin{pmatrix}3\\2\\1\end{pmatrix},\lambda\in\Re$, Two vectors // to π_2 are $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ and $\begin{pmatrix}3\\2\\1\end{pmatrix}$, normal to π_2 is // to $\begin{pmatrix}1\\-4\\5\end{pmatrix}\times\begin{pmatrix}3\\2\\1\end{pmatrix}=\begin{pmatrix}-14\\14\\14\end{pmatrix}=14\begin{pmatrix}-1\\1\\1\end{pmatrix}$ Equation of π_2: $r\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=0$, ie. $-x+y+z=0$ To find line of intersection: $-x+y+z=0$ $2x+y-2z=6$ The augmented matrix, M is $\begin{pmatrix}-1&1&1&0\\2&1&-2&6\end{pmatrix}$, RREF (M) = $\begin{pmatrix}1&0&-1&2\\0&1&0&2\end{pmatrix}$ $x-z=2$ $y=2$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2+z\\2\\z\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}+\begin{pmatrix}1\\0\\1\end{pmatrix}z$ ie. Equation of line: $r=\begin{pmatrix}2\\2\\0\end{pmatrix}+\mu\begin{pmatrix}1\\0\\1\end{pmatrix},\mu\in\Re$ [Alternatively, Cartesian equation of line: $x-2=z, y=2$]

14	$\mathbf{p}\times\mathbf{q}=3\mathbf{p}\times\mathbf{r}$ $\mathbf{p}\times(\mathbf{q}-3\mathbf{r})=\mathbf{0}$ $\mathbf{p}\,//\,(\mathbf{q}-3\mathbf{r})$

	$\mathbf{q}-\mathbf{3r}=\lambda\mathbf{p}$, where λ is a scalar. $(\mathbf{q}-3\mathbf{r}).(\mathbf{q}-3\mathbf{r})=\lambda\mathbf{p}.\lambda\mathbf{p}$ $\|\mathbf{q}\|^2-6\mathbf{q}.\mathbf{r}+9\|\mathbf{r}\|^2=\lambda^2\|\mathbf{p}\|^2$ $5^2-6\times5\times2\left(\frac{5}{6}\right)+9(2)^2=\lambda^2(1)^2$ $\lambda^2=11$ $\lambda=\pm\sqrt{11}$

15 (i)	$\overrightarrow{AC}=\begin{pmatrix}5\\2\\6\end{pmatrix}-\begin{pmatrix}-5\\-2\\3\end{pmatrix}=\begin{pmatrix}10\\4\\3\end{pmatrix}$ line AC : $\mathrm{r}=\begin{pmatrix}5\\2\\6\end{pmatrix}+\lambda\begin{pmatrix}10\\4\\3\end{pmatrix}$ or $\mathrm{r}=\begin{pmatrix}-5\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}10\\4\\3\end{pmatrix}$, $\lambda\in\mathbb{R}$
(ii)	Let point R be the top of the pillar $\overrightarrow{OR}=\begin{pmatrix}15\\6\\h\end{pmatrix}$ lies on line AC $\begin{pmatrix}15\\6\\h\end{pmatrix}=\begin{pmatrix}5\\2\\6\end{pmatrix}+\lambda\begin{pmatrix}10\\4\\3\end{pmatrix}$ $15=5+\lambda10\Rightarrow\lambda=1$ $6=2+4\lambda\Rightarrow\lambda=1$ $h=6+3\lambda=9$ Height of pillar is 9m.
(iii)	$\overrightarrow{OX}=\begin{pmatrix}5\\2\\6\end{pmatrix}+\lambda\begin{pmatrix}10\\4\\3\end{pmatrix}$ for some λ $\overrightarrow{DX}=\begin{pmatrix}5\\2\\6\end{pmatrix}+\lambda\begin{pmatrix}10\\4\\3\end{pmatrix}-\begin{pmatrix}-5\\2\\6\end{pmatrix}=\begin{pmatrix}10+10\lambda\\4\lambda\\3\lambda\end{pmatrix}$

$$\begin{pmatrix}10+10\lambda\\4\lambda\\3\lambda\end{pmatrix}\bullet\begin{pmatrix}10\\4\\3\end{pmatrix}=0$$



$$100+100\lambda+16\lambda+9\lambda=0$$

$$\lambda=-\frac{4}{5}$$

$$\overrightarrow{OX}=\begin{pmatrix}5\\2\\6\end{pmatrix}-\frac{4}{5}\begin{pmatrix}10\\4\\3\end{pmatrix}=\frac{3}{5}\begin{pmatrix}-5\\-2\\6\end{pmatrix}$$

---

16 (i)

$$\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}\Rightarrow\begin{array}{ll}\mu=-\lambda & \text{---(1)}\\ \mu=\lambda & \text{---(2)}\\ \mu=\lambda-1 & \text{---(3)}\end{array}$$

The first and second equation has only 1 solution i.e. $\lambda=0$ and $\mu=0$ and it is obvious that equation (3) will be inconsistent for this solution; this implies that l_1 and l_2 are non-intersecting lines.

Since l_1 and l_2 are non-parallel lines as $\begin{pmatrix}1\\2\\2\end{pmatrix}\neq k\begin{pmatrix}-1\\2\\2\end{pmatrix}$ where k is a scalar

Since l_1 and l_2 are non-parallel and non-intersecting lines, l_1 and l_2 are skew lines.

(ii)

Let $\overrightarrow{OX}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}$ and $\overrightarrow{OY}=\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}$

$$\overrightarrow{OZ}=\frac{1}{2}\left(\overrightarrow{OX}+\overrightarrow{OY}\right)=\frac{1}{2}\left[\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}+\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}\right]=\begin{pmatrix}0\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}-\frac{1}{2}\\1\\1\end{pmatrix}+\mu\begin{pmatrix}\frac{1}{2}\\1\\1\end{pmatrix}$$

Since λ and μ can be any real number, the locus of Z is a plane that passes through (0, 0, -1) and parallel to both $-\frac{1}{2}\boldsymbol{i}+\mathbf{j}+\boldsymbol{k}$ and $\frac{1}{2}$ i + j + k ,

Therefore $\begin{pmatrix}-\frac{1}{2}\\1\\1\end{pmatrix}\times\begin{pmatrix}\frac{1}{2}\\1\\1\end{pmatrix}=\begin{pmatrix}0\\1\\-1\end{pmatrix}$ is a normal to the plane p. The equation in scalar product form is

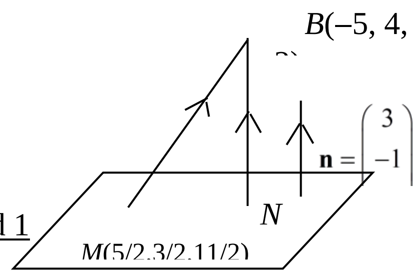
	$p: \ \mathbf{r}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=\begin{pmatrix}0\\0\\-1\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=1$
(iii)	Let $\overrightarrow{OS}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}$ and $\overrightarrow{OS'}=\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}$ Method 1: $\overrightarrow{S'S}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}-\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}=\begin{pmatrix}-\lambda-\mu\\2\lambda-2\mu\\-2+2\lambda-2\mu\end{pmatrix}$ This vector will be parallel to the normal of p. $\overrightarrow{S'S}=\begin{pmatrix}-\lambda-\mu\\2\lambda-2\mu\\-2+2\lambda-2\mu\end{pmatrix}=k\left[\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]=k\begin{pmatrix}0\\1\\-1\end{pmatrix}\Rightarrow \begin{matrix}\lambda+\mu=0\\2\lambda-2\mu=k\\-2+2\lambda-2\mu=-k\end{matrix}$ Solving, $\lambda=\frac{1}{4}$ => $\overrightarrow{OS}=\begin{pmatrix}-\frac{1}{4}\\\frac{1}{2}\\-\frac{3}{2}\end{pmatrix}$ Coordinates of S is $\left(-\frac{1}{4},\frac{1}{2},-\frac{3}{2}\right)$ Method 2: Let F be the midpoint between S and S’, $\overrightarrow{OF}=\frac{1}{2}\left(\overrightarrow{OS}+\overrightarrow{OS'}\right)=\frac{1}{2}\begin{pmatrix}-\lambda-\mu\\2\lambda-2\mu\\-2+2\lambda-2\mu\end{pmatrix}$ and $\overrightarrow{OF}=\overrightarrow{OS}+k\,\underset{\sim}{n}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}+k\begin{pmatrix}0\\1\\-1\end{pmatrix}=\begin{pmatrix}-\lambda\\2\lambda+k\\-2+2\lambda-k\end{pmatrix}$ Equating the position vector of point F, $\frac{1}{2}\begin{pmatrix}-\lambda-\mu\\2\lambda-2\mu\\-2+2\lambda-2\mu\end{pmatrix}=\begin{pmatrix}-\lambda\\2\lambda+k\\-2+2\lambda-k\end{pmatrix}\Rightarrow \begin{matrix}\lambda+\mu=0\\2\lambda-2\mu=k\\-2+2\lambda-2\mu=-k\end{matrix}$

	Solving, $\lambda = \frac{1}{4}$ => $\overrightarrow{OS} = \begin{pmatrix} -\frac{1}{4} \\ \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix}$ Coordinates of S is $\left(-\frac{1}{4}, \frac{1}{2}, -\frac{3}{2}\right)$

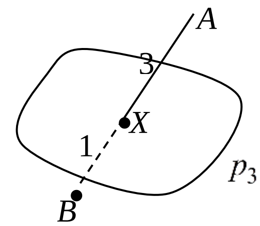
17 (i)	$c = 0$ as the line l is on π_2
(ii)	Since the line l is on π_2, l is perpendicular to the normal of π_2. $\begin{pmatrix} 1 \\ a \\ -1 \end{pmatrix} \bullet \begin{pmatrix} b \\ 5 \\ -1 \end{pmatrix} = 0$ $b + 5a + 1 = 0$ $b + 5a = -1$ (shown)
(iii)	Method 1: $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ a \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ a-2 \end{pmatrix}$ is a vector $\perp$ to π_1 $\begin{pmatrix} -2 \\ 1 \\ a-2 \end{pmatrix} \bullet \begin{pmatrix} b \\ 5 \\ -1 \end{pmatrix} = 0 \Rightarrow -2b + 5 - a + 2 = 0 \Rightarrow a + 2b = 7$ Method 2: $\begin{pmatrix} 1 \\ a \\ -1 \end{pmatrix} \times \begin{pmatrix} b \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 5-a \\ 1-b \\ 5-ab \end{pmatrix}$ is a vector $\perp$ to π_1 $\begin{pmatrix} 5-a \\ 1-b \\ 5-ab \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = 0 \Rightarrow 5 - a + 2 - 2b = 0 \Rightarrow a + 2b = 7$ Method 3: $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} b \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 5-2b \end{pmatrix}$ is a vector $\perp$ to π_1

	$\begin{pmatrix}-2\\1\\5-2b\end{pmatrix}\bullet\begin{pmatrix}1\\a\\-1\end{pmatrix}=0\Rightarrow -2+a-5+2b=0\Rightarrow a+2b=7$ Method 4: $\begin{pmatrix}1\\2\\0\end{pmatrix}\times\begin{pmatrix}1\\a\\-1\end{pmatrix}=\begin{pmatrix}-2\\1\\a-2\end{pmatrix}$ is a vector $\perp$ to π_1 $\begin{pmatrix}1\\2\\0\end{pmatrix}\times\begin{pmatrix}b\\5\\-1\end{pmatrix}=\begin{pmatrix}-2\\1\\5-2b\end{pmatrix}$ is a vector $\perp$ to π_1 $\therefore \begin{pmatrix}-2\\1\\a-2\end{pmatrix}=k\begin{pmatrix}-2\\1\\5-2b\end{pmatrix}$ $\therefore k=1$ and $a-2=5-2b$ $\therefore a+2b=7$ Using $b=-5a-1$ from part (ii) and solving simultaneous equation, we get a = -1 and b = 4.
(iv)	Possible answers are: $\mathbf{r}\bullet\begin{pmatrix}4\\5\\-1\end{pmatrix}=4\sqrt{42}$ or $\mathbf{r}\bullet\begin{pmatrix}4\\5\\-1\end{pmatrix}=-4\sqrt{42}$

18(i)	For plane p: $\mathbf{n}=\begin{pmatrix}1\\17\\7\end{pmatrix}\times\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}-21\\7\\-14\end{pmatrix}=-7\begin{pmatrix}3\\-1\\2\end{pmatrix}$ $\mathbf{r}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}0\\-17\\0\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}=17$ $3x-y+2z=17$
(ii)	$\overrightarrow{AB}=\begin{pmatrix}-5\\4\\-2\end{pmatrix}-\begin{pmatrix}1\\2\\4\end{pmatrix}=\begin{pmatrix}-6\\2\\-6\end{pmatrix}=-2\begin{pmatrix}3\\-1\\3\end{pmatrix}$

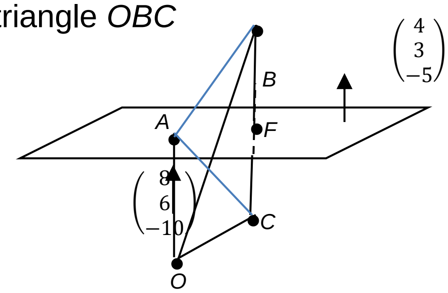
	line l: $\mathbf{r}=\begin{pmatrix}1\\2\\4\end{pmatrix}+s\begin{pmatrix}3\\-1\\3\end{pmatrix},\ s\in\mathbb{R}$ Let M be the point of intersection. $\overrightarrow{OM}=\begin{pmatrix}1\\2\\4\end{pmatrix}+s\begin{pmatrix}3\\-1\\3\end{pmatrix}$ for some $s\in\mathbb{R}$ Substitute $\overrightarrow{OM}$ into p: $\left[\begin{pmatrix}1\\2\\4\end{pmatrix}+s\begin{pmatrix}3\\-1\\3\end{pmatrix}\right]\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}=17$ $(3-2+8)+(9+1+6)s=17$ $s=\frac{8}{16}=\frac{1}{2}$ Substitute $s=\frac{1}{2}$ into $\overrightarrow{OM}$. $\overrightarrow{OM}=\begin{pmatrix}1\\2\\4\end{pmatrix}+\frac{1}{2}\begin{pmatrix}3\\-1\\3\end{pmatrix}=\begin{pmatrix}5/2\\3/2\\11/2\end{pmatrix}$ Coordinates of M: (5/2, 3/2, 11/2)
(iii)	Let θ be the acute angle between l and p. $\sin\theta=\left\lvert\frac{\begin{pmatrix}3\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}}{\sqrt{9+1+9}\sqrt{9+1+4}}\right\rvert\left\lvert\frac{9+1+6}{\sqrt{19}\sqrt{14}}\right\rvert=\frac{16}{\sqrt{19}\sqrt{14}}$ $=$ $\theta=78.8°$
(iv)	 Method 1

	Perpendicular distance from B to p $= \left\| \dfrac{\overrightarrow{MB} \cdot \mathbf{n}}{\|\mathbf{n}\|} \right\| = \left\| \dfrac{\begin{pmatrix} -15/2 \\ 5/2 \\ -15/2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{9+1+4}} \right\| = \left\| \dfrac{-40}{\sqrt{14}} \right\| = \dfrac{20}{7}\sqrt{14}$ (or 10.7) <u>Method 2</u> $MB = \left\| \begin{pmatrix} -15/2 \\ 5/2 \\ -15/2 \end{pmatrix} \right\| = \dfrac{\sqrt{475}}{2}$ Perpendicular distance from B to p = $MB\sin\theta = \dfrac{\sqrt{475}\cdot}{2}\dfrac{16}{\sqrt{19}\sqrt{14}} = \dfrac{20}{7}\sqrt{14}$ (or 10.7)
	<u>Method 3</u> Let N be the foot of the perpendicular from B to p. line BN: $\mathbf{r} = \begin{pmatrix} -5 \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix},\ t \in \mathbb{R}$ $\overrightarrow{ON} = \begin{pmatrix} -5 \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ for some $t \in \mathbb{R}$ Substitute $\overrightarrow{ON}$ into p: $\left[\begin{pmatrix} -5 \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}\right] \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 17$ $(-15-4-4)+(9+1+4)t = 17$ $t = \dfrac{40}{14} = \dfrac{20}{7}$ $\overrightarrow{BN} = \begin{pmatrix} -5 \\ 4 \\ -2 \end{pmatrix} + \dfrac{20}{7}\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \\ -2 \end{pmatrix} = \dfrac{20}{7}\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ Perpendicular distance from B to p $= \left\|\overrightarrow{BN}\right\| = \dfrac{20}{7}\sqrt{9+1+4} = \dfrac{20}{7}\sqrt{14}$ (or 10.7)

19a	$\overrightarrow{OX}=\dfrac{\overrightarrow{OA}+3\overrightarrow{OB}}{4}=\dfrac{1}{4}\left[\begin{pmatrix}1\\1\\2\end{pmatrix}+3\begin{pmatrix}3\\5\\6\end{pmatrix}\right]=\begin{pmatrix}\frac{5}{2}\\4\\5\end{pmatrix}$  *X* lies in π: $\begin{pmatrix}\frac{5}{2}\\4\\5\end{pmatrix}\cdot\begin{pmatrix}2\\\lambda\\0\end{pmatrix}=\mu$ $5+4\lambda=\mu$
b	$\overrightarrow{AB}=\begin{pmatrix}3\\5\\6\end{pmatrix}-\begin{pmatrix}1\\1\\2\end{pmatrix}=\begin{pmatrix}2\\4\\4\end{pmatrix}$ If the plane π contains the line *AB*, then p_3 is parallel to *AB*: $\begin{pmatrix}2\\4\\4\end{pmatrix}\cdot\begin{pmatrix}2\\\lambda\\0\end{pmatrix}=0$ $4+4\lambda=0$ $\lambda=-1$ $\lambda=-1$ and the point *A* lies in the plane p_3: $\begin{pmatrix}1\\1\\2\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\0\end{pmatrix}=\mu$ $2-1=\mu$ $\mu=1$ <u>Alternatively:</u> Since the points A and B lie in the plane p_3: $\begin{pmatrix}1\\1\\2\end{pmatrix}\cdot\begin{pmatrix}2\\\lambda\\0\end{pmatrix}=\mu$ and $\begin{pmatrix}3\\5\\6\end{pmatrix}\cdot\begin{pmatrix}2\\\lambda\\0\end{pmatrix}=\mu$ $2+\lambda=\mu \quad ---(1)$ $\qquad 6+5\lambda=\mu \quad ---(2)$ Solving (1) and (2) $\lambda=-1$, $\mu=1$

20 (i)	$\mathbf{n}=\begin{pmatrix}1\\2\\2\end{pmatrix}\times\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}4\\3\\-5\end{pmatrix}$

	$\mathbf{r}\cdot\begin{pmatrix}4\\3\\-5\end{pmatrix}=\begin{pmatrix}8\\6\\-10\end{pmatrix}\cdot\begin{pmatrix}4\\3\\-5\end{pmatrix}=100$ $\mathbf{r}\cdot\begin{pmatrix}4\\3\\-5\end{pmatrix}=100$
(ii)	Let F be the foot of perpendicular from *B* to plane π. Line *BF*: $\mathbf{r}=\begin{pmatrix}0\\0\\-10\end{pmatrix}+\lambda\begin{pmatrix}4\\3\\-5\end{pmatrix},\quad \lambda\in\mathbb{R}$ $\overrightarrow{OF}=\begin{pmatrix}0\\0\\-10\end{pmatrix}+\lambda\begin{pmatrix}4\\3\\-5\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\left[\begin{pmatrix}0\\0\\-10\end{pmatrix}+\lambda\begin{pmatrix}4\\3\\-5\end{pmatrix}\right]\cdot\begin{pmatrix}4\\3\\-5\end{pmatrix}=100$ $50+50\lambda=100$ $\lambda=1$ $\overrightarrow{OF}=\begin{pmatrix}0\\0\\-10\end{pmatrix}+\begin{pmatrix}4\\3\\-5\end{pmatrix}=\begin{pmatrix}4\\3\\-15\end{pmatrix}$ $\overrightarrow{OF}=\dfrac{\overrightarrow{OB}+\overrightarrow{OC}}{2}$ $\overrightarrow{OC}=2\overrightarrow{OF}-\overrightarrow{OB}=2\begin{pmatrix}4\\3\\-15\end{pmatrix}-\begin{pmatrix}0\\0\\-10\end{pmatrix}=\begin{pmatrix}8\\6\\-20\end{pmatrix}$
(iii)	Area of triangle *OBC* $=\dfrac{1}{2}\left\|\overrightarrow{OB}\times\overrightarrow{OC}\right\|$ $=\dfrac{1}{2}\left\|\overrightarrow{OB}\times\overrightarrow{OC}\right\|=\dfrac{1}{2}\left\|\begin{pmatrix}0\\0\\-10\end{pmatrix}\times\begin{pmatrix}8\\6\\-20\end{pmatrix}\right\|=\dfrac{1}{2}\left\|\begin{pmatrix}60\\80\\0\end{pmatrix}\right\|=50\text{ sq. units}$

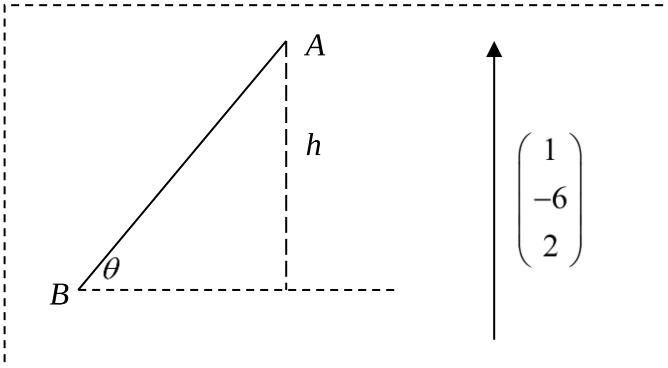
(iv)	Since OA is parallel to *BC*, triangles *ABC* and *OBC* have the same height. Hence area of triangle *ABC* = area of triangle *OBC* $= 50$ sq. units 

21(i)	$\begin{pmatrix}1\\6\\-2\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=2-6+4=0$. Therefore l_1 is parallel to p_1 $\begin{pmatrix}9\\1\\1\end{pmatrix}.\begin{pmatrix}1\\-6\\2\end{pmatrix}=9-6+2=5$. Therefore (9,1,1) lies in p_1 Therefore l_1 lies in p_1. **Alternatively**, $\begin{pmatrix}9+2\lambda\\1+\lambda\\2+4\lambda\end{pmatrix}.\begin{pmatrix}1\\-6\\2\end{pmatrix}$ $= 9+2\lambda-6-6\lambda+2+4\lambda = 5$
(ii)	$\mathbf{r}.\begin{pmatrix}1\\-6\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\1\end{pmatrix}.\begin{pmatrix}1\\-6\\2\end{pmatrix}$ Therefore $\mathbf{r}.\begin{pmatrix}1\\-6\\2\end{pmatrix}=1+6+2=9$ The Cartesian equation is $x-6y+2z=9$
(iii)	$\overrightarrow{BA}=\begin{pmatrix}-8\\-2\\0\end{pmatrix}$ which is parallel to $\begin{pmatrix}4\\1\\0\end{pmatrix}$ Let θ be the acute angle between l_2 and p_1 $\left\|\begin{pmatrix}4\\1\\0\end{pmatrix}.\begin{pmatrix}1\\-6\\2\end{pmatrix}\right\|=\sqrt{16+1}\sqrt{1+36+4}\,\sin\theta$

$\Rightarrow |4-6| = \sqrt{17}\sqrt{41}\ \sin\theta$

Therefore, $\sin\theta = \dfrac{2}{\sqrt{17}\sqrt{41}} = \dfrac{2\sqrt{697}}{697}$

Hence mtd



Hence mtd 1

Let h be the perpendicular distance between the planes

$$\frac{h}{AB} = \sin\theta$$

Therefore $h = \dfrac{4}{\sqrt{41}}$

Otherwise mtd 2(find length of projection on normal)

$$h = \left| \overrightarrow{BA} . \frac{\begin{pmatrix} 1 \\ 6 \\ -2 \end{pmatrix}}{\sqrt{41}} \right| = \frac{4}{\sqrt{41}}$$

Otherwise mtd 3 (find perpendicular distance fr O to each plane)

$$h = \left| \frac{5}{\sqrt{41}} - \frac{9}{\sqrt{41}} \right| = \frac{4}{\sqrt{41}}$$

22(i) l_1 is perpendicular to l_2

$$\Rightarrow \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} . \begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix} = 0$$

$\Rightarrow 2 + 2k + 4 = 0$

$\Rightarrow k=-3$

(ii)

$$\begin{pmatrix}1\\2\\4\end{pmatrix}\times\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}14\\7\\-7\end{pmatrix} \quad // \quad \begin{pmatrix}2\\1\\-1\end{pmatrix}$$

Equation of the line l_3 is $\mathbf{r} = \begin{pmatrix}4\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}, \quad \lambda \in IR$

(iii)

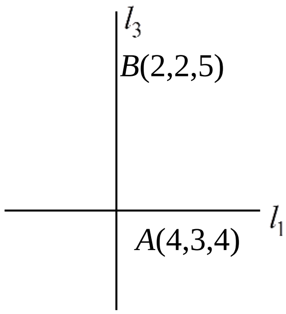
Let $\begin{pmatrix}2\\2\\5\end{pmatrix}=\begin{pmatrix}4+2\lambda\\3+\lambda\\4-\lambda\end{pmatrix}$

$\Rightarrow \lambda=-1$

$\lambda=-1$

$\lambda=-1$

Therefore A lies on l_3



Therefore , perpendicular distance from B to l_1 is $\sqrt{(4-2)^2+(3-2)^2+(4-5)^2}$

$=\sqrt{6}$ units

23(i)

$\overrightarrow{OM}=\dfrac{\mathbf{a}+\mathbf{c}}{2}$

Length of Projection $=\left|\left(\dfrac{\mathbf{a}+\mathbf{c}}{2}\right)\bullet\left(\dfrac{\mathbf{c}}{|\mathbf{c}|}\right)\right|$

$=\left|\dfrac{\mathbf{a}\bullet\mathbf{c}+\mathbf{c}\bullet\mathbf{c}}{2|\mathbf{c}|}\right|$

$=\left|\dfrac{|\mathbf{a}||\mathbf{c}|\cos 60^\circ+|\mathbf{c}|^2}{2|\mathbf{c}|}\right|$

	$=\left\|\dfrac{0.5\|\mathbf{a}\|\|\mathbf{c}\|+\|\mathbf{c}\|^2}{2\|\mathbf{c}\|}\right\|$ **Note:** $\|\mathbf{a}\|=\|\mathbf{c}\|$ $=\left\|\dfrac{0.5\|\mathbf{c}\|\|\mathbf{c}\|+\|\mathbf{c}\|^2}{2\|\mathbf{c}\|}\right\|$ $=\dfrac{3}{4}\|\mathbf{c}\|\,(Shown)$
(ii)	$Area\ of\ \Delta OMC=\left(\dfrac{1}{4}\|\mathbf{a}\times\mathbf{c}\|\right)$ $=\dfrac{1}{4}\|\mathbf{a}\|\|\mathbf{c}\|\sin 60^\circ$ $=\dfrac{\sqrt{3}}{8}\|\mathbf{c}\|\|\mathbf{c}\|$ **Note:** $\|\mathbf{a}\|=\|\mathbf{c}\|$ $=\dfrac{\sqrt{3}}{8}\|\mathbf{c}\|^2$ $\therefore k=\dfrac{\sqrt{3}}{8}$
(iii)	$\overrightarrow{OD}=\dfrac{5}{2}\mathbf{c}$ $Shortest\ \Delta OMC=\left\|\overrightarrow{OD}\times\dfrac{\mathbf{a}}{\|\mathbf{a}\|}\right\|$ $=\dfrac{5}{2}\left\|\dfrac{\mathbf{c}\times\mathbf{a}}{\|\mathbf{a}\|}\right\|=\dfrac{5}{2}\left\|\dfrac{\|\mathbf{c}\|\|\mathbf{a}\|\sin 60^\circ}{\|\mathbf{a}\|}\right\|=\dfrac{5\sqrt{3}}{4}\left\|\dfrac{\|\mathbf{c}\|\|\mathbf{a}\|}{\|\mathbf{a}\|}\right\|=\dfrac{5\sqrt{3}}{4}\|\mathbf{c}\|$ $\therefore t=\dfrac{5\sqrt{3}}{4}$

24(i)	Using ratio theorem, $\overrightarrow{OB}=\dfrac{4\overrightarrow{OM}+\overrightarrow{OA}}{5}$ $\overrightarrow{OM}=\dfrac{5\overrightarrow{OB}-\overrightarrow{OA}}{4}$ $=\dfrac{1}{4}\left(5\begin{pmatrix}-1\\3\\2\end{pmatrix}-\begin{pmatrix}1\\-1\\2\end{pmatrix}\right)=\dfrac{1}{2}\begin{pmatrix}-3\\8\\4\end{pmatrix}$

<table>
<tr><td>(ii)</td><td>$\dfrac{\left|\overrightarrow{OA}\times\overrightarrow{OB}\right|}{\left|\overrightarrow{OB}\right|}$ represents the shortest/perpendicular distance from point A to the line OB.</td></tr>
<tr><td>(iii)</td><td>

$$\overrightarrow{OA}\times\overrightarrow{OB}=\begin{pmatrix}1\\-1\\2\end{pmatrix}\times\begin{pmatrix}-1\\3\\2\end{pmatrix}$$
$$=\begin{pmatrix}-8\\-4\\2\end{pmatrix}$$
$$=2\begin{pmatrix}-4\\-2\\1\end{pmatrix}$$

$\therefore$ Normal to the plane containing O, A and B is $\begin{pmatrix}-4\\-2\\1\end{pmatrix}$.

Method 1:

$$\text{shortest distance }=\frac{\left|\begin{pmatrix}1\\3\\8\end{pmatrix}\cdot\begin{pmatrix}-4\\-2\\1\end{pmatrix}\right|}{\sqrt{21}}$$
$$=\frac{2}{\sqrt{21}}\ \frac{\sqrt{21}}{\sqrt{21}}$$
$$=\frac{2\sqrt{21}}{21}\text{ units}$$

Method 2:
Let N be the foot of perpendicular from C to the plane.

Equation of line CN is $\mathbf{r}=\begin{pmatrix}1\\3\\8\end{pmatrix}+\lambda\begin{pmatrix}-4\\-2\\1\end{pmatrix}, \lambda\in\mathbb{R}$

Equation of plane: $\mathbf{r}\cdot\begin{pmatrix}-4\\-2\\1\end{pmatrix}=0$

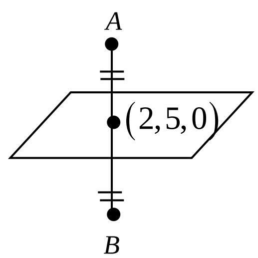
</td></tr>
</table>

	Since N lies on the plane, $\begin{pmatrix} 1-4\lambda \\ 3-2\lambda \\ 8+\lambda \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix} = 0$ $\Rightarrow \lambda = \frac{2}{21}$ $\overrightarrow{ON} = \begin{pmatrix} 1-4\left(\frac{2}{21}\right) \\ 3-2\left(\frac{2}{21}\right) \\ 8+\left(\frac{2}{21}\right) \end{pmatrix} = \begin{pmatrix} \frac{13}{21} \\ \frac{59}{21} \\ \frac{170}{21} \end{pmatrix}$ So $\overrightarrow{CN} = \overrightarrow{ON} - \overrightarrow{OC} = \begin{pmatrix} \frac{13}{21} \\ \frac{59}{21} \\ \frac{170}{21} \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} = \begin{pmatrix} -\frac{8}{21} \\ -\frac{4}{21} \\ \frac{2}{21} \end{pmatrix}$ $CN = \sqrt{\left(-\frac{8}{21}\right)^2 + \left(-\frac{4}{21}\right)^2 + \left(\frac{2}{21}\right)^2} = \sqrt{\frac{4}{21}} = \frac{2\sqrt{21}}{21}$ units

25(i)	l_1: $\mathbf{r} = \begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}, \lambda \in \mathbb{R}$ p_1: $x + 2y - 3z = 4$ p_1: $\mathbf{r} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 4$ Since $\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} = -\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, the line l_1 is parallel to n_1. The line l_1 is perpendicular to the plane p_1.

	$\begin{pmatrix} 4-\lambda \\ 5-2\lambda \\ -6+3\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 4$ $4-\lambda+10-4\lambda+18-9\lambda = 4$ $28 = 14\lambda$ $\lambda = 2$ Coordinates of foot of perpendicular is $(2, 1, 0)$.
(ii)	l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \ \mu \in \mathbb{R}$ $\mathbf{n}_2 = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ p_2: $\mathrm{r} \bullet \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ p_2: $x+4y+3z = 2$ (shown) Since $\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} \neq k \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, l_1 and l_2 are not parallel and since $\begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = 6 \neq 2$, l_1 is not on p_2, the two lines are on different planes, Hence, they are skew lines.
(iii)	p_1: $x+2y-3z = 4$ p_2: $x+4y+3z = 2$ Using GC, l_3: $\mathbf{r} = \begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}, \ \gamma \in \mathbb{R}$

26 (i)	l: $\mathbf{r} == \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \ \lambda \in \Re$ —— (1) p: $\mathbf{r} \cdot \begin{pmatrix} 3 \\ \alpha \\ 0 \end{pmatrix} = 26$ —— (2) If l is parallel to p, then normal of $p \perp l$

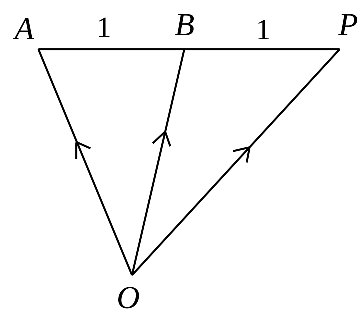
	i.e. $\begin{pmatrix}3\\\alpha\\0\end{pmatrix}.\begin{pmatrix}4\\-3\\1\end{pmatrix}=0$ $12-3\alpha=0$ $\alpha=4$
(ii)	Let $\overrightarrow{ON}$ be $\begin{pmatrix}-1\\1\\0\end{pmatrix}+\lambda'\begin{pmatrix}3\\4\\0\end{pmatrix}$ for a particular $\lambda'\in\Re$. Then $[\begin{pmatrix}-1\\1\\0\end{pmatrix}+\lambda'\begin{pmatrix}3\\4\\0\end{pmatrix}].\begin{pmatrix}3\\4\\0\end{pmatrix}=26$ $\Rightarrow -3+9\lambda'+4+16\lambda'=26$ $\Rightarrow \lambda'=1$ $\overrightarrow{ON}=\begin{pmatrix}-1\\1\\0\end{pmatrix}+1\begin{pmatrix}3\\4\\0\end{pmatrix}=\begin{pmatrix}2\\5\\0\end{pmatrix}$
(iii)	Using Ratio Theorem, $\begin{pmatrix}2\\5\\0\end{pmatrix}=\dfrac{\overrightarrow{OA}+\overrightarrow{OB}}{2}$  $\overrightarrow{OB}=2\begin{pmatrix}2\\5\\0\end{pmatrix}-\begin{pmatrix}-1\\1\\0\end{pmatrix}=\begin{pmatrix}5\\9\\0\end{pmatrix}$ Coordinates of *B*: (5, 9, 0)
(iv)	Area of triangle *OAB* $=\frac{1}{2}\lvert\overrightarrow{OA}\times\overrightarrow{OB}\rvert=\frac{1}{2}\left\lvert\begin{pmatrix}-1\\1\\0\end{pmatrix}\times\begin{pmatrix}5\\9\\0\end{pmatrix}\right\rvert=\frac{1}{2}\left\lvert\begin{pmatrix}0\\0\\-14\end{pmatrix}\right\rvert=7$

27 (i)	$\overrightarrow{OA}=\begin{pmatrix}6\\0\\0\end{pmatrix}$; $\overrightarrow{OB}=\begin{pmatrix}0\\4\\0\end{pmatrix}$; $\overrightarrow{OP}=\begin{pmatrix}0\\0\\5\end{pmatrix}$; $\overrightarrow{OR}=\begin{pmatrix}0\\4\\2\end{pmatrix}$; $\overrightarrow{PR}=\begin{pmatrix}0\\4\\-3\end{pmatrix}$ Hence the vector equation of line *PR* is $\mathbf{r}=\begin{pmatrix}0\\0\\5\end{pmatrix}+\lambda\begin{pmatrix}0\\4\\-3\end{pmatrix}$, where $\lambda\in\mathbb{R}$.

(ii)	$\overrightarrow{OQ} = \dfrac{\begin{pmatrix}0\\4\\0\end{pmatrix}+3\begin{pmatrix}6\\0\\0\end{pmatrix}}{4} = \frac{1}{4}\begin{pmatrix}18\\4\\0\end{pmatrix} = \begin{pmatrix}9/2\\1\\0\end{pmatrix}$
(iii)	$l: \boldsymbol{r} = \begin{pmatrix}0\\1\\8\end{pmatrix} + \mu\begin{pmatrix}0\\-3\\1\end{pmatrix}$, $\mu \in \mathbb{R}$ Equate: $\begin{pmatrix}0\\0\\5\end{pmatrix} + \lambda\begin{pmatrix}0\\4\\-3\end{pmatrix} = \begin{pmatrix}0\\1\\8\end{pmatrix} + \mu\begin{pmatrix}0\\-3\\1\end{pmatrix}$ $\lambda = -2 \;\; \mu = 3$ $\overrightarrow{OX} = \begin{pmatrix}0\\1\\8\end{pmatrix} + 3\begin{pmatrix}0\\-3\\1\end{pmatrix} = \begin{pmatrix}0\\-8\\11\end{pmatrix}$
(iv)	$\overrightarrow{QX} = \begin{pmatrix}-9/2\\-9\\11\end{pmatrix}$ $\lvert \overrightarrow{QX} \times \mathbf{m} \rvert = \dfrac{\left\lvert\begin{pmatrix}-9/2\\-9\\11\end{pmatrix}\times\begin{pmatrix}-3\\2\\0\end{pmatrix}\right\rvert}{\sqrt{9+4}} = \dfrac{\left\lvert\begin{pmatrix}-22\\33\\-36\end{pmatrix}\right\rvert}{\sqrt{13}} = 14.855$ $\lvert \overrightarrow{QX} \times \mathbf{m} \rvert$ is the perpendicular/shortest distance from point X to line AB. Area of triangle AXB = $\frac{1}{2}\lvert\overrightarrow{AB}\rvert 14.855$ $= \frac{1}{2}\sqrt{36+16}(14.855) = 53.6$ (3 s.f.)

28	$\dfrac{\mathbf{r}\cdot\mathbf{a}}{\mathbf{r}\cdot\mathbf{b}} = \dfrac{\mu\lvert\mathbf{a}\rvert^2 + \lambda\mathbf{a}\cdot\mathbf{b}}{\lambda\lvert\mathbf{b}\rvert^2 + \mu\mathbf{a}\cdot\mathbf{b}}$ Given $\lambda : \mu = \lvert\mathbf{a}\rvert : \lvert\mathbf{b}\rvert \Rightarrow \dfrac{\lambda}{\mu} = \dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}$ $\dfrac{\mathbf{r}\cdot\mathbf{a}}{\mathbf{r}\cdot\mathbf{b}} = \dfrac{\mu\left(\lvert\mathbf{a}\rvert^2 + \dfrac{\lambda}{\mu}\mathbf{a}\cdot\mathbf{b}\right)}{\mu\left(\dfrac{\lambda}{\mu}\lvert\mathbf{b}\rvert^2 + \mathbf{a}\cdot\mathbf{b}\right)} = \dfrac{\lvert\mathbf{a}\rvert^2 + \dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}\mathbf{a}\cdot\mathbf{b}}{\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}\lvert\mathbf{b}\rvert^2 + \mathbf{a}\cdot\mathbf{b}} = \dfrac{\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}\left(\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert + \mathbf{a}\cdot\mathbf{b}\right)}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert + \mathbf{a}\cdot\mathbf{b}} = \dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}$ deduced From the above result, $\lvert\mathbf{b}\rvert(\mathbf{r}\cdot\mathbf{a}) = \lvert\mathbf{a}\rvert(\mathbf{r}\cdot\mathbf{b})$ Let θ_1 and θ_2 be the angle between $\mathbf{a}$ and $\mathbf{r}$, $\mathbf{b}$ and $\mathbf{r}$ respectively. Then

	$\lvert\mathbf{b}\rvert\left(\lvert\mathbf{r}\rvert\lvert\mathbf{a}\rvert\cos\theta_1\right)=\lvert\mathbf{a}\rvert\left(\lvert\mathbf{r}\rvert\lvert\mathbf{b}\rvert\cos\theta_2\right)$ $\cos\theta_1=\cos\theta_2$ $\theta_1=\theta_2$ Since cos θ is a 1-1 function from $0\le\theta\le\pi$. $\therefore$ the line *OR* bisects angle *AOB* (shown).

29(a)	$10\overrightarrow{OF}=3\overrightarrow{OE}+7\overrightarrow{OG}$ $3\overrightarrow{OF}+7\overrightarrow{OF}=3\overrightarrow{OE}+7\overrightarrow{OG}$ $3\overrightarrow{OF}-3\overrightarrow{OE}=7\overrightarrow{OG}-7\overrightarrow{OF}$ $\Rightarrow 3\overrightarrow{EF}=7\overrightarrow{FG}$ $\quad\therefore E, F$ & G are collinear
(b) (i)	**<u>Method 1</u>** Using $\overrightarrow{AP}=2\overrightarrow{AB}$, $\overrightarrow{OP}-\overrightarrow{OA}=2\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ $\overrightarrow{OP}=2\overrightarrow{OB}-\overrightarrow{OA}$ $=2\begin{pmatrix}5\\1\\-2\end{pmatrix}-\begin{pmatrix}12\\2\\6\end{pmatrix}=\begin{pmatrix}-2\\0\\-10\end{pmatrix}$  **<u>Method 2</u>** Using mid-pt theorem, $\overrightarrow{OB}=\frac{1}{2}\left[\overrightarrow{OA}+\overrightarrow{OP}\right]$ $\overrightarrow{OP}=2\overrightarrow{OB}-\overrightarrow{OA}=2\begin{pmatrix}5\\1\\-2\end{pmatrix}-\begin{pmatrix}12\\2\\6\end{pmatrix}=\begin{pmatrix}-2\\0\\-10\end{pmatrix}$
(ii)	$\overrightarrow{AP}=\begin{pmatrix}-2\\0\\-10\end{pmatrix}-\begin{pmatrix}12\\2\\6\end{pmatrix}=\begin{pmatrix}-14\\-2\\-16\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}1+2\lambda\\5+2\lambda\\-2+\lambda\end{pmatrix}-\begin{pmatrix}5\\1\\-2\end{pmatrix}=\begin{pmatrix}2\lambda-4\\2\lambda+4\\\lambda\end{pmatrix}$ $\overrightarrow{AP}\bullet\overrightarrow{BC}=0$ $(\because AP\perp BC)$

	$\begin{pmatrix}-14\\-2\\-16\end{pmatrix}\bullet\begin{pmatrix}2\lambda-4\\2\lambda+4\\\lambda\end{pmatrix}=0\Rightarrow 24\lambda=24\Rightarrow\lambda=1$
(iii)	**<u>Method 1</u>** Using λ=1, $\overrightarrow{BC}=\begin{pmatrix}2-4\\2+4\\1\end{pmatrix}=\begin{pmatrix}-2\\6\\1\end{pmatrix}$ Area of $\Delta ACP=\frac{1}{2}\left\|\overrightarrow{AP}\right\|\left\|\overrightarrow{BC}\right\|=\frac{1}{2}\left\|\begin{pmatrix}-14\\-2\\-16\end{pmatrix}\right\|\left\|\begin{pmatrix}-2\\6\\1\end{pmatrix}\right\|=\frac{1}{2}\times\sqrt{456}\times\sqrt{41}=68.4\text{ unit}^2$ **<u>Method 2</u>** Using λ=1, $\overrightarrow{AC}=\begin{pmatrix}1+2-12\\5+2-2\\-2+1-6\end{pmatrix}=\begin{pmatrix}-9\\5\\-7\end{pmatrix}$ Area of $\Delta ACP=\frac{1}{2}\left\|\overrightarrow{AC}\times\overrightarrow{AP}\right\|=\frac{1}{2}\left\|\begin{pmatrix}-9\\5\\-7\end{pmatrix}\times\begin{pmatrix}-14\\-2\\-16\end{pmatrix}\right\|=\frac{1}{2}\left\|\begin{pmatrix}-94\\-46\\88\end{pmatrix}\right\|=\frac{1}{2}\sqrt{18696}=68.4\text{ unit}^2$ \\

30 (i)	Normal of plane $OBDC$ = $\mathbf{b}\times\mathbf{c}$ $PE=\frac{\left\|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right\|}{\left\|\mathbf{b}\times\mathbf{c}\right\|}$
(ii)	Volume of the parallelepiped = Base area $\times$ height = Area of $OCDB$ $\times PE=\left\|\mathbf{b}\times\mathbf{c}\right\|\frac{\left\|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right\|}{\left\|\mathbf{b}\times\mathbf{c}\right\|}=\left\|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right\|$

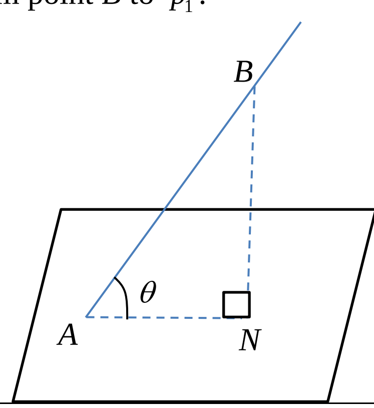
(iii)	$\lvert\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\rvert=18$ $\left\lvert\begin{pmatrix}-10\\3\\0\end{pmatrix}\cdot\left[\begin{pmatrix}2\\0\\3\end{pmatrix}\times\begin{pmatrix}p\\1\\5\end{pmatrix}\right]\right\rvert=18$ $\left\lvert\begin{pmatrix}-10\\3\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\3p-10\\2\end{pmatrix}\right\rvert=18$ $\lvert 30+9p-30\rvert=18$ $p=\pm 2$
	Yes since $\lvert\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\rvert=\lvert\mathbf{b}\cdot(\mathbf{a}\times\mathbf{c})\rvert=$ Volume of the given parallelpiped

31 (i)	Length of projection of **a** onto **b**= $\left\lvert\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert}\right\rvert=\dfrac{\left\lvert 2\lvert\mathbf{b}\rvert\cos\frac{\pi}{6}\right\rvert}{\lvert\mathbf{b}\rvert}=\sqrt{3}$
(ii)	$\mathbf{a}\cdot(\mathbf{b}-2\mathbf{a})=0$ $\mathbf{a}\cdot\mathbf{b}-2\mathbf{a}\cdot\mathbf{a}=0$ $\mathbf{a}\cdot\mathbf{b}=2\lvert\mathbf{a}\rvert^2$ $\mathbf{a}\cdot\mathbf{b}=8$ $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\dfrac{\pi}{6}=8$ $\lvert\mathbf{b}\rvert=\dfrac{8}{2\left(\frac{\sqrt{3}}{2}\right)}=\dfrac{8}{\sqrt{3}}$
(iii)	$\overrightarrow{OP}=\dfrac{3}{4}\mathbf{b}$ Area ΔAPB= $\dfrac{1}{2}\left\lvert\overrightarrow{AP}\times\overrightarrow{AB}\right\rvert$ $=\dfrac{1}{2}\left\lvert(\dfrac{3}{4}\mathbf{b}-\mathbf{a})\times(\mathbf{b}-\mathbf{a})\right\rvert=\dfrac{1}{2}\left\lvert\dfrac{3}{4}\mathbf{a}\times\mathbf{b}-\mathbf{a}\times\mathbf{b}\right\rvert=\dfrac{1}{8}\lvert\mathbf{a}\times\mathbf{b}\rvert=\dfrac{1}{8}\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta=\dfrac{1}{8}(2)\left(\dfrac{8}{\sqrt{3}}\right)\sin\left(\dfrac{\pi}{6}\right)=\dfrac{1}{\sqrt{3}}$

32(i)	$\cos 30^\circ = \left\lvert \dfrac{\begin{pmatrix} b \\ 1 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{3}\sqrt{b^2+2}} \right\rvert$ $\dfrac{\sqrt{3}}{2} = \dfrac{\lvert -b-2 \rvert}{\sqrt{3}\sqrt{b^2+2}} = \dfrac{b+2}{\sqrt{3}\sqrt{(b^2+2)}}$ $3\sqrt{(b^2+2)} = 2(b+2)$ $9(b^2+2) = 4(b+2)^2$ $5b^2 - 16b + 2 = 0$ $b = 3.07$ (2 d.p.) $(\because b > 1)$ or Alternative $0.86603 = \dfrac{\lvert -b-2 \rvert}{\sqrt{3}\sqrt{b^2+2}} = \dfrac{b+2}{\sqrt{3}\sqrt{(b^2+2)}}$ $1.50\sqrt{(b^2+2)} = (b+2)$ $2.25(b^2+2) = (b+2)^2$ $1.25b^2 - 4b + 0.5 = 0$ $b = 3.07$ (2 d.p.) $(\because b > 1)$
(ii)	Using b = 3 $l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3+3\lambda \\ 1+\lambda \\ 2-\lambda \end{pmatrix}$ At xy plane, $z = 0 \Rightarrow \lambda = 2$ Point $A(9,3,0)$ $l_2 : \mathbf{r} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4-\mu \\ -\mu \\ 1+\mu \end{pmatrix}$ At xy plane, $z = 0 \Rightarrow \mu = -1$ Point $B(5,1,0)$
(iii)	Using points A(9, 3, 0), B(5, 1, 0) & C(2, 7, 3) which lie on p_1, $\overrightarrow{AB} = \begin{pmatrix} 5-9 \\ 1-3 \\ 0-0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 2-5 \\ 7-1 \\ 3-0 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} = 3\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 2-9 \\ 7-3 \\ 3-0 \end{pmatrix} = \begin{pmatrix} -7 \\ 4 \\ 3 \end{pmatrix}$ Normal of p_1, $n_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ or $\begin{pmatrix} -7 \\ 4 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ -15 \end{pmatrix} = -3\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$

	$\therefore$ eqn of plane $p_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} = 3$ $\Rightarrow x - 2y + 5z = 3$ (shown)
(iv)	$\text{rref}\left(\begin{array}{ccc\|c} 1 & -2 & 5 & 3 \\ 1 & 2 & 3 & 2 \end{array}\right) = \begin{pmatrix} 1 & 0 & 4 & \frac{5}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{4} \end{pmatrix}$ $x + 4z = \frac{5}{2}$ $y - \frac{1}{2}z = -\frac{1}{4}$ $\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2.5 \\ -0.25 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -4 \\ \frac{1}{2} \\ 1 \end{pmatrix}$ $\therefore\ l_3 :\ \mathbf{r} = \begin{pmatrix} 2.5 \\ -0.25 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -8 \\ 1 \\ 2 \end{pmatrix},\ \ \beta \in \mathbb{R}$
(v)	Since l_2 and l_3 have different direction vector, they are not parallel. Equating l_2 and l_3 $\begin{pmatrix} 4-\mu \\ -\mu \\ 1+\mu \end{pmatrix} = \begin{pmatrix} 2.5-8\beta \\ -0.25+\beta \\ 2\beta \end{pmatrix}$ yields no consistent values for β or μ $\Rightarrow$ l_2 and l_3 do not intersect $\Rightarrow$ skew lines.

33 (i)	Vector equation of L is $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix},\ \ \lambda \in \mathbb{R}.$ ----(1) $p_1 : \text{r} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = 5$ ---(2) Let A the point of intersection between L and p_1. Substitute (1) into (2)

	$\begin{pmatrix} 2+\lambda \\ 4-2\lambda \\ 2+\lambda \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = 5$ $6+3\lambda+4-2\lambda+4+2\lambda=5$ $3\lambda=-9$ $\lambda=-3$ Substitute $\lambda=-3$ into (1), $\overrightarrow{OA}=\begin{pmatrix} -1 \\ 10 \\ -1 \end{pmatrix}$.
(ii)	Let the acute angle between L and p_1 be θ. $\theta=\sin^{-1}\dfrac{\left\lvert \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right\rvert}{\sqrt{6}\sqrt{14}}=\sin^{-1}\dfrac{3}{\sqrt{6}\sqrt{14}}=19.107^{\circ}$ (3 d.p) $\theta=19.1^{\circ}$ (1 d.p)
(iii)	Observe that the point $B\ (2,4,2)$ lies on L. $\overrightarrow{AB}=\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}-\begin{pmatrix} -1 \\ 10 \\ -1 \end{pmatrix}=\begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$ <u>Method 1 (Hence)</u> $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{9+36+9}=3\sqrt{6}$ Let N be the foot of perpendicular from point B to p_1. $\sin\theta=\dfrac{BN}{AB}$ $BN=\left(\dfrac{3}{\sqrt{6}\sqrt{14}}\right)\left(3\sqrt{6}\right)$ $BN=\dfrac{9}{\sqrt{14}}=2.41$ (3 s.f) 

ℓ p_1

Method 2 (Otherwise)

$$BN = \frac{\left|\begin{pmatrix}3\\-6\\3\end{pmatrix}\bullet\begin{pmatrix}3\\1\\2\end{pmatrix}\right|}{\sqrt{14}} = \frac{9}{\sqrt{14}} = 2.41 \text{ (3 s.f)}$$

(iv)

$$\mathbf{n} = \begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\-2\\3\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix}$$

$\therefore$ Vector equation of the plane is

$$\mathbf{r}\bullet\begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}2\\1\\0\end{pmatrix} = 1$$

34(i)

A normal vector of p_1 is $\begin{pmatrix}1\\0\\1\end{pmatrix}\times\begin{pmatrix}-2\\1\\-2\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$.

Equation of line AN: $\mathbf{r} = \begin{pmatrix}0\\3\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\0\\1\end{pmatrix}, \lambda \in \mathbb{R}$

Sub. equation of line AN into equation of plane p_1:

$$\begin{pmatrix}0\\3\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\0\\1\end{pmatrix} = \begin{pmatrix}4\\5\\-1\end{pmatrix} + s\begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}-2\\1\\-2\end{pmatrix}$$

Solving, we have $t = -2$, $s = -5$, $\lambda = -3$

$$\therefore \overrightarrow{ON} = \begin{pmatrix}0\\3\\1\end{pmatrix} - 3\begin{pmatrix}-1\\0\\1\end{pmatrix} = \begin{pmatrix}3\\3\\-2\end{pmatrix}$$

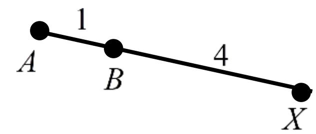
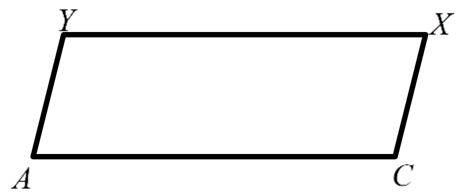
(ii)

Length of projection

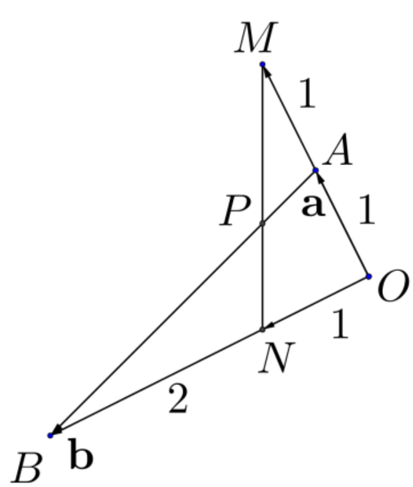
	$= = \dfrac{\left\lvert \overrightarrow{AB} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{2}} = \dfrac{\left\lvert \left[\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} \right] \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{2}} = \dfrac{\left\lvert \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{2}} = \dfrac{\left\lvert \begin{pmatrix} -3 \\ -2 \\ -3 \end{pmatrix} \right\rvert}{\sqrt{2}} = \sqrt{11}$ units
(iii)	$\overrightarrow{BN} = \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$ Equation of p_2: $\mathbf{r} \bullet \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 23$ Cartesian equation of p_2 is $2x + 3y - 4z = 23$.
(iv)	A vector parallel to l is $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$.

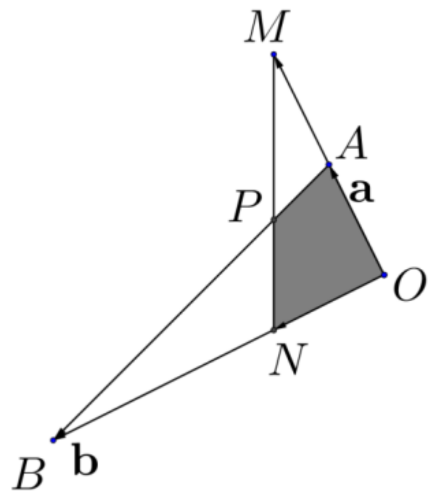
35	$\overrightarrow{OS} = \dfrac{5\mathbf{a} + 4\mathbf{b}}{9}$ $\overrightarrow{MS} = \dfrac{2\mathbf{a} + 4\mathbf{b}}{9}$ $\overrightarrow{MN} = \dfrac{\mathbf{b} - \mathbf{a}}{3}$ Area of $\Delta OAB = \dfrac{1}{2}\lvert \mathbf{a} \times \mathbf{b} \rvert$ Area of ΔMNS $= \dfrac{1}{2}\left\lvert \overrightarrow{MS} \times \overrightarrow{MN} \right\rvert$ $= \dfrac{1}{2}\left\lvert \dfrac{2\mathbf{a} + 4\mathbf{b}}{9} \times \dfrac{\mathbf{b} - \mathbf{a}}{3} \right\rvert$ $= \dfrac{1}{2}\left(\dfrac{1}{27}\right)\left\lvert (2\mathbf{a} + 4\mathbf{b}) \times (\mathbf{b} - \mathbf{a}) \right\rvert$ $= \dfrac{1}{2}\left(\dfrac{1}{27}\right)\left\lvert (2\mathbf{a} \times \mathbf{b}) + (4\mathbf{b} \times \mathbf{b}) - (2\mathbf{a} \times \mathbf{a}) - (4\mathbf{b} \times \mathbf{a}) \right\rvert$

	$=\frac{1}{2}\left(\frac{1}{27}\right)\left\|(2\mathbf{a}\times\mathbf{b})+(4\mathbf{a}\times\mathbf{b})\right\|$ $=\frac{1}{2}\left(\frac{2}{9}\right)\left\|\mathbf{a}\times\mathbf{b}\right\|$ ratio of the area of triangle *MNS* to the area of triangle *OAB* $=\frac{2}{9}:1=2:9$

36(i)	Using ratio theorem,  $\overrightarrow{OB}=\frac{4\overrightarrow{OA}+\overrightarrow{OX}}{5}$ $\Rightarrow \overrightarrow{OX}=5\overrightarrow{OB}-4\overrightarrow{OA}=5\mathbf{b}-4\mathbf{a}$  Since $ACXY$ forms a parallelogram, we have $\overrightarrow{AC}=\overrightarrow{YX}$ $\Rightarrow \overrightarrow{OY}=\overrightarrow{OX}-\overrightarrow{AC}$ $\Rightarrow \overrightarrow{OY}=(5\mathbf{b}-4\mathbf{a})-\mathbf{b}=4(\mathbf{b}-\mathbf{a})$ Since $\overrightarrow{OY}=4(\mathbf{b}-\mathbf{a})\Rightarrow\overrightarrow{OY}=4\overrightarrow{AB}$ $\Rightarrow \overrightarrow{OY}$ is parallel to $\overrightarrow{AB}\Rightarrow OABY$ is a trapezium.
(ii)	As $\mathbf{b}$ is a unit vector, thus $\left\|(\mathbf{a}+\mathbf{b})\cdot\mathbf{b}\right\|$ is the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OB}$.
(iii)	Area of $ACXY$ $=\left\|\overrightarrow{AC}\times\overrightarrow{AY}\right\|$ $=\left\|\mathbf{b}\times(4\mathbf{b}-5\mathbf{a})\right\|$ $=5\left\|\mathbf{b}\times\mathbf{a}\right\|$ $=5\times2=10$ Area of $OAB=\frac{1}{2}\left\|\mathbf{b}\times\mathbf{a}\right\|=1$ $\Rightarrow\left\|\mathbf{b}\times\mathbf{a}\right\|=2$ Let the shortest distance be d. Then area of $ACXY$ = $AC\times d$ and $AC=OB=1$. Therefore, $d=10$.

37 (i)	Given *A*, *B* and *C* are collinear, $\overrightarrow{AC} = k\overrightarrow{AB}$ $\mathbf{c}-\mathbf{a} = k(\mathbf{b}-\mathbf{a})$ $\mathbf{c} = k\mathbf{b}+(1-k)\mathbf{a}$ (shown)
(ii)	$\lvert\mathbf{a}\times\mathbf{c}\rvert = \lvert\mathbf{a}\times[k\mathbf{b}+(1-k)\mathbf{a}]\rvert = \lvert k(\mathbf{a}\times\mathbf{b})+(1-k)(\mathbf{a}\times\mathbf{a})\rvert = \lvert k\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin 90^\circ\hat{\mathbf{n}}+(1-k)\mathbf{0}\rvert = 9\lvert k\rvert$
	It is the area of a parallelogram with sides *OA* and *OC*.
(iii)	Area of triangle *OAC* = 3 x area of triangle *OAB* $\frac{1}{2}\lvert\mathbf{a}\times\mathbf{c}\rvert = \frac{3}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert$ $9\lvert k\rvert = 3\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin 90^\circ = 27$ $\lvert k\rvert = 3$ $k = \pm 3$
(iv)	Length of projection of *OC* onto *OA* = 12 $\frac{\lvert\mathbf{c}\bullet\mathbf{a}\rvert}{\lvert\mathbf{a}\rvert} = 12$ $\lvert\mathbf{c}\bullet\mathbf{a}\rvert = 12\lvert\mathbf{a}\rvert = 36$
	When $k=3$, $\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ $\lvert\mathbf{c}\bullet\mathbf{a}\rvert = \lvert(3\mathbf{b}-2\mathbf{a})\bullet\mathbf{a}\rvert = \lvert 3(\mathbf{b}\bullet\mathbf{a}) - 2\mathbf{a}\bullet\mathbf{a}\rvert = \lvert 3(0)-2(3)^2\rvert = 18$
	When $k=-3$, $\mathbf{c} = -3\mathbf{b}+4\mathbf{a}$ $\lvert\mathbf{c}\bullet\mathbf{a}\rvert = \lvert(-3\mathbf{b}+4\mathbf{a})\bullet\mathbf{a}\rvert = \lvert -3(\mathbf{b}\bullet\mathbf{a})+4\mathbf{a}\bullet\mathbf{a}\rvert = \lvert -3(0)+4(3)^2\rvert = 36$
	$\therefore \mathbf{c} = -3\mathbf{b}+4\mathbf{a}$

38 (i)	Since *A*, *B* and *P* are collinear, $\overrightarrow{OP} = \lambda\mathbf{a}+(1-\lambda)\mathbf{b}$. Since *M*, *N* and *P* are collinear, $\overrightarrow{OP} = \mu 2\mathbf{a}+(1-\mu)\frac{1}{3}\mathbf{b}$. Comparing, we have $\begin{cases}\lambda = 2\mu \\ 1-\lambda = \frac{1}{3}(1-\mu)\end{cases}$. Solving, we have $\mu = \frac{2}{5}$ and $\lambda = \frac{4}{5}$. Thus, $\overrightarrow{OP} = \frac{4}{5}\mathbf{a}+\frac{1}{5}\mathbf{b} = \frac{1}{5}(4\mathbf{a}+\mathbf{b})$. 

(ii)	Method 1 Area of $OMN = \frac{1}{2}\left\|2\mathbf{a}\times\frac{1}{3}\mathbf{b}\right\| = \frac{1}{3}\|\mathbf{a}\times\mathbf{b}\|$ Area of $APM = \frac{1}{2}\left\|\mathbf{a}\times\left(\frac{1}{5}\mathbf{b}-\frac{1}{5}\mathbf{a}\right)\right\| = \frac{1}{10}\|\mathbf{a}\times\mathbf{b}\|$ Area of the quadrilateral $OAPN$ $= \frac{1}{3}\|\mathbf{a}\times\mathbf{b}\| - \frac{1}{10}\|\mathbf{a}\times\mathbf{b}\| = \frac{7}{30}\|\mathbf{a}\times\mathbf{b}\|$  Method 2 Area of $OAB = \frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|$ Area of $BPN = \frac{1}{2}\left\|\frac{2}{3}\mathbf{b}\times\left(\frac{4}{5}\mathbf{a}-\frac{4}{5}\mathbf{b}\right)\right\| = \frac{4}{15}\|\mathbf{a}\times\mathbf{b}\|$ Area of the quadrilateral $OAPN$ $= \frac{1}{2}\|\mathbf{a}\times\mathbf{b}\| - \frac{4}{15}\|\mathbf{a}\times\mathbf{b}\| = \frac{7}{30}\|\mathbf{a}\times\mathbf{b}\|$ Method 3 Area of $OAP = \frac{1}{2}\left\|\mathbf{a}\times\frac{1}{5}(4\mathbf{a}+\mathbf{b})\right\| = \frac{1}{10}\|\mathbf{a}\times\mathbf{b}\|$ Area of ONP $= \frac{1}{2}\left\|\frac{1}{3}\mathbf{b}\times\frac{1}{5}(4\mathbf{a}+\mathbf{b})\right\| = \frac{2}{15}\|\mathbf{a}\times\mathbf{b}\|$ Area of the quadrilateral $OAPN$ $= \frac{1}{10}\|\mathbf{a}\times\mathbf{b}\| + \frac{2}{15}\|\mathbf{a}\times\mathbf{b}\| = \frac{7}{30}\|\mathbf{a}\times\mathbf{b}\|$

39(i)	$\begin{pmatrix}2\\0\\3\end{pmatrix}\times\begin{pmatrix}0\\-4\\5\end{pmatrix} = \begin{pmatrix}12\\-10\\-8\end{pmatrix} = 2\begin{pmatrix}6\\-5\\-4\end{pmatrix} \Rightarrow$ Normal to $p_1 = \begin{pmatrix}6\\-5\\-4\end{pmatrix}$ Vector equation of p_1 is $\mathbf{r}\bullet\begin{pmatrix}6\\-5\\-4\end{pmatrix} = \begin{pmatrix}1\\2\\7\end{pmatrix}\bullet\begin{pmatrix}6\\-5\\-4\end{pmatrix} = -32$ Hence the Cartesian equation of p_1 and p_2 are

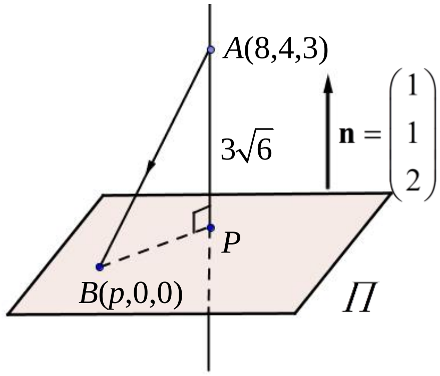
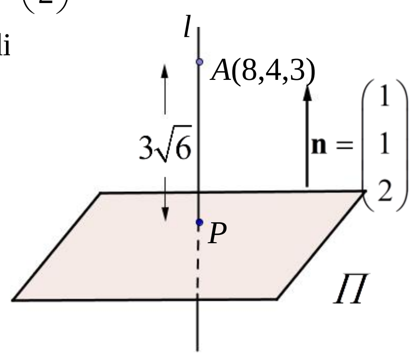
	$6x-5y-4z=-32$ $5x-y+3z=24$ Since the lines intersect, using GC, Vector equation of line l is $r=\begin{pmatrix}8\\16\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\-2\\1\end{pmatrix}, \lambda\in\mathbb{R}$
(ii)	$\begin{pmatrix}3\\-4\\1\end{pmatrix}-\begin{pmatrix}8\\16\\0\end{pmatrix}=\begin{pmatrix}-5\\-20\\1\end{pmatrix}$ Therefore 2 direction vectors parallel to p_3 are $\begin{pmatrix}-5\\-20\\1\end{pmatrix}$ and $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ $\begin{pmatrix}-5\\-20\\1\end{pmatrix}\times\begin{pmatrix}1\\2\\-1\end{pmatrix}=\begin{pmatrix}18\\-4\\10\end{pmatrix}=2\begin{pmatrix}9\\-2\\5\end{pmatrix}$ Normal to p_3 is $\begin{pmatrix}9\\-2\\5\end{pmatrix}$ Therefore, vector equation of plane p_3: $\mathbf{r}\bullet\begin{pmatrix}9\\-2\\5\end{pmatrix}=\begin{pmatrix}3\\-4\\1\end{pmatrix}\bullet\begin{pmatrix}9\\-2\\5\end{pmatrix}=40$ Thus, Cartesian equation of plane p_3: $9x-2y+5z=40$

40(i)	$\left\lvert\mathbf{a}\bullet\frac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right\rvert=\lvert\mathbf{a}\bullet\mathbf{b}\rvert$ represents the length of projection of $\overrightarrow{OA}$ onto $\overrightarrow{OB}$.
(ii)	$\lvert 3\mathbf{a}-\mathbf{b}\rvert^2=10^2$ $(3\mathbf{a}-\mathbf{b})\bullet(3\mathbf{a}-\mathbf{b})=100$ $9\mathbf{a}\bullet\mathbf{a}+\mathbf{b}\bullet\mathbf{b}-6\mathbf{a}\bullet\mathbf{b}=100$ $9\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2-6\mathbf{a}\bullet\mathbf{b}=100$ $6\mathbf{a}\bullet\mathbf{b}=9(3)^2+(5)^2-100=6$ Therefore $\mathbf{a}\bullet\mathbf{b}=1$.
(iii)	Let N be the foot of the perpendicular from A to the line OB.

	$ON=\left\|\mathbf{a}\bullet\frac{\mathbf{b}}{\|\mathbf{b}\|}\right\|=\frac{\|\mathbf{a}\bullet\mathbf{b}\|}{\|\mathbf{b}\|}=\frac{1}{5}$. Using Pythagoras Theorem, $AN^2=OA^2-ON^2=3^2-\left(\frac{1}{5}\right)^2=\frac{224}{25}$. $AN=\sqrt{\frac{224}{25}}=\frac{4}{5}\sqrt{14}$ Area of triangle $OAB=\frac{1}{2}OB\times AN=2\sqrt{14}$ units2
(iv)	$(\mu\mathbf{a}+2\mathbf{b})-\mathbf{a}=k\left[(2\mathbf{a}+3\mathbf{b})-\mathbf{a}\right]$ for some constant k. $(\mu-1)\mathbf{a}+2\mathbf{b}=k\mathbf{a}+3k\mathbf{b}$ Now, $\|\mathbf{a}\|\neq 0$, $\|\mathbf{b}\|\neq 0$ and $\|\mathbf{a}\bullet\mathbf{b}\|=1\neq\|\mathbf{a}\|\|\mathbf{b}\|$, so **a** and **b** are non-zero and non-parallel vectors. $\therefore\begin{cases}\mu-1=k\\2=3k\end{cases}$ Hence $k=\frac{2}{3}$ and $\mu=k+1=\frac{5}{3}$.

41(a)	We consider a plane Π first, $\begin{pmatrix}1\\1\\2\end{pmatrix}$ is a normal to Π. Equation of Π is $\mathbf{r}.\begin{pmatrix}1\\1\\2\end{pmatrix}=p$ and $B(p,0,0)$ lies in Π. Perpendicular distance = $=3\sqrt{6}$

l

	$AP = \left\|\overrightarrow{AB}.\underset{\sim}{\hat{n}}\right\| = 3\sqrt{6}$ $\left\|\begin{pmatrix} p-8 \\ -4 \\ -3 \end{pmatrix}.\frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\right\| = 3\sqrt{6}$ $\frac{1}{\sqrt{6}}\|p-18\| = 3\sqrt{6}$ $\|p-18\| = 18$ $p-18 = -18$ or 18 $p = 0$ or 36  Equations of the planes are $x+y+2z=0$ and $x+y+2z=36$. **<u>Alternative Solution</u>** We consider a plane Π first**,** $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ is a normal to Π. Let P be the foot of perpendi $\overrightarrow{AP} = \pm 3\sqrt{6}\left[\frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\right] = \pm\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$ $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$ $= \begin{pmatrix} 8 \\ 4 \\ 3 \end{pmatrix} \pm \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} 11 \\ 7 \\ 9 \end{pmatrix}$  $\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}.\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$ or $\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 7 \\ 9 \end{pmatrix}.\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 36$ Equations of the planes are $x+y+2z=0$ and $x+y+2z=36$.
(b)(i)	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R}, \quad l_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 8 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix}, \mu \in \mathbb{R}$ Consider $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix}$,

	$2+\lambda=3+\mu$ ------- (1) $-\lambda=8+2\mu$ ------- (2) Solving (1) & (2), $\lambda=-2, \mu=-3$ $1+2\lambda=a\mu$ ------- (3) For skew lines, $\lambda=-2, \mu=-3$ do not satisfy (3). $-3\neq-3a$, ie $a\neq1$. $\therefore a\in\mathbb{R}, a\neq1$
(ii)	Since p and l_2 have no common point, l_2 is parallel to p. $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}1\\2\\-3\end{pmatrix}$ are parallel to p. $\begin{pmatrix}1\\-1\\2\end{pmatrix}\times\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-1\\5\\3\end{pmatrix}$ is a normal to p. $\mathbf{r}.\begin{pmatrix}-1\\5\\3\end{pmatrix}=\begin{pmatrix}2\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\3\end{pmatrix}=1$ $\therefore$ the required equation is $\mathbf{r}.\begin{pmatrix}-1\\5\\3\end{pmatrix}=1$

42(i)	$\mathbf{a}\bullet(\mathbf{a}+3\mathbf{b})=0$ $\lvert\mathbf{a}\rvert^2+3\mathbf{a}\bullet\mathbf{b}=0$ $1+3\mathbf{a}\bullet\mathbf{b}=0$ since $\mathbf{a}$ is a unit vector $\mathbf{a}\bullet\mathbf{b}=-\frac{1}{3}$ Since angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{2\pi}{3}$, $\cos\left(\frac{2\pi}{3}\right)=\frac{\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ $-\frac{1}{2}=\frac{-1/3}{(1)\lvert\mathbf{b}\rvert}$ $\lvert\mathbf{b}\rvert=\frac{2}{3}$

(ii)	$(\mathbf{b}-2\mathbf{a})\bullet(\mathbf{b}-2\mathbf{a})=\lvert\mathbf{b}-2\mathbf{a}\rvert^2$ $\lvert\mathbf{b}\rvert^2+4\lvert\mathbf{a}\rvert^2-4\mathbf{a}\bullet\mathbf{b}=\lvert\mathbf{b}-2\mathbf{a}\rvert^2$ $\frac{4}{9}+4-4\left(-\frac{1}{3}\right)=\lvert\mathbf{b}-2\mathbf{a}\rvert^2$ $\lvert\mathbf{b}-2\mathbf{a}\rvert=\sqrt{\frac{52}{9}}=\frac{2\sqrt{13}}{3}$
(iii)	By Ratio theorem, $\overrightarrow{OP}=\lambda\mathbf{b}+(1-\lambda)\mathbf{a}$ Note that $\lvert\mathbf{a}\times\mathbf{b}\rvert=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{3}$ Area of triangle *OAP* $=\frac{1}{2}\lvert\mathbf{a}\times\mathbf{p}\rvert=\frac{1}{2}\lvert\mathbf{a}\times(\lambda\mathbf{b}+(1-\lambda)\mathbf{a})\rvert=\frac{1}{2}\lvert\lambda\mathbf{a}\times\mathbf{b}+(1-\lambda)\mathbf{a}\times\mathbf{a}\rvert=\frac{1}{2}\lambda\lvert\mathbf{a}\times\mathbf{b}\rvert=\frac{\lambda\sqrt{3}}{6}$

43(a)	Equation of the line : $\mathbf{r}=\mathbf{a}+\lambda\mathbf{u},\ \lambda\in\mathbb{R}$ $\mathbf{r}-\mathbf{a}=\lambda\mathbf{u},\lambda\in\mathbb{R}$ $\Rightarrow\mathbf{r}-\mathbf{a}$ is parallel to $\mathbf{u}$ $(\mathbf{r}-\mathbf{a})\times\mathbf{u}=\mathbf{0}$
(b)	$\lvert\mathbf{b}-\mathbf{a}\rvert^2=(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})$ $=\mathbf{b}\cdot\mathbf{b}-2\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{a}$ $\lvert\mathbf{b}-\mathbf{a}\rvert^2=\lvert\mathbf{b}\rvert^2-2\mathbf{a}\cdot\mathbf{b}+\lvert\mathbf{a}\rvert^2$ $\left(\sqrt{7}\lvert\mathbf{a}\rvert\right)^2=\left(\sqrt{3}\lvert\mathbf{a}\rvert\right)^2-2\mathbf{a}\cdot\mathbf{b}+\lvert\mathbf{a}\rvert^2$ $7\lvert\mathbf{a}\rvert^2=4\lvert\mathbf{a}\rvert^2-2\mathbf{a}\cdot\mathbf{b}$ $2\mathbf{a}\cdot\mathbf{b}=-3\lvert\mathbf{a}\rvert^2$ $\mathbf{a}\cdot\mathbf{b}=-\frac{3}{2}\lvert\mathbf{a}\rvert^2\neq 0$ $\Rightarrow\overrightarrow{OA}$ is not perpendicular to $\overrightarrow{OB}$ Hence, $\angle AOB$ is not a right angle. Therefore, points *O*, *A* and *B* are not points that lie on the circumference of a circle with diameter *AB*.

44(i)	(i) $(\underset{\sim}{r}-\underset{\sim}{a})\times\underset{\sim}{b}=\underset{\sim}{0}$ Either $\underset{\sim}{b}=\underset{\sim}{0}$ (reject since $\underset{\sim}{b}$ is a non-zero vector) or $(\underset{\sim}{r}-\underset{\sim}{a})=\underset{\sim}{0}$ or $(\underset{\sim}{r}-\underset{\sim}{a})//\underset{\sim}{b}$ $\Rightarrow \underset{\sim}{r}=\underset{\sim}{a}$ OR $(\underset{\sim}{r}-\underset{\sim}{a})=k\underset{\sim}{b}$ for $k\in\mathbb{R}\setminus\{0\}$ $\Rightarrow \underset{\sim}{r}=\underset{\sim}{a}+k\underset{\sim}{b}$ for $k\in\mathbb{R}$ (Note that $\underset{\sim}{r}=\underset{\sim}{a}$ is included in these solutions) This is the equation of a line passing through point with position vector $\underset{\sim}{a}$ and parallel to the vector $\underset{\sim}{b}$.
(ii)	$\underset{\sim}{n}=\begin{pmatrix}0\\1\\1\end{pmatrix}\times\begin{pmatrix}m\\0\\1\end{pmatrix}=\begin{pmatrix}1\\m\\-m\end{pmatrix}$ p1: $\underset{\sim}{r}\bullet\begin{pmatrix}1\\m\\-m\end{pmatrix}=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\m\\-m\end{pmatrix}=2+2m$ □p2: $\underset{\sim}{r}\bullet\begin{pmatrix}1\\m\\-m\end{pmatrix}=\begin{pmatrix}-1\\2\\3\end{pmatrix}\bullet\begin{pmatrix}1\\m\\-m\end{pmatrix}=-1-m$
(iii)	Let θ be the acute angle between the plane and the line. $\sin\theta=\left\|\frac{\underset{\sim}{d}\bullet\underset{\sim}{n}}{\|\underset{\sim}{d}\|\|\underset{\sim}{n}\|}\right\|=\frac{\left\|\begin{pmatrix}-3\\1\\4\end{pmatrix}\bullet\begin{pmatrix}1\\m\\-m\end{pmatrix}\right\|}{\sqrt{26}\sqrt{1+2m^2}}=\frac{\|-3-3m\|}{\sqrt{26+52m^2}}$ $\theta=\sin^{-1}\left\|\frac{3+3m}{\sqrt{26+52m^2}}\right\|$
(iv)	For p_3 to contain both L_1 and L_2, $\theta=0$. $\Rightarrow 3+3m=0$ $\Rightarrow m=-1$ Substituting $m=-1$ into either plane equation above, $\underset{\sim}{r}\bullet\begin{pmatrix}1\\-1\\1\end{pmatrix}=0$ $x-y+z=0$ is the Cartesian equation of p_3.

45(i)	Shortest distance $=\lvert\mathbf{b}\times\hat{\mathbf{a}}\rvert$ or $\dfrac{\lvert\mathbf{b}\times\mathbf{a}\rvert}{\lvert\mathbf{a}\rvert}$
(ii)	$\lvert\mathbf{b}\times\mathbf{c}\rvert$ is the area of a parallelogram with sides *OB* and *OC*. $\lvert\mathbf{b}\times\mathbf{c}\rvert=\lvert\mathbf{b}\times(\mathbf{a}+\mathbf{b})\rvert=\lvert(\mathbf{b}\times\mathbf{a})+(\mathbf{b}\times\mathbf{b})\rvert=\lvert\mathbf{b}\times\mathbf{a}\rvert=\lvert-(\mathbf{a}\times\mathbf{b})\rvert=\lvert\mathbf{a}\times\mathbf{b}\rvert$ (shown) Note that the area of the parallelogram with sides *OB* and *OC* is the same as the area of the parallelogram with sides *OA* and *OB* (with same base and vertical height)
(iii)	$\mathbf{a}\times 2\mathbf{b}=\mathbf{d}\times 3\mathbf{a}$ $(\mathbf{a}\times 2\mathbf{b})-(\mathbf{d}\times 3\mathbf{a})=0$ $\mathbf{a}\times(2\mathbf{b}+3\mathbf{d})=0$ Vector **a** is parallel to $2\mathbf{b}+3\mathbf{d}$. $\mathbf{a}=k(2\mathbf{b}+3\mathbf{d}),k\in\mathbb{R}$

46 **(i)**	$\overrightarrow{AB}=\begin{pmatrix}5\\4\\-1\end{pmatrix}-\begin{pmatrix}5\\-2\\5\end{pmatrix}=6\begin{pmatrix}0\\1\\-1\end{pmatrix}\Rightarrow\left\lvert\overrightarrow{AB}\right\rvert=6\sqrt{2}$ $\overrightarrow{AC}=\begin{pmatrix}-1\\-2\\-1\end{pmatrix}-\begin{pmatrix}5\\-2\\5\end{pmatrix}=-6\begin{pmatrix}1\\0\\1\end{pmatrix}\Rightarrow\left\lvert\overrightarrow{AC}\right\rvert=6\sqrt{2}$ $\overrightarrow{BC}=\begin{pmatrix}-1\\-2\\-1\end{pmatrix}-\begin{pmatrix}5\\4\\-1\end{pmatrix}=-6\begin{pmatrix}1\\1\\0\end{pmatrix}\Rightarrow\left\lvert\overrightarrow{AC}\right\rvert=6\sqrt{2}$ Since *AB* = *BC* = *CA*, triangle *ABC* is an equilateral triangle.
(ii)	$\begin{pmatrix}0\\1\\-1\end{pmatrix}\times\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$
(iii)	Since π_1 is perpendicular to $\overrightarrow{AB}$, the normal vector of π_1 is $\begin{pmatrix}0\\1\\-1\end{pmatrix}$. By symmetry, π_1 will pass through *C*. $\pi_1:\ \underset{\sim}{r}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=\begin{pmatrix}-1\\-2\\-1\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=-1$ Cartesian equation of π_1 is: $y-z=-1$

(iv)	Since π_2 is perpendicular to $\overrightarrow{BC}$, the normal vector of π_2 is $\begin{pmatrix}1\\1\\0\end{pmatrix}$. By symmetry, π_2 will pass through *A*. $\pi_2 : \underset{\sim}{r} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}5\\-2\\5\end{pmatrix} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} = 3$ Cartesian equation of π_2 is: $x + y = 3$ Using GC, the equation of the line of intersection of the two planes is $l: \ \underset{\sim}{r} = \begin{pmatrix}4\\-1\\0\end{pmatrix} + \lambda \begin{pmatrix}-1\\1\\1\end{pmatrix}$, where $\lambda \in \mathbb{R}$.
(v)	Note that point *G* lies on the line *l* found in part **(iv)**. Since *G* lies on *l*, $\overrightarrow{OG} = \begin{pmatrix}4\\-1\\0\end{pmatrix} + \lambda \begin{pmatrix}-1\\1\\1\end{pmatrix}$ for some $\lambda \in \mathbb{R}$ $\overrightarrow{AG} = \begin{pmatrix}-1-\lambda\\1+\lambda\\-5+\lambda\end{pmatrix}$ & $\overrightarrow{DG} = \begin{pmatrix}-3-\lambda\\3+\lambda\\\lambda+3\end{pmatrix}$ Given that $\left\|\overrightarrow{DG}\right\| = \left\|\overrightarrow{AG}\right\|$, $2(1+\lambda)^2 + (\lambda-5)^2 = 3(\lambda+3)^2$ $2(\lambda^2 + 2\lambda + 1) + (\lambda^2 - 10\lambda + 25) = 3(\lambda^2 + 6\lambda + 9)$ $4\lambda + 2 - 10\lambda + 25 = 18\lambda + 27$ $\lambda = 0$ $\overrightarrow{OG} = \begin{pmatrix}4\\-1\\0\end{pmatrix}$ Thus, coordinates of *G* are (4, -1, 0).
(vi)	$\cos\angle AGD = \dfrac{\overrightarrow{DG} \bullet \overrightarrow{AG}}{\left\|\overrightarrow{DG}\right\|\left\|\overrightarrow{AG}\right\|} = \dfrac{3\begin{pmatrix}-1\\1\\1\end{pmatrix} \bullet \begin{pmatrix}-1\\1\\-5\end{pmatrix}}{(3\sqrt{3})(\sqrt{27})} = -\dfrac{1}{3}$ $\angle AGD = 109.5^\circ$
	<u>Alternative:</u> Let angle *AGX* be α. $\cos\alpha = \dfrac{1}{3} \Rightarrow \alpha = 70.52^\circ \ \Rightarrow \ \theta = 180^\circ - \alpha = 109.5^\circ$

46. 2022 ASRJC Prelim/II/2

Relative to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. It is given that λ and μ are non-zero numbers such that $\lambda\mathbf{a} + \mu\mathbf{b} - \mathbf{c} = \mathbf{0}$ and $\lambda + \mu = 1$.

(i) Show that the points A, B and C are collinear. [3]

The angle between $\mathbf{a}$ and $\mathbf{b}$ is known to be obtuse and that $|\mathbf{a}| = 2$.

(ii) If k denotes the area of triangle OAB, show that $(\mathbf{a}\cdot\mathbf{b})^2 = 4\left(|\mathbf{b}|^2 - k^2\right)$. [3]

D is a point on the line segment AB with position vector $\mathbf{d}$.

(iii) It is given that area of triangle OAB is 6 units^2, $|\mathbf{b}| = 10$ and that AOD is 90^0. By finding the value of $\mathbf{a}\cdot\mathbf{b}$, find $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

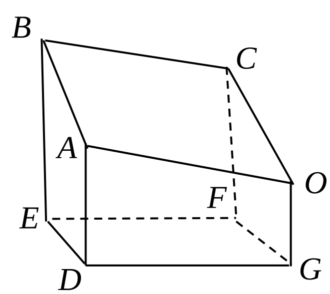
ASRJC Prelim 9758/2022/02/Q2

	Solution	
	(i) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ $= \lambda\mathbf{a} + \mu\mathbf{b} - \mathbf{a}$	
	$= (\lambda - 1)\mathbf{a} + \mu\mathbf{b}$	
	$= -\mu\mathbf{a} + \mu\mathbf{b}$	
	$= \mu(\mathbf{b} - \mathbf{a})$	
	Since $\overrightarrow{AC} = \mu\overrightarrow{AB}$ for some $\mu \in \mathbb{R}$, and A is a common point, therefore A, B, C are collinear.	
	(ii) $k = \frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert$	
	$k = \frac{1}{2}\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\lvert\sin\theta\rvert$, where θ is the obtuse angle between $\mathbf{a}$ and $\mathbf{b}$	
	$k^2 = \lvert\mathbf{b}\rvert^2 \sin^2\theta$	
	$k^2 = \lvert\mathbf{b}\rvert^2\left(1 - \cos^2\theta\right)$	
	$k^2 = \lvert\mathbf{b}\rvert^2\left[1 - \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)^2\right]$	
	$k^2 = \lvert\mathbf{b}\rvert^2 - \frac{(\mathbf{a}\cdot\mathbf{b})^2}{4}$	

	$(\mathbf{a}\cdot\mathbf{b})^2 = 4\left(\lvert\mathbf{b}\rvert^2 - k^2\right)$	
	(iii) Since *D* lies on line *AB*, $\mathbf{d} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ for some $\lambda \in \mathbb{R}$	
	OD is perpendicular to *OA* $\Rightarrow \left[\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})\right]\cdot\mathbf{a} = 0$ for some $\lambda \in \mathbb{R}$	
	$\Rightarrow (1-\lambda)\lvert\mathbf{a}\rvert^2 + \lambda(\mathbf{b}\cdot\mathbf{a}) = 0$ $4(1-\lambda) + \lambda(\mathbf{b}\cdot\mathbf{a}) = 0$	
	As $(\mathbf{a}\cdot\mathbf{b})^2 = 4\left(\lvert\mathbf{b}\rvert^2 - k^2\right)$	
	$(\mathbf{a}\cdot\mathbf{b})^2 = 4\left(10^2 - 6^2\right)$	
	$\mathbf{a}\cdot\mathbf{b} = -16 (\because \theta \text{ is obtuse})$	
	$\Rightarrow 4(1-\lambda) - 16\lambda = 0$	
	$\lambda = \frac{1}{5}$	
	$\mathbf{d} = \frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$	

47. 2022 CJC Prelim/I/10

The diagram below shows the structure of a building.



The slanted rooftop is modelled by the plane *OABC* where *O* is taken as the origin. The horizontal ground is modelled by the plane *DEFG* which has a normal vector in the direction of $\mathbf{k}$. It is given that the position vectors of points *A*, *B* and *C* are $-7\mathbf{j}+\mathbf{k}$, $3\mathbf{i}-9\mathbf{j}+3\mathbf{k}$ and $4\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ respectively where the units are in metres.

(i) Find the cartesian equation of the rooftop. [3]

(ii) Find the acute angle between the rooftop and the horizontal ground. [2]

(iii) Find the area of the slanted rooftop. [3]

A point H has position vector $-2\mathbf{i}-5\mathbf{j}+9\mathbf{k}$.

(iv) find the coordinates of the point on the rooftop which is nearest to H. [4]

CJC Prelim 9758/2022/01/Q10

(i) Normal to plane OABC

$=\overrightarrow{OA}\times\overrightarrow{OC}$

$=\begin{pmatrix}0\\-7\\1\end{pmatrix}\times\begin{pmatrix}4\\-5\\3\end{pmatrix}$

$=\begin{pmatrix}-21+5\\-(0-4)\\0+28\end{pmatrix}$

$=\begin{pmatrix}-16\\4\\28\end{pmatrix}=4\begin{pmatrix}-4\\1\\7\end{pmatrix}$

Equation of rooftop plane is $\mathbf{r}\bullet\begin{pmatrix}-4\\1\\7\end{pmatrix}=0$

Hence cartesian equation is $-4x+y+7z=0$

(ii)

Angle between rooftop and ground

$=\cos^{-1}\dfrac{\left|\begin{pmatrix}-4\\1\\7\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}\right|}{\sqrt{16+1+49}\sqrt{1}}$

$=\cos^{-1}\dfrac{|7|}{\sqrt{66}}$

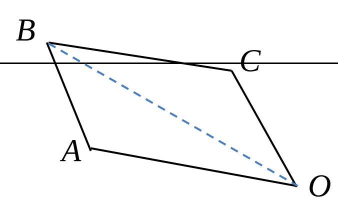
$=30.5^{0}$

(iii)

Note that rooftop OABC is **<u>not</u>** a parallelogram.

Surface area of rooftop (where units are in tens of metres)
= sum of area of triangle OAB and triangle OBC

$=\dfrac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OB}\right|+\dfrac{1}{2}\left|\overrightarrow{OC}\times\overrightarrow{OB}\right|$



$$=\frac{1}{2}\left|\begin{pmatrix}0\\-7\\1\end{pmatrix}\times\begin{pmatrix}3\\-9\\3\end{pmatrix}\right|+\frac{1}{2}\left|\begin{pmatrix}4\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-9\\3\end{pmatrix}\right|$$

$$=\frac{3}{2}\left|\begin{pmatrix}0\\-7\\1\end{pmatrix}\times\begin{pmatrix}1\\-3\\1\end{pmatrix}\right|+\frac{3}{2}\left|\begin{pmatrix}4\\-5\\3\end{pmatrix}\times\begin{pmatrix}1\\-3\\1\end{pmatrix}\right|$$

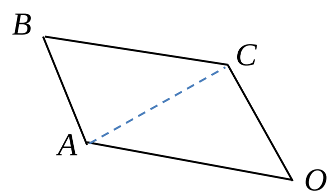
$$=\frac{3}{2}\left|\begin{pmatrix}-4\\1\\7\end{pmatrix}\right|+\frac{3}{2}\left|\begin{pmatrix}4\\-1\\-7\end{pmatrix}\right|$$

$$=3\sqrt{66}\ \text{units}^2$$

<u>Alternative Method</u>

Surface area of rooftop (where units are in tens of metres)
= sum of area of triangle OAC and triangle ABC

$$=\frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OC}\right|+\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$



$$=\frac{1}{2}\left|\begin{pmatrix}0\\-7\\1\end{pmatrix}\times\begin{pmatrix}4\\-5\\3\end{pmatrix}\right|+\frac{1}{2}\left|\begin{pmatrix}3-0\\-9+7\\3-1\end{pmatrix}\times\begin{pmatrix}4-0\\-5+7\\3-1\end{pmatrix}\right|$$

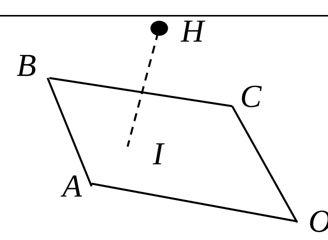
$$=\frac{1}{2}\left|\begin{pmatrix}0\\-7\\1\end{pmatrix}\times\begin{pmatrix}4\\-5\\3\end{pmatrix}\right|+\frac{1}{2}\left|\begin{pmatrix}3\\-2\\2\end{pmatrix}\times\begin{pmatrix}4\\2\\2\end{pmatrix}\right|$$

$$=\frac{1}{2}\left|\begin{pmatrix}-16\\4\\28\end{pmatrix}\right|+\frac{1}{2}\left|\begin{pmatrix}-8\\2\\14\end{pmatrix}\right|$$

$$=\frac{1}{2}\sqrt{1056}+\frac{1}{2}\sqrt{264}=3\sqrt{66}\ \text{units}^2$$

Note:
Students may notice that the cross product is the same as that for the normal of plane *OABC* and so can use the result from (i) instead of solving the cross product from scratch.
Students may notice that plane *OABC* is a kite and so the area can be taken as twice area of triangle OAB.

(iv)
For the pole to be shortest, the pole must be perpendicular to the rooftop from point *H*.
Let *I* be the foot of perpendicular from *H* to plane *OABC*.



L_{HI} : $\mathbf{r}=\begin{pmatrix}-2\\-5\\9\end{pmatrix}+\lambda\begin{pmatrix}-4\\1\\7\end{pmatrix}$

Since I lies on plane $OABC$,

$$\begin{pmatrix}-2-4\lambda\\-5+\lambda\\9+7\lambda\end{pmatrix}\bullet\begin{pmatrix}-4\\1\\7\end{pmatrix}=0$$

$$8+16\lambda-5+\lambda+63+49\lambda=0$$

$$\lambda=-1$$

So, $\vec{OI}=\begin{pmatrix}-2+4\\-5-1\\9-7\end{pmatrix}=\begin{pmatrix}2\\-6\\2\end{pmatrix}$

So point is $(2,-6,2)$

Alternative method 1:

$$\pi_{OABC}:\ \mathbf{r}=s\begin{pmatrix}0\\-7\\1\end{pmatrix}+t\begin{pmatrix}1\\-3\\1\end{pmatrix}=\begin{pmatrix}0+\\-49/5\\7/5\end{pmatrix}$$

$$\overrightarrow{IH}=\begin{pmatrix}-2-t\\-5+7s+3t\\9-s-t\end{pmatrix}$$

Since $\overrightarrow{IH}\perp\pi_{OABC}$,

$$\begin{pmatrix}-2-t\\-5+7s+3t\\9-s-t\end{pmatrix}\bullet\begin{pmatrix}0\\-7\\1\end{pmatrix}=0$$

$$35-49s-21t+9-s-t=0 \quad \text{-----}(1)$$

$$\begin{pmatrix}-2-t\\-5+7s+3t\\9-s-t\end{pmatrix}\bullet\begin{pmatrix}1\\-3\\1\end{pmatrix}=0$$

$$-2-t+15-21s-9t+9-s-t=0 \quad \text{-----}(2)$$

Solving, $s=0,\ t=2$

Coordinates: $(2,-6,2)$

Alternative method 2:

$$\overrightarrow{BH} = \begin{pmatrix} -5 \\ 4 \\ 6 \end{pmatrix}$$

$$\overrightarrow{IH} = \left(\overrightarrow{BH} \bullet \mathbf{n}\right)\mathbf{n}$$

$$= \left(\frac{\begin{pmatrix} -5 \\ 4 \\ 6 \end{pmatrix} \bullet \begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}}{\sqrt{4^2 + 1^2 + 7^2}} \right) \frac{\begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}}{\sqrt{4^2 + 1^2 + 7^2}}$$

$$= \begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}$$

$$\overrightarrow{OI} = \overrightarrow{OH} - \overrightarrow{IH}$$

$$= \begin{pmatrix} -2 \\ -5 \\ 9 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}$$

Coordinates: $\left(2, -6, 2\right)$