1 Without using a calculator, solve

$$x < \frac{3}{x-2}.$$ [3]

Hence solve

$$e^{-x} < \frac{3}{e^{-x}-2}.$$ [2]

2 The curve C has equation $y=\dfrac{1-2x}{2x+3}$.

(i) Express the equation of C in the form $y=a+\dfrac{b}{2x+3}$, where a and b are constants to be determined. [2]

(ii) Describe a sequence of three transformations which transform the graph of $y=\dfrac{1}{x+3}$ onto the graph of C. [3]

3 By writing $\dfrac{1}{4r^2-1}$ in partial fractions, find $\displaystyle\sum_{r=1}^{n} \frac{4}{4r^2-1}$, giving your answer in the form $M-\mathrm{f}(n)$, where M is a real constant to be determined. [5]

Explain why $\displaystyle\sum_{r=1}^{\infty} \frac{4}{4r^2-1}$ is a convergent series, and state the value of the sum to infinity. [2]

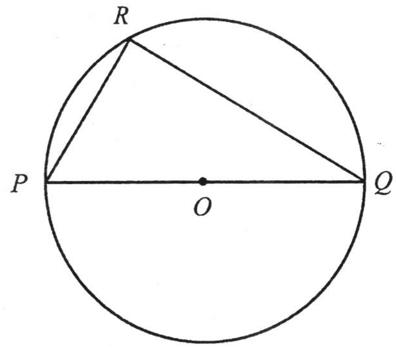
4 Find the derivative of the following expressions with respect to x, leaving your answers in terms of x only.

(a) $\ln\sqrt{x^2+1}$, [2]

(b) $\tan^{-1}\left(e^{2x}\right)$, [2]

(c) $x^{\sec 2x}$. [3]

5 The diagram shows a circle with centre O and diameter PQ. The point R lies on the circumference of the circle. Taking the centre O as the origin, the position vectors of the points P, Q and R are $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

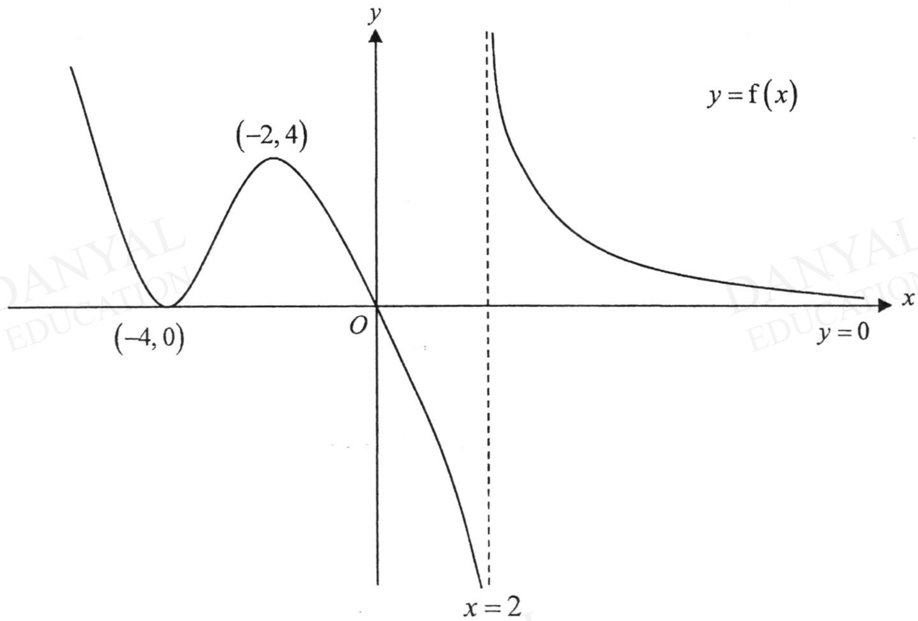


(i) By first writing down $\overrightarrow{PR}$ and $\overrightarrow{QR}$ in terms of $\mathbf{p}$ and $\mathbf{r}$, prove that the lines PR and QR are perpendicular, showing your working clearly. [4]

It is given that $\mathbf{q} \times \mathbf{r} = (\mathbf{q} - \mathbf{r}) \times \mathbf{s}$ where $\mathbf{s}$ is the position vector of the point S.

(ii) By considering $\overrightarrow{SR} \times \overrightarrow{QR}$, show that $\overrightarrow{SR} = k\overrightarrow{QR}$, $k \in \mathbb{R}$, $k \neq 0$. [3]

6 The diagram shows the graph of $y=f(x)$. The curve passes through the origin, has a minimum point at $(-4,0)$ and a maximum point at $(-2,4)$. The lines $y=0$ and $x=2$ are asymptotes of the curve.



On separate diagrams, sketch the graphs of

(i) $y=f'(x)$, [2]

(ii) $y=f(3x+2)$, and [3]

(iii) $y=\dfrac{1}{f(x)}$, [3]

indicating clearly the equations of any asymptotes, the coordinates of any turning points and any points where the curve crosses the x- and y-axes whenever possible.

7 A curve has equation $y=\sqrt{2-xy}$.

(i) Without using a calculator, find the equations of the tangent and normal to the curve at the point P where $x=-1$. [7]

(ii) Hence find the exact area of the region bounded by the tangent and normal to the curve at the point P and the y-axis. [2]

8 The curve C has equation

$$y=\frac{3x^2-2x-1}{x+1} \text{ for } x\in\mathbb{R},\ x\neq -1.$$

(i) Find, using an algebraic method, the range of values that y cannot take. [3]

(ii) Sketch C, stating clearly the equations of asymptotes, the coordinates of the axial intercepts and the coordinates of the turning points. [4]

(iii) Verify that $(-1,\ -8)$ lies on the graph $y=m(x+1)-8$, where $m\in\mathbb{R}$. Hence find the range of values of m for which the equation $\frac{3x^2-2x-1}{x+1}=m(x+1)-8$ has 2 distinct real roots. [2]

9 A sequence $u_1,\ u_2,\ u_3,\ \ldots$ is such that $u_{n+1} = 3u_n + Pn$, where P is a constant and $n \in \mathbb{Z}^+$. The terms of the sequence are defined by their previous terms, for example, $u_2 = 3u_1 + P$.

(i) Given that $u_1 = 1$ and $u_2 = 7$, find P and u_3. [2]

It is known that the nth term of this sequence is given by

$$u_n = a\left(3^n\right) + bn + c,$$

where a, b and c are constants.

(ii) Find a, b and c. [4]

(iii) Find $\sum_{r=1}^{n} u_r$ in terms of n. [4]

10 A function f is said to be a self-inverse function if $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$.

The function f is defined by

$$\mathrm{f}(x)=\frac{ax+b}{cx-2}, \quad x\in\mathbb{R},\ x\neq\frac{2}{c},$$

where a, b, c and d are non-zero constants.

(i) By finding $\mathrm{f}^{-1}(x)$, show that $a=2$ for f to be a self-inverse function. [2]

For the rest of the question, use $a=2$, $b=3$ and $c=5$.

(ii) Find $\mathrm{f}^{2}(x)$. [1]

(iii) Evaluate $\mathrm{f}^{71}(4)$. [2]

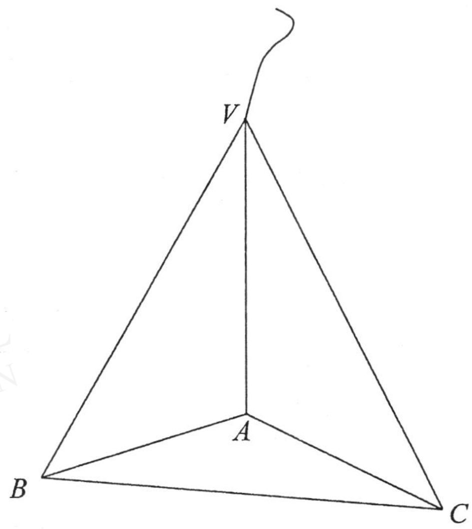
The function g is defined by

$$\mathrm{g}(x)=2x^2-3, \quad x\in\mathbb{R}$$

(iv) Explain why composite function fg does not exist. [2]

(v) Find the exact solutions of $\mathrm{gf}(x)=5$. [3]

11 A hanging ornament is constructed by connecting six metal rods to form a tetrahedron $VABC$ as shown in the diagram.



The points (x, y, z) are defined relative to a reference point $O(0,0,0)$, where units are in centimetres. The coordinates of the points A, B, C and V are $(1,2,0)$, $(6,3,0)$, $(2,7,0)$ and $(3,4,5)$ respectively.

(i) Find the equation of the plane VBC in scalar-product form. [3]
(ii) Find the coordinates of the point P on the plane VBC which is nearest to the point A. [3]
(iii) Hence or otherwise, find the shortest distance from the point A to the plane VBC. [2]
(iv) Find the acute angle between the line VA and the plane VBC. [3]

12 **(a)** Mr Lam saves \$120 on 1 January 2019. On the first day of each subsequent month, he saves \$10 more than in the previous month, so he saves \$130 on 1 February 2019, \$140 on 1 March 2019, and so on. On what date will he have first saved over \$10 000 in total? [4]

(b) Mr Lui has opened a special investment account that guarantees an interest rate of 8% per annum, starting from the first year. He deposits \$50 000 into the account at the start of the first year and a fixed amount, $\$x$, at the beginning of each year from the second year onwards. The interest earned is based on the total amount in the account at the end of that particular year and it is deposited into the account.

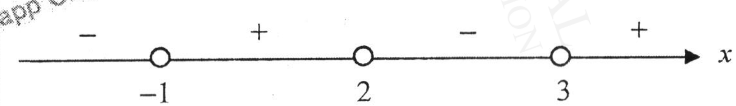
For example,
In Year One, at the beginning of the year there are 50000 dollars in the account, and at the end of that year there are (1.08)(50000) dollars.
In Year Two, at the beginning of the year there are $\left[(1.08)(50000)+x\right]$ dollars in the account, and at the end of that year there are $(1.08)\left[(1.08)(50000)+x\right]$ dollars, which simplifies to $\left[(1.08)^2(50000)+(1.08)x\right]$ dollars.

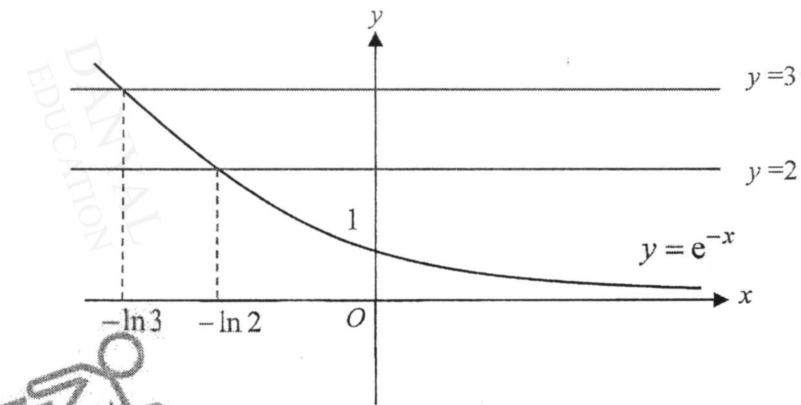
(i) Show that the total amount of money, in dollars, in the investment account at the end of the nth year is

$$50000(1.08)^n+13.5x\left(1.08^{n-1}-1\right).$$ [3]

(ii) If the yearly fixed amount that Mr Lui puts into the account from the second year onwards is \$5 000, find the total interest that he would earn at the end of the 10th year, correct to the nearest dollar. [2]

(iii) Mr Lui is interested in opening another investment account that is similar to the special investment account but with an interest rate of r% per annum. If his deposit for the first year is still \$50 000, and the yearly fixed deposit thereafter is \$5 000, find the value of r that would enable Mr Lui to have \$300 000 in the account after 10 years. [3]

Q1. Inequalities		
Assessment Objectives	**Solution**	**Examiner's Feedback**
Solving questions involving inequality	$x < \frac{3}{x-2}$ $x - \frac{3}{x-2} < 0$ $\frac{x(x-2)-3}{x-2} < 0$ $\frac{x^2-2x-3}{x-2} < 0$ $\frac{(x-3)(x+1)}{x-2} < 0$  $x < -1$ or $2 < x < 3$	The first part was generally well done. However, a handful of them cross-multiplied the inequality without considering the fact that $x-2>0$.

Identify replacement and solve accordingly	Replace x with e^{-x}, $e^{-x} < -1$ $\left(\text{rej.} \because e^{-x} > 0 \text{ for } x \in \mathbb{R}\right)$ or $2 < e^{-x} < 3$  $\therefore -\ln 3 < x < -\ln 2$	Almost everyone knows that they need to substitute e^{-x} for x in part (i). Then, very few of them used graphical method to solve for x. The majority of the students just take ln to both sides of the inequality, and this becomes $\ln 2 < \ln e^{-x} < \ln 3$ $\Rightarrow \ln 2 < -x < \ln 3$ $\Rightarrow -\ln 2 > x > -\ln 3$(Answer) A few forgot to change the inequality signs when this is divided by -1.

Q2. Transformation

Assessment Objectives	Solution		Examiner's Feedback
Carry out long division	(i)	$y=\frac{1-2x}{2x+3}$ $=-1+\frac{4}{2x+3}$ $\therefore a=-1$ and $b=4$ Long division: $(-2x+1)\div(2x+3)$: quotient -1; $-2x+1-(-2x-3)=4$	Almost all students can do long division to get the correct answer,
Work backwards for transformations	(ii)	$y=\frac{1}{x+3}$ $\downarrow$ scale parallel to x-axis by a scale factor of $\frac{1}{2}$ $y=\frac{1}{2x+3}$ $\downarrow$ scale parallel to y-axis by a scale factor of 4 $\frac{1}{4}y=\frac{1}{2x+3}$ $y=\frac{4}{2x+3}$ $\downarrow$ translate 1 units in the negative y-direction $y+1=\frac{4}{2x+3}$ $y=-1+\frac{4}{2x+3}$ **Alternative** 1st: Scale parallel to x-axis by a scale factor of ½ 2nd: Translate ¼ units in negative y-direction 3rd: Scale parallel to y-axis by a factor of 4	This part was generally well answered. Some made careless mistakes Such as replacement of x with $\frac{x}{2}$ instead of $2x$. Students also need to note the key terms to use, such as 'scale', 'translate', 'scale factors', 'positive/negative x, y – direction etc. [Instead of move/shift to y – direction etc] A number actually read the question carelessly and reversed the order of transformations.

Q3. Method of Difference

Assessment Objectives	Solution	Examiner's Feedback
Perform the procedure of method of difference.	$\frac{1}{4r^2-1}=\frac{1}{(2r-1)(2r+1)}=\frac{A}{(2r-1)}+\frac{B}{(2r+1)}$ $\therefore\ 1 = A(2r+1) + B(2r-1)$ By means of substitution or comparision, $A=\frac{1}{2}$ and $B=-\frac{1}{2}$. Hence $\frac{1}{4r^2-1}=\frac{1}{2}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$	Generally well done. For students who could not recall the decomposition of partial fractions, they can always refer to MF26.
	$\sum_{r=1}^{n}\left(\frac{4}{4r^2-1}\right)=4\sum_{r=1}^{n}\left(\frac{1}{2}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]\right)$ $=2\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$ $=2\left[\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2n-3}-\frac{1}{2n-1}+\frac{1}{2n-1}-\frac{1}{2n+1}\right]$ $=2\left(1-\frac{1}{2n+1}\right)$ $=2-\frac{2}{2n+1}$	Generally well done in the second part. Some common mistake are: • Some students wrote $\frac{1}{4}$ rather than 4 when they tried to re-express the given summation using the previous result which affected their final answers. • A small percentage of students tried to use Method of Differences without substituting r-values carefully. • A small percentage of students did not demonstrate Method of Differences in their workings clearly. • A small percentage of students did not carry out expansion to leave their answers in the required form $M-f(n)$. • A small percentage of students did not express the final answer in terms of n even though the given form $M-f(n)$ is given. Final answer should be simplified to the lowest terms.

Identify sum to infinity	As $n \to \infty$, $\frac{2}{2n+1} \to 0$. $\therefore \sum_{r=1}^{\infty} \frac{4}{4r^2-1}$ is a convergent series. Hence the sum to infinity is 2.	Answers are accepted based on students' results in the previous part. A common misconception is that students tend to associate sum to infinity to geometric progression (GP) only. The sum to infinity can exist for other series besides GP. Another common mistake is that students considered the scenario where *r* approaches infinity. It should be the case where *n* approaches infinity instead.

Q4. Techniques of Differentiation

Assessment Objectives	Solution		Examiner's Feedback
Differentiate logarithmic functions	(a)	**Method ①:** $\frac{\mathrm{d}}{\mathrm{d}x}\ln\sqrt{x^2+1}$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(x^2+1)^{\frac{1}{2}}\right]$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{2}\ln(x^2+1)\right]$ $=\frac{1}{2}\left(\frac{2x}{x^2+1}\right)$ $=\frac{x}{x^2+1}$ **Method ②:** $\frac{\mathrm{d}}{\mathrm{d}x}\ln\sqrt{x^2+1}$ $=\frac{1}{\sqrt{x^2+1}}\left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right]$ $=\frac{x}{x^2+1}$	Generally well done. It is generally easier to differentiate after simplifying the logarithm using laws of logarithms. Some common mistakes are: • Some students mixed up the two methods as seen in their workings. • Some students were not able to simplify their answers correctly as they applied the law of indices wrongly. • Some students could not recall that $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)=\frac{1}{x}$ and this following answer $\frac{\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)}{\ln(x^2+1)}$ was commonly seen.

<table>
<tr>
<td>Differentiate inverse trigonometric functions</td>
<td>(b)</td>
<td>
<u>Method ①:</u>
Let $y=\tan^{-1}\left(e^{2x}\right)$
$$\frac{dy}{dx}=\left(\frac{1}{1+\left(e^{2x}\right)^2}\right)\left(2e^{2x}\right)$$
$$=\frac{2e^{2x}}{1+e^{4x}}$$
<u>Method ②:</u>
$$\tan y=e^{2x}$$
$$\sec^2 y\frac{dy}{dx}=2e^{2x}$$
$$\frac{dy}{dx}=\frac{2e^{2x}}{\sec^2 y}$$
$$=\frac{2e^{2x}}{1+\tan^2 y}$$
$$=\frac{2e^{2x}}{1+e^{4x}}$$
</td>
<td>
Well-attempted.

Some common mistakes are:
• Some students did not perform Chain Rule which is an essential step of differentiation.
• Some students simplified $\left(e^{2x}\right)^2$ as e^{4x^2} as they applied the law of indices wrongly.
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<tr>
<td>Perform implicit differentiation</td>
<td>(c)</td>
<td>

<u>Method ①:</u>
Let $y = x^{\sec 2x}$
Take ln on both sides:
$\ln y = (\sec 2x)(\ln x)$
Differentiate with respect to x:
$\frac{1}{y}\left(\frac{dy}{dx}\right) = \underbrace{(\sec 2x)\left(\frac{1}{x}\right) + (2\sec 2x \cdot \tan 2x)(\ln x)}_{\text{Product Rule}}$
$\frac{dy}{dx} = y\sec 2x\left[\frac{1}{x} + 2(\tan 2x)(\ln x)\right]$
$= x^{\sec 2x}(\sec 2x)\left[\frac{1}{x} + 2(\tan 2x)(\ln x)\right]$

<u>Method ②:</u>
$\frac{d}{dx}\left(x^{\sec 2x}\right)$
$= \frac{d}{dx}\left(e^{\ln\left(x^{\sec 2x}\right)}\right)$
$= \frac{d}{dx}\left(e^{(\sec 2x)(\ln x)}\right)$
$= e^{(\sec 2x)(\ln x)}\underbrace{(\sec 2x)\left(\frac{1}{x}\right) + (2\sec 2x \cdot \tan 2x)(\ln x)}_{\text{Product Rule}}$
$= x^{\sec 2x}(\sec 2x)\left[\frac{1}{x} + 2(\tan 2x)(\ln x)\right]$

</td>
<td>

Poorly attempted.

Some common mistakes are:
• A significant percentage of students assumed $x^{\sec 2x} = (\sec 2x)(\ln x)$. These two expressions are not equal!
• Another group of students tried to apply the differentiation rule $\frac{d}{dx}(x^n) = nx^{n-1}$ when it did not apply in this part – the variable appeared in both the base and the index.
• Students tended to omit the constant 2 in their differentiation of $\sec 2x$.
• Some students did not read the question carefully as they expressed their final answers in terms of x and y.

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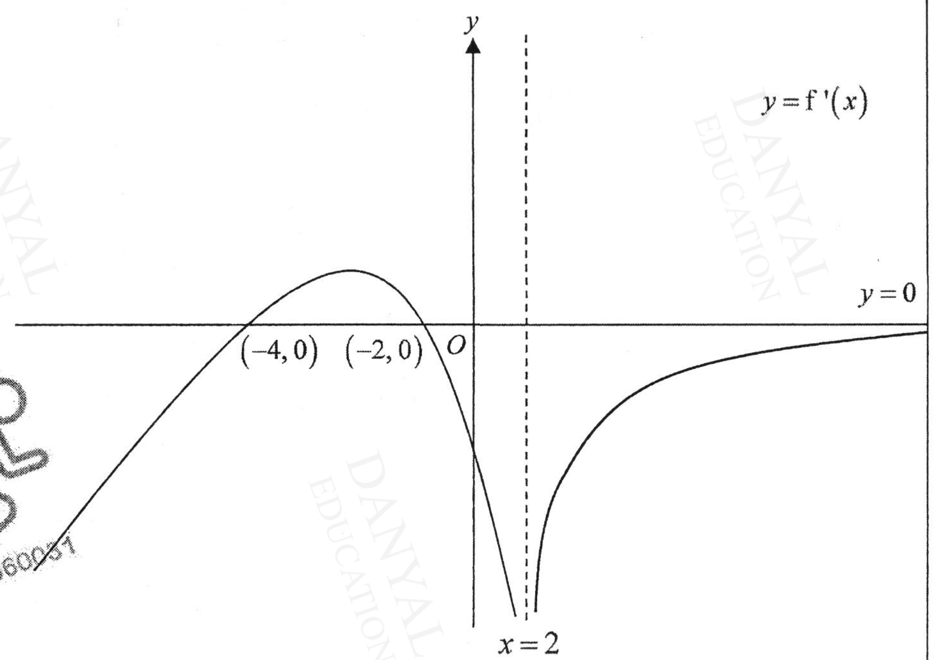
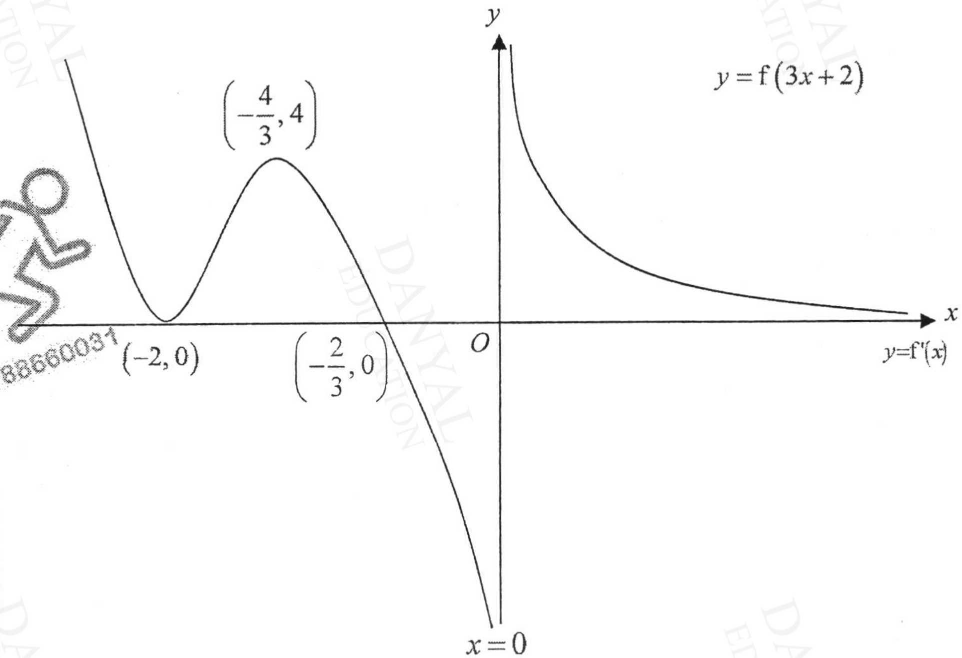
Q5. Vectors (Basic)

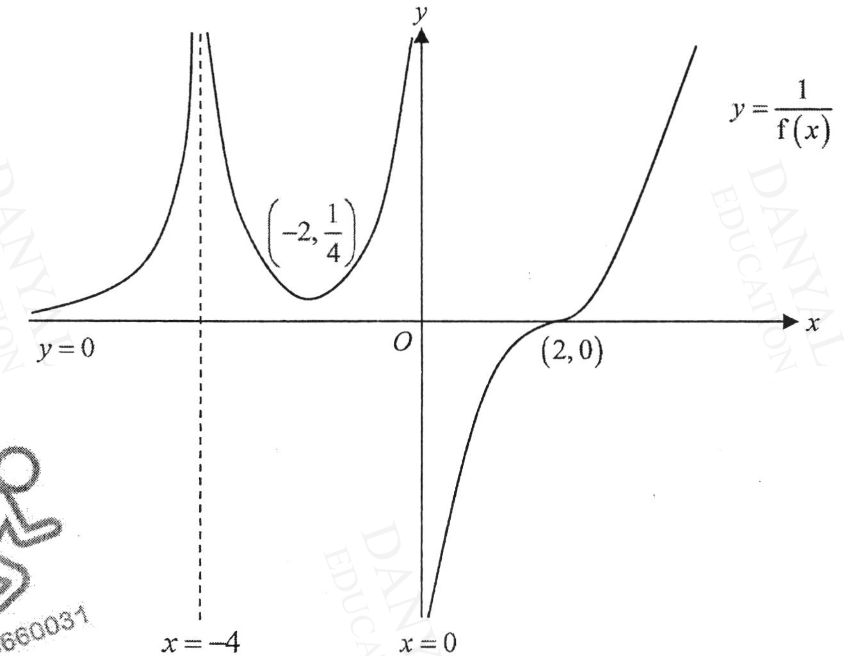
Assessment Objectives	Solution		Examiner's Feedback
Use triangle law of vector addition Apply the property of scalar product for vectors that are perpendicular	(i)	$\overrightarrow{PR} = \mathbf{r} - \mathbf{p}$ $\overrightarrow{QR} = \mathbf{r} - \mathbf{q} = \mathbf{r} + \mathbf{p}$ $\overrightarrow{PR} \cdot \overrightarrow{QR} = (\mathbf{r}+\mathbf{p}) \cdot (\mathbf{r}-\mathbf{p})$ $= \mathbf{r}\cdot\mathbf{r} - \mathbf{r}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{r} - \mathbf{p}\cdot\mathbf{p}$ $= \lvert\mathbf{r}\rvert^2 - \lvert\mathbf{p}\rvert^2$ $= 0 \quad (\lvert\mathbf{p}\rvert = \lvert\mathbf{r}\rvert = \text{radius of circle})$ Therefore lines PR and QR are perpendicular. (shown)	Students generally did not read question carefully. It is expected that students first write down $\overrightarrow{PR}$ and $\overrightarrow{QR}$ in terms of $\mathbf{p}$ and $\mathbf{r}$. However students simply left and $\overrightarrow{QR}$ in terms of $\mathbf{q}$ and $\mathbf{r}$. Students also did not realise that $\mathbf{p} = -\mathbf{q}$, i.e. they have the same magnitude but in opposite direction. Many assumed $\mathbf{p}$, $\mathbf{r}$ and $\mathbf{q}$ are equal vectors, i.e. same magnitude and same direction. This is clearly not true based on the given diagram. Students should also understand that in proving questions, clear steps must be written leading to the 'shown' part. There was omission of essential working in many scripts. Good scripts seen were those that explained $\lvert\mathbf{p}\rvert = \lvert\mathbf{r}\rvert$ because they are the radius of the circle.

Apply the property of cross product for vectors that are parallel. [HOT]	(ii)	$\overrightarrow{SR}\times\overrightarrow{QR}=(\mathbf{r}-\mathbf{s})\times(\mathbf{r}-\mathbf{q})$ $=\mathbf{r}\times\mathbf{r}-\mathbf{r}\times\mathbf{q}-\mathbf{s}\times(\mathbf{r}-\mathbf{q})$ $=\mathbf{0}+\mathbf{q}\times\mathbf{r}-\left[-(\mathbf{r}-\mathbf{q})\times\mathbf{s}\right]$ $=\mathbf{q}\times\mathbf{r}-(\mathbf{q}-\mathbf{r})\times\mathbf{s}$ $=\mathbf{0}$ (since $\mathbf{q}\times\mathbf{r}=(\mathbf{q}-\mathbf{r})\times\mathbf{s}$) $\Rightarrow$ SR and QR are parallel $\Rightarrow \overrightarrow{SR}=k\overrightarrow{QR}$, $k\in\mathbb{R}$, $k\neq 0$	Poorly attempted. Students did not consider $\overrightarrow{SR}\times\overrightarrow{QR}$ even though it is written in the question. Many started off with $\mathbf{q}\times\mathbf{r}=(\mathbf{q}-\mathbf{r})\times\mathbf{s}$, often leading to nowhere. Students who considered $\overrightarrow{SR}\times\overrightarrow{QR}$ were not careful with the expansion. For cross product, $-\mathbf{r}\times\mathbf{q}=\mathbf{q}\times\mathbf{r}$ (anti-commutative property) .

Q6. Transformations

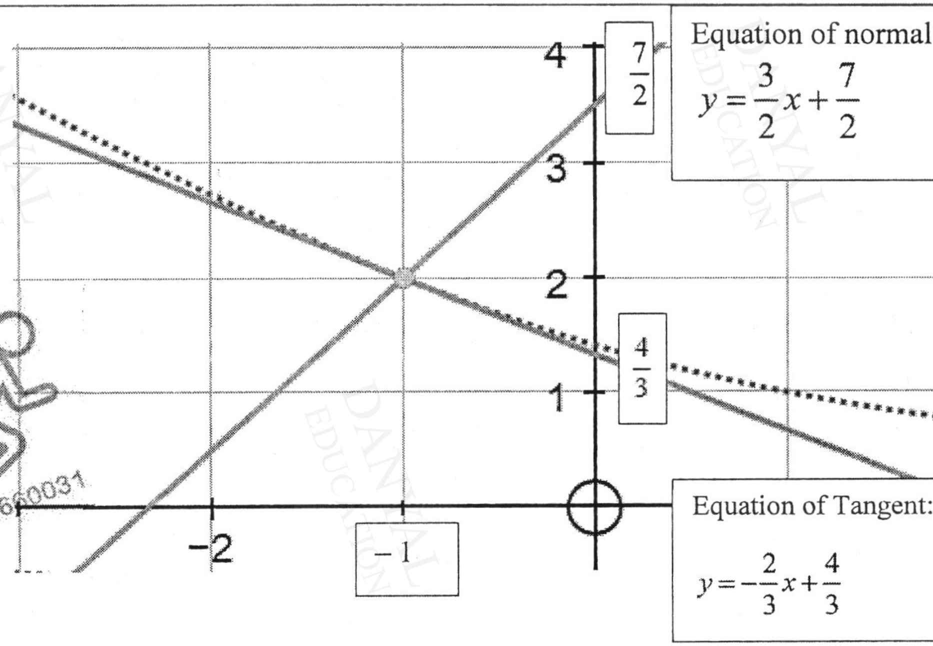
Assessment Objectives	Solution		Examiner's Feedback
Sketch derivative graph	(i)		Generally well done. Most students were able to identify the x-intercepts and asymptotes but lost mark for the shape.

Apply a series of transformations	**(ii)**	$y=\mathrm{f}(x)$ $\downarrow$ translate 2 units in negative x-direction $y=\mathrm{f}(x+2)$ $\downarrow$ scale // x-axis, factor $\frac{1}{3}$ $y=\mathrm{f}(3x+2)$  	Students who made mistakes are those who chose to scale first then translate. Others used the wrong scale factor.

Sketching of reciprocal graph	**(iii)**		Generally well done. Most students were able to identify the features of the reciprocal graph but lost mark for the shape.

Q7. Applications of Differentiation				
Assessment Objectives	**Solution**			**Examiner's Feedback**
Perform Implicit Differentiation	(i)	**Method ①:** $y=\sqrt{2-xy}=(2-xy)^{\frac{1}{2}}$ Differentiating both sides wrt x: $\frac{dy}{dx}=\frac{1}{2}(2-xy)^{-\frac{1}{2}}\left(0-\left(x\frac{dy}{dx}+y\right)\right)$ $\frac{dy}{dx}=-\frac{1}{2\sqrt{2-xy}}\left(x\frac{dy}{dx}+y\right)$ Substitute $y=\sqrt{2-xy}$ $\frac{dy}{dx}=-\frac{1}{2y}\left(x\frac{dy}{dx}+y\right)$ $\frac{dy}{dx}+\frac{x}{2y}\frac{dy}{dx}=-\frac{1}{2}$ $\left(1+\frac{x}{2y}\right)\frac{dy}{dx}=-\frac{1}{2}$ $\left(\frac{x+2y}{2y}\right)\frac{dy}{dx}=-\frac{1}{2}$ $\frac{dy}{dx}=\frac{-y}{x+2y}$	**Method ②:** Squaring both sides and rearranging, $y^2=2-xy$ $xy+y^2=2$ Differentiating both sides wrt x: $\left(x\frac{dy}{dx}+y\right)+2y\left(\frac{dy}{dx}\right)=0$ $\left(x\frac{dy}{dx}+2y\frac{dy}{dx}\right)=-y$ $(x+2y)\frac{dy}{dx}=-y$ $\frac{dy}{dx}=\frac{-y}{x+2y}$	It is much easier to do the second method to find the derivative of y. There were a lot of careless mistakes in algebraic manipulation. Quite a lot of students missed out the negative sign while differentiating $(2-xy)^{\frac{1}{2}}$ and ending up with $\frac{dy}{dx}=\frac{1}{2}(2-xy)^{-\frac{1}{2}}\left(x\frac{dy}{dx}+y\right)$ while some students forgot to differentiate the number "2" and ended up with $\frac{dy}{dx}=\frac{1}{2}(2-xy)^{-\frac{1}{2}}\left(2-\left(x\frac{dy}{dx}+y\right)\right)$.

Without a calculator, find the gradient at a given point		When $x=-1$, $y=\sqrt{2+y}$ $y^2=2+y$ $y^2-y-2=0$ $(y+1)(y-2)=0$ $y=2$ or $y=-1$ (rejected $\because y>0$) When $x=-1, y=2$, $\frac{dy}{dx}=\frac{-y}{x+2y}=\frac{-2}{-1+2(2)}=-\frac{2}{3}$ Equation of Tangent at $(-1, 2)$: $y-y_1=f'(x_1)(x-x_1)$ $y-2=\left(-\frac{2}{3}\right)(x-(-1))$ $y=-\frac{2}{3}(x+1)+2$ $y=-\frac{2}{3}x+\frac{4}{3}$ Equation of Normal $(-1, 2)$: $y-y_1=-\frac{1}{f'(x_1)}(x-x_1)$ $y-2=-\left(-\frac{3}{2}\right)(x-(-1))$	Some students wrote the equation of tangent as $y-2=\left(\frac{-y}{x+2y}\right)(x-(-1))$ without first finding the gradient at $x=-1$. A lot of students substitute $x=-1$ into $y=\sqrt{2-xy}$ and get $y=\sqrt{2-y}$, hence giving the wrong values for y. Quite a number of students did not reject $y=-1$ and ended up with two sets of equations of tangent and normal.

		$y=\frac{3}{2}(x+1)+2$ $y=\frac{3}{2}x+\frac{7}{2}$	
	(ii)	 Area of triangle $=\frac{1}{2}\left(\frac{7}{2}-\frac{4}{3}\right)(1)$ $=\frac{1}{2}\left(\frac{7}{2}-\frac{4}{3}\right)$ $=\frac{13}{12}$ units^2	Students should draw diagram to see where is the region of the area that they need to find. A lot of students identified the wrong region. Instead of the region bounded with the y-axis, they found the region bounced with the x-axis. The question asked for "the exact area of the region …", this should suggest only one answer to this part of the question. However, students who found two sets of tangent and normal, continued with two sets of areas. Some students did not leave their answer in ***exact*** value as requested by the question.

Q8. Graphing Techniques

Assessment Objectives	Solution		Examiner's Feedback
Use algebraic method to find the range of values for which y cannot take	(i)	$y=\dfrac{3x^2-2x-1}{x+1}$ $yx+y=3x^2-2x-1$ $3x^2+(-2-y)x+(-1-y)=0$ Consider $b^2-4ac<0$, $(-2-y)^2-4(3)(-1-y)<0$ $4+4y+y^2+12+12y<0$ $y^2+16y+16<0$ Let $y^2+16y+16=0$ **Method 1:** From GC, $y=-14.9$ or -1.07 **Method 2:** $y=\dfrac{-16\pm\sqrt{16^2-4(1)(16)}}{2}$ $=\dfrac{-16\pm\sqrt{192}}{2}$ $=-8\pm4\sqrt{3}$ **Method 3:** $y^2+16y+16=0$ $(y+8)^2-64+16=0$ $(y+8)^2=48$ $y+8=\pm4\sqrt{3}$ $y=-8\pm4\sqrt{3}$ $\therefore -14.9<y<-1.07$ OR $-8-4\sqrt{3}<y<-8+4\sqrt{3}$	Generally well done. Common mistakes (overall) • Unsuitable methods such as factorise numerator or carry out long division, so unable to proceed further • Writing $-14.9<y<-1.07$ as $-1.07<y<-14.9$ Common mistakes (discriminant) • Discriminant was set to be non-positive (≤ 0) or positive (>0) • Wrong expansion: $(-2-y)^2=y^2-4y+4$ or $(-2-y)^2=y^2+2y+4$ • Finding roots using inequality rather than equation, resulting in incorrect expressions $y<-8\pm4\sqrt{3}$ or $y+8<\pm\sqrt{48}$ • Simplifying $\dfrac{-16\pm\sqrt{192}}{2}$ as $-8\pm\sqrt{192}$
Use differentiation to find the range of values for which y cannot take when function is of the form $\dfrac{\text{quadratic}}{\text{linear}}$		Alternative Method	

Method 1	Method 2
$y=\frac{3x^2-2x-1}{x+1}$ $\frac{dy}{dx}=\frac{(x+1)(6x-2)-(3x^2-2x-1)}{(x+1)^2}$ $=\frac{3x^2+6x-1}{(x+1)^2}$	From long division, $y=3x-5+\frac{4}{x+1}$ $\frac{dy}{dx}=3-\frac{4}{(x+1)^2}$

At stationary points, $\frac{dy}{dx}=0$

Method 1	Method 2
$\frac{3x^2+6x-1}{(x+1)^2}=0$ $3x^2+6x-1=0$	$3-\frac{4}{(x+1)^2}=0$ $(x+1)^2=\frac{4}{3}$

Using GC, $x=0.15470$ or -2.1546

First derivative test

x	0.15470^-	0.15470	0.15470^+
$\frac{dy}{dx}$	$-$	0	+
shape	\	-	/

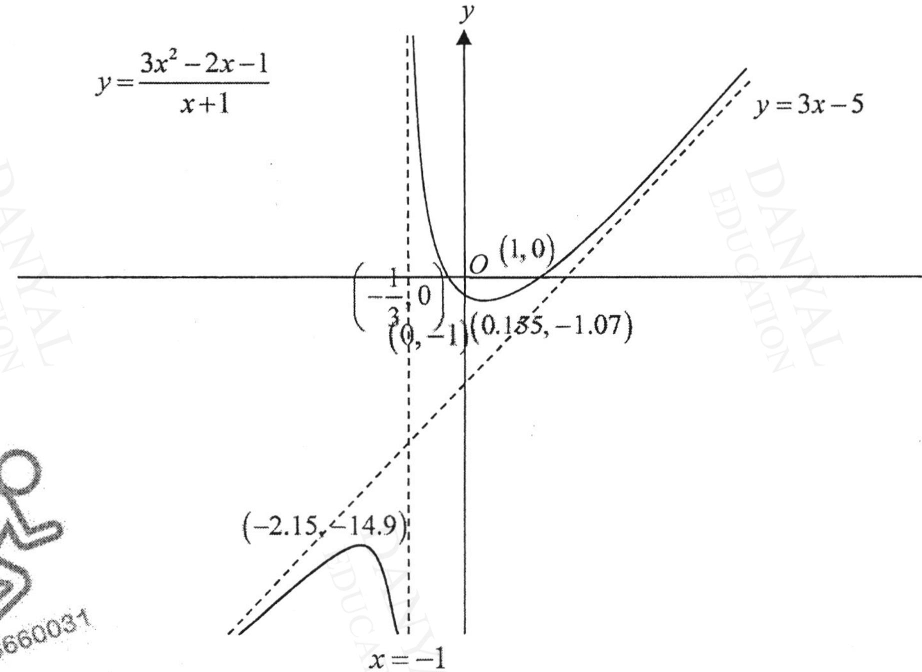
Therefore, $(0.15470,-1.0717)$ is a minimum point

x	$(-2.1546)^-$	-2.1546	$(-2.1546)^+$
$\frac{dy}{dx}$	+	0	$-$
shape	/	-	\

Therefore, $(-2.1546,-14.928)$ is a maximum point

Common mistakes (differentiation)
- Missing first/second derivative test to support conclusion

		Second derivative test $\frac{dy}{dx}=\frac{3x^2+6x-1}{(x+1)^2}$ $\frac{d^2y}{dx^2}$ $=\frac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-1)}{(x+1)^4}$ $=\frac{(x+1)(6x+6)-2(3x^2+6x-1)}{(x+1)^3}$ $=\frac{8}{(x+1)^3}$ Alternatively: $\frac{dy}{dx}=3-\frac{4}{(x+1)^2}$, $\frac{d^2y}{dx^2}=\frac{8}{(x+1)^3}$ At $x=0.15470$, $\frac{d^2y}{dx^2}=5.1961>0$. Therefore, $(0.15470,-1.0717)$ is a minimum point At $x=-2.1546$, $\frac{d^2y}{dx^2}=-5.1961<0$. Therefore, $(-2.1546,-14.928)$ is a maximum point Since the numerator is quadratic and denominator is linear, we deduce from the shape of the graph that the range y cannot take is $-14.9<y<-1.07$.	

Use a G.C. to aid in sketching of a graph	**(ii)**		Generally well done. Common mistakes • A: $y=3x+5$ • I: Missing one or both x-intercepts • I: Expressing $\left(-\frac{1}{3}, 0\right)$ as (−0.33, 0) • S: Missing negative signs in one or more coordinates • S: Expressing $(0.155, -1.07)$ as $(0.15, -1.07)$ or $(0.154, -1.07)$ • I, S: Drawing y-intercept and minimum as one and the same point • Shape: Missing bottom portion when part (i) suggests that the graph has y-values less than or equal to $-8-4\sqrt{3}$ • Shape: Tail ends turning away from asymptotes • Shape: Minimum point in 3rd instead of 4th quadrant

[HOT]	(iii)	When $x=-1$, $y=m(-1+1)-8$ $=-8$ $\therefore$ $(-1, -8)$ lies on the graph.	Generally well done but poorly presented. Common mistakes • Not showing any substitution • Substituting both $x=-1$, $y=-8$ at once $y=m(x+1)-8$ $-8=m(-1+1)-8$ rather than showing from one side to the other side
		$m>3$	Very poorly done as most attempts did not make use of their graph despite the question requirement 'hence'. Common mistakes • Finding *m* using discriminant, ignoring the 'hence' requirement • $m>3$ is part of the answer but not the full answer e.g. $m>3$ or m... e.g. $3<m<$... • $m<3$

Q9. System of Linear Equations + Sigma Notation

Assessment Objectives	Solution		Examiner's Feedback
	(i)	$u_2 = 3u_1 + P(1) \Rightarrow 7 = 3 + P \Rightarrow P = 4$ $u_3 = 3u_2 + 4(2) = 3(7) + 8 = 29$	Some students read Pn as P_n Students are generally able to find P, but those who did not read carefully did not get u_3 correct.
Solve system of linear equations	(ii)	Using $u_1 = 1$, $u_2 = 7$ and $u_3 = 29$ $1 = 3a + b + c$ $7 = 9a + 2b + c$ $29 = 27a + 3b + c$ Solving, $a = \frac{4}{3}, b = -2, c = -1$	Some students did not realise this is a system of linear equations. They tried other methods such as comparing coefficients, table of values etc without much success.
Relate sigma notation with sum of arithmetic and geometric progression [HOT]	(iii)	$\sum_{r=1}^{n} u_r = \sum_{r=1}^{n}\left[\frac{4}{3}(3^r) - 2r - 1\right]$ $= \sum_{r=1}^{n}\left[4(3^{r-1})\right] - \sum_{r=1}^{n}[2r+1]$ $= \frac{4(3^n - 1)}{3-1} - \frac{n}{2}(3 + 2n + 1)$ $= 2(3^n - 1) - n(n+2)$	Many students erroneously tried to do Method of Difference Some students still do know the form of a GP, AP and constant term in Sigma Notation. Some students put n in the sigma notation instead of r, which is conceptually wrong.

Q10. Functions			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Find inverse function	(i)	$y=\frac{ax+b}{cx-2}$ $cxy-2y=ax+b$ $x(cy-a)=b+2y$ $x=\frac{b+2y}{cy-a}=\frac{2y+b}{cy-a}$ Hence, $f^{-1}(x)=\frac{2x+b}{cx-a}$ Comparing, $a=2$.	A self-inverse function is defined in the question i.e. $f(x)=f^{-1}(x)$. Some candidates wrote $f(x)=f^{-1}(x)=x$ which is incorrect. Majority of the candidates managed to find $f^{-1}(x)$ while some still fumble at this basic skill. Candidates are supposed to compare coefficients by observing the rules of $f(x)$ and $f^{-1}(x)$.
Apply properties of self-inverse functions	(ii)	$f(x)=f^{-1}(x)$ $ff(x)=ff^{-1}(x)$ $f^2(x)=ff^{-1}(x)$ $=x$ Alternative method:	Many candidates failed to see that since $f(x)=f^{-1}(x)$, then the first function f can be equated to $f^{-1}(x)$, thus $ff(x)=ff^{-1}(x)$ which is equals to x in theory. Instead, candidates went to use the alternative method, which is longer. Some realized the above fact, some did not as they made algebraic mistakes which is alarming. This include:

		$ff(x)=f\left(\frac{2x+3}{5x-2}\right)$ $=\frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2}$ $=\frac{\frac{4x+6+3(5x-2)}{5x-2}}{\frac{10x+15-2(5x-2)}{5x-2}}$ $=\frac{19x}{19}$ $=x$	• $2(2x+3)=4x+3$ instead of $4x+6$ • Transferring $f(x)$ wrongly i.e. $\frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)+3}$ instead of $\frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2}$. There were also candidates who did this: $f^2(x)=(f(x))^2$ which is conceptually wrong!
[HOT]	(ii)	$f^{71}(x)=f^{(35\times2+1)}(x)=f(x)$ Hence, $f^{71}(4)=f(4)=\frac{2(4)+3}{5(4)-2}=\frac{11}{18}$ OR 0.611 (3 s.f.)	This part was poorly done. Many candidates did not understand that f and f^{-1} cancels the effect out and it does not equals to x multiplied to a composite function i.e. $fff(x)=ff^{-1}f(x)\neq xf(x)$. They also make conceptual mistakes i.e. $ff(x)=f(x)\cdot f(x)$ which is not true. Some candidates did not what they are doing, or left this blank.

Criteria for composite function to exist	**(iv)**	For fg to exist, $R_g \subseteq D_f$. $R_g = [-3, \infty)$ $D_f = \left(-\infty, \frac{2}{5}\right) \cup \left(\frac{2}{5}, \infty\right)$ Since $R_g \not\subseteq D_f$, fg does not exist.	Generally well done for candidates, showing that they studied. They are expected to write R_g and D_f accurately and properly in the interval notation, and state the correct conclusion i.e. $R_g \not\subseteq D_f$ to be awarded full credit. Candidates who state $R_g \subseteq D_f$ are not given full credit. However, some candidates missed R_g and wrote $R_g = (-3, \infty)$. The *y*-coordinate −3 of the minimum point is included in the range. Majority of the candidates did not present D_f properly although stated in the question and they are reminded to present clear working. They should use $\cup$ which represents "OR" (union) instead of $\cap$ ("AND" – intersection)

Find rule for a composite function	(v)	$\text{gf}(x)=5$ $\text{g}\left(\frac{2x+3}{5x-2}\right)=5$ $2\left(\frac{2x+3}{5x-2}\right)^2-3=5$ $\left(\frac{2x+3}{5x-2}\right)^2=4$ **Method ①:** $2x+3=\pm 2(5x-2)$ $2x+3=2(5x-2)$ or $2x+3=-2(5x-2)$ $8x=7$ $\quad$ $12x=1$ $x=\frac{7}{8}$ $\quad$ $x=\frac{1}{12}$ **Method ②:** $\left(\frac{2x+3}{5x-2}\right)^2=4$ $4x^2+12x+9=4(25x^2-20x+4)$ $96x^2-92x+7=0$ $x=\frac{92\pm\sqrt{8464-4(96)(7)}}{2(96)}$ $=\frac{92\pm 76}{192}$ $=\frac{7}{8}$ or $\frac{1}{12}$	Candidates are expected to score in this question. However, many failed to obtain full credit due to algebraic errors in solving quadratic equation. Candidates should always avoid expanding the square in this equation when the term on the right, i.e. 4, is a perfect square and can be square rooted. Some missed out on ± and thus, miss another solution.

Q11. Vectors (Lines & Planes)			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Find cartesian equation of a plane with 3 points	(i)	$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ $=\begin{pmatrix}2\\7\\0\end{pmatrix}-\begin{pmatrix}6\\3\\0\end{pmatrix}$ $=\begin{pmatrix}-4\\4\\0\end{pmatrix}$ $=4\begin{pmatrix}-1\\1\\0\end{pmatrix}$ $\overrightarrow{VC}=\overrightarrow{OC}-\overrightarrow{OV}$ $=\begin{pmatrix}2\\7\\0\end{pmatrix}-\begin{pmatrix}3\\4\\5\end{pmatrix}$ $=\begin{pmatrix}-1\\3\\-5\end{pmatrix}$ $=-\begin{pmatrix}1\\-3\\5\end{pmatrix}$	Most candidates were able to calculate 2 direction vectors that are parallel to the plane. Candidates can choose any 2 vectors $\overrightarrow{BC}$, $\overrightarrow{VC}$, or $\overrightarrow{VB}$

		A vector perpendicular to the plane *VBC*: $\begin{pmatrix}-1\\1\\0\end{pmatrix}\times\begin{pmatrix}1\\-3\\5\end{pmatrix}=\begin{pmatrix}5\\5\\2\end{pmatrix}$ Equation of the plane *VBC* in scalar-product form: $\underset{\sim}{r}\cdot\begin{pmatrix}5\\5\\2\end{pmatrix}=\begin{pmatrix}3\\4\\5\end{pmatrix}\cdot\begin{pmatrix}5\\5\\2\end{pmatrix}$ $\underset{\sim}{r}\cdot\begin{pmatrix}5\\5\\2\end{pmatrix}=45$	Most candidates attempted to calculate the normal vector by using cross product. However, a significant number of candidates made errors in the evaluation of the normal vector to the plane. This normal vector $\begin{pmatrix}5\\5\\2\end{pmatrix}$ was crucial in the calculation for subsequent parts of the question. Accuracy of answers for subsequent parts were affected for these candidates.
Find foot of perpendicular from a point to a plane	**(ii)**	Equation of the line *AP*: $\underset{\sim}{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}5\\5\\2\end{pmatrix}, \lambda\in\mathbb{R}$ Since *P* lies on the line *AP*, $\overrightarrow{OP}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}5\\5\\2\end{pmatrix}$, for a λ value Since *P* also lies on the plane *VBC*,	Finding the foot of perpendicular from a point to a plane is a standard procedure, and candidates who understood the question and followed the procedure were able to obtain the foot correctly. There was a sizable group of candidates who erroneously assumed that the foot of perpendicular lie on $\overrightarrow{BC}$. Some candidates tried to use projection method or perpendicular distance method to find the vector $\overrightarrow{AP}$,

		$\left[\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}5\\5\\2\end{pmatrix}\right]\cdot\begin{pmatrix}5\\5\\2\end{pmatrix}=45$ $15+54\lambda=45$ $\lambda=\frac{5}{9}$ $\overrightarrow{OP}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\frac{5}{9}\begin{pmatrix}5\\5\\2\end{pmatrix}$ $=\begin{pmatrix}\frac{34}{9}\\\frac{43}{9}\\\frac{10}{9}\end{pmatrix}$ Coordinates of the point P are $\left(\frac{34}{9},\frac{43}{9},\frac{10}{9}\right)$.	followed by $\overrightarrow{OP}$. They were usually unsuccessful.

Find shortest distance between a point and a plane	**(iii)**	Shortest distance between A and P $\left\lvert\overrightarrow{AP}\right\rvert = \left\lvert\begin{pmatrix}\frac{34}{9}\\ \frac{43}{9}\\ \frac{10}{9}\end{pmatrix} - \begin{pmatrix}1\\2\\0\end{pmatrix}\right\rvert$ $= \left\lvert\begin{pmatrix}\frac{25}{9}\\ \frac{25}{9}\\ \frac{10}{9}\end{pmatrix}\right\rvert$ $= \sqrt{\left(\frac{25}{9}\right)^2 + \left(\frac{25}{9}\right)^2 + \left(\frac{10}{9}\right)^2}$ $= \sqrt{\frac{1350}{81}}$ $= \sqrt{\frac{50}{3}}$ $= \frac{5\sqrt{6}}{3}$ units or 4.08 units (3 s.f.)	Most errors for this part was caused by the error in calculation for the normal vector in (i). A small number of candidates assumed that the shortest distance was $\overrightarrow{OP}$. This is a conceptual error.

“Otherwise” Method 1

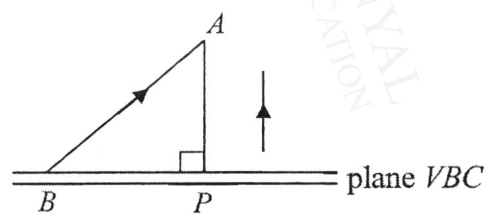
$$\overrightarrow{BA}=\begin{pmatrix}1\\2\\0\end{pmatrix}-\begin{pmatrix}6\\3\\0\end{pmatrix}=\begin{pmatrix}-5\\-1\\0\end{pmatrix}$$

Shortest distance between A and P

$$\left|\overrightarrow{AP}\right|=\left|\frac{\begin{pmatrix}-5\\-1\\0\end{pmatrix}\cdot\begin{pmatrix}5\\5\\2\end{pmatrix}}{\left|\begin{pmatrix}5\\5\\2\end{pmatrix}\right|}\right|$$

$$=\left|\frac{-30}{\sqrt{54}}\right|$$

$$=\frac{5\sqrt{6}}{3}\text{ units} \quad \text{or} \quad 4.08 \text{ units (3 s.f.)}$$

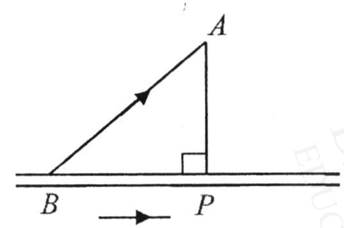


“Otherwise” Method 2

$$\overrightarrow{BA}=\begin{pmatrix}1\\2\\0\end{pmatrix}-\begin{pmatrix}6\\3\\0\end{pmatrix}=\begin{pmatrix}-5\\-1\\0\end{pmatrix}$$

$$\overrightarrow{BP}=\begin{pmatrix}\frac{34}{9}\\\frac{43}{9}\\\frac{10}{9}\end{pmatrix}-\begin{pmatrix}6\\3\\0\end{pmatrix}=\begin{pmatrix}-\frac{20}{9}\\\frac{16}{9}\\\frac{10}{9}\end{pmatrix}$$

Shortest distance between A and P



$$\left|\overrightarrow{AP}\right| = \frac{\left|\begin{pmatrix}-5\\-1\\0\end{pmatrix} \times \begin{pmatrix}-\frac{20}{9}\\\frac{16}{9}\\\frac{10}{9}\end{pmatrix}\right|}{\left|\begin{pmatrix}-\frac{20}{9}\\\frac{16}{9}\\\frac{10}{9}\end{pmatrix}\right|}$$

$$= \frac{1}{\sqrt{\left(-\frac{20}{9}\right)^2 + \left(\frac{16}{9}\right)^2 + \left(\frac{10}{9}\right)^2}} \left|\begin{pmatrix}\frac{10}{9}\\-\frac{50}{9}\\\frac{100}{9}\end{pmatrix}\right|$$

$$= \frac{\sqrt{\left(\frac{10}{9}\right)^2 + \left(-\frac{50}{9}\right)^2 + \left(\frac{100}{9}\right)^2}}{\sqrt{\frac{28}{3}}}$$

$$= \frac{\sqrt{\frac{1400}{9}}}{\sqrt{\frac{28}{3}}} = \sqrt{\frac{50}{3}} = \frac{5\sqrt{6}}{3} \text{ units} \quad \text{or } 4.08 \text{ units (3 s.f.)}$$

<table>
<tr>
<td>Find the acute angle between a line and a plane</td>
<td>(iv)</td>
<td>$\overrightarrow{VA} = \overrightarrow{OA} - \overrightarrow{OV}$
$= \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$
$= \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix}$
$= -\begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$
Acute angle between the line VA and the normal vector of the plane VBC
$= \cos^{-1}\left(\dfrac{\begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}\right| \left|\begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix}\right|} \right)$
$= \cos^{-1}\left(\dfrac{30}{\sqrt{33}\sqrt{54}} \right)$
$= 44.711^\circ$ (3 d.p.) or 0.78035 (5 s.f.)

Acute angle between the line VA and the plane VBC
$= 90^\circ - 44.711^\circ$ or $\dfrac{\pi}{2} - 0.78035$
$= 45.3^\circ$ (1 d.p.) or 0.790 (3 s.f.)</td>
<td>The majority of candidates used the formula
$\sin\alpha = \left| \dfrac{\underset{\sim}{m} \bullet \underset{\sim}{n}}{|\underset{\sim}{m}||\underset{\sim}{n}|} \right|$ without defining the angle or any supporting diagrams.

As a result, it was not sure if candidates were aware of what the angle α represented; or did they remember the Scalar Product formula wrongly; or was it a use of formula without understanding.
Regardless, Benefit of Doubt was given and they were awarded the credit.

Most candidates who made errors used the wrong vectors in the calculation, showing a lack of understanding of the question of a lack of visualization skills.</td>
</tr>
</table>

Q12. Geometric Progression

<table>
<tr><th>Assessment Objectives</th><th colspan="2">Solution</th><th>Examiner's Feedback</th></tr>
<tr><td>Use of A.P. sum</td><td>(a)</td><td>$a=120,\ d=10$
$$S_n > 10000$$
$\frac{n}{2}\left[2(120)+(n-1)(10)\right] > 10000$
Using G.C.,
<table>
<tr><th>n</th><th>$\frac{n}{2}\left[2(120)+(n-1)(10)\right]$</th></tr>
<tr><td>34</td><td>9690</td></tr>
<tr><td>35</td><td>10150</td></tr>
<tr><td>36</td><td>10620</td></tr>
</table>
$\therefore n=35$
Mr Lam will have saved over $10,000 on 1 Nov 2021.</td><td>A few candidates painstakingly and literally listed out the monthly amounts to obtain the solution. This is highly discouraged as the number of terms may not be as small the next time.

Some candidates set up using the incorrect formula (U_n instead of S_n) and simplified the calculations significantly.

The setup should ideally be an inequality so that students can take the following integer value of n from the resulting solution inequality. However, a significant number of candidates set up an equation instead, and seemingly arrive at the value of n by means of simple rounding off. For these case, students will be penalized if there is no explicit indication of finding "Least n" or "rounding upwards to the next integer".</td></tr>
</table>

<table>
<tr>
<td>Systematically set up expressions to represent a situation

Observe patterns/sequences within expression and applying relevant formulae</td>
<td>(b)(i)</td>
<td>
<table>
<tr><td>n</td><td>Beginning</td><td>End</td></tr>
<tr><td>1</td><td>50000</td><td>50000(1.08)</td></tr>
<tr><td>2</td><td>$50000(1.08) + x$</td><td>$50000(1.08)^2 + 1.08x$</td></tr>
<tr><td>3</td><td>$50000(1.08)^2 + 1.08x + x$</td><td>$50000(1.08)^3 + (1.08)^2 x + 1.08x$</td></tr>
</table>

Amount at the end of nth month
$= 50000(1.08)^n + 1.08^{n-1}x + 1.08^{n-2}x + \ldots + 1.08^2 x + 1.08x$
$= 50000(1.08)^n + x\left[1.08 + 1.08^2 + \ldots + 1.08^{n-2} + 1.08^{n-1}\right]$
$= 50000(1.08)^n + \dfrac{1.08x\left(1.08^{n-1} - 1\right)}{1.08 - 1}$
$= 50000(1.08)^n + 13.5x\left(1.08^{n-1} - 1\right)$ (shown)
</td>
<td>Students who could not do this particular portion are those who should work on systematically tracking the sequence of events, writing out expressions that represent the situation at different stages and trying to spot a pattern.

As this is a "Show" type of question, detailed steps would have to be presented leading to the final result.</td>
</tr>
<tr>
<td>See relevance in formula obtained and using it accordingly</td>
<td>(ii)</td>
<td>If $x = 5000$,
Interest earned at the end of the 10th year
$= 50000(1.08)^{10} + 13.5(5000)\left(1.08^{10-1} - 1\right) - \left[50000 + 9(5000)\right]$
$= 175379 - 95000$
$= 80379$
The interest earned at the end of the 10th year is \$80379.</td>
<td>Interest earned can be found by using the general knowledge:
(total amount) – (all deposits)

A common mistake made in the "deposits" part was that only \$50000 was considered, or that 10 deposits of \$5000 (instead of 9) was considered.</td>
</tr>
</table>

Link formula obtained to given parameters and adjust the formula to reflect changes in parameters. Use G.C. to solve complex equations **[HOT]**	**(iii)**	Let $R=\left(1+\frac{r}{100}\right)$ When interest is 8%, amount after 10 years $=50000(1.08)^{10}+\frac{1.08(5000)\left(1.08^{10-1}-1\right)}{1.08-1}$ When interest is r%, amount after 10 years $=50000(R)^{10}+\frac{R(5000)\left(R^{10-1}-1\right)}{R-1}$ Hence, $50000(R)^{10}+\frac{R(5000)\left(R^{10-1}-1\right)}{R-1}=300000$ By graphical method, using GC, $R=1.151$ Therefore $r=15.1$	This section is not well done. The key setup is this: $50000(R)^{10}+\frac{R(5000)\left(R^{10-1}-1\right)}{R-1}=300000$ Almost all candidates mistook the R to be r, which, if they are able to work out, would lead them to conclude the interest rate to be 1.15% A lot of candidates also used this $50000(1.08)^{10}+13.5(5000)\left(1.08^{10-1}-1\right)$ without realizing that that 13.5 comes from the 8% interest, which should not be used as it is not the case now. A number of candidates used the table of values in the GC to try to obtain the value of the unknown. However, they should realise that the answer is not an integer, so the table does not work unless you take the trouble to adjust and refine the increments.