RAFFLES INSTITUTION
PHYSICS DEPARTMENT

14 CURRENT OF ELECTRICITY

Content
- **Electric Current**
- **Potential Difference**
- **Resistance and resistivity**
- **Sources of electromotive force**

Learning Outcomes

Candidates should be able to:

(a) show an understanding that electric current is the rate of flow of charge.

(b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity.

(c) recall and solve problems using the equation $Q = It$.

(d) recall and solve problems using the equation $V = W/Q$.

(e) recall and solve problems using the equation $P = IV$, $P = I^2R$ and $P = V^2/R$.

(f) recall and solve problems using the equation $V = IR$.

(g) sketch and explain the *I-V* characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor.

(h) sketch the resistance-temperature characteristic of an NTC thermistor.

(i) recall and solve problems using $R = \rho l / A$.

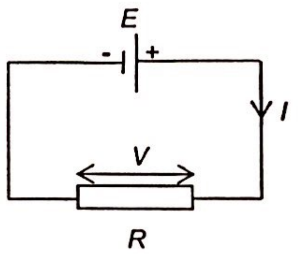
(j) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations.

(k) show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.

there is no electric field within an isolated conductor

14.1 Introduction

Electrical Circuit



Circuit Element	Symbol	Units
Source of electromotive force (e.m.f.), *E*		Volt V
Conductor of resistance *R*		Ohm Ω
Direction of conventional current, I		Ampere A

The figure above shows a closed electrical circuit. There is a continuous conducting path, provided by the connecting wires, for electrical charges to flow from one terminal of the source of e.m.f. to one or more devices, like a resistor, and then back to the other terminal. To analyse such a closed circuit, we shall now look at the different parts of the electrical circuit.

14.2 Electric Current I & Charge Q

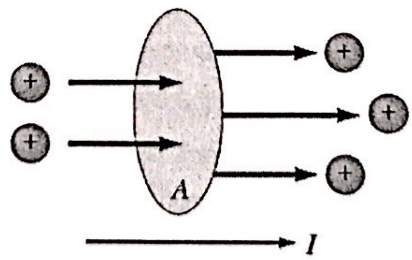
Electric Current I and Its Unit

Whenever charged particles move, an electric current is said to exist.

Definition

Electric current is defined as the rate of flow of charges.

Suppose charges are moving perpendicular to a surface of area *A* as shown in the figure below. (This area *A* could be the cross sectional area of a wire, for example.)



If ΔQ is the amount of charge passing through this surface in a time interval Δt, the **average electric current I_{ave}** is equal to the net charges that pass through the area per unit time.

$$I_{ave} = \frac{\Delta Q}{\Delta t} \qquad (1)$$

If the rate at which charges flow varies with time, the **instantaneous current I** is defined as the differential limit of average current.

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad (2)$$

(~~net charge~~ diff charges don't cancel out, they travel in diff direction)

current due to flow of protons + current due to flow of electrons

- The S.I. unit for current is the **ampere,** A (it is one of the seven base units).

Charge Q and Its Unit

Electric charge is a fundamental property of matter which causes a charged object to experience a force when placed in an electric field. When a constant current I flows through a cross-section of a conductor for a duration t, the amount of **electric charge** Q passing through it is given by

$$Q = It \qquad (3)$$

charge: $(3.0\times10^{18})(-1.6\times10^{-19})$
$= -0.48$

The S.I. unit for electric charge is the **coulomb,** C.

Definition

One **coulomb** is defined as the amount of electric charges that passes through a point in one second when there is a constant current of one ampere ($1\ \text{C} = 1\ \text{A s}$).

If the current flow is not constant, from equation (2),

$$Q = \int I\,dt \qquad (4)$$

Example 1 (a) If a current of 0.320 mA flows through a calculator, how many electrons pass through it in a second? (charge of an electron, e = 1.60 x 10^{-19} C)
(b) How long does it take for 1.00 C of charge to pass through the calculator?

(a)

$$Q = It$$
$$= (0.320\times10^{-3})(1.00)$$
$$= 0.320\times10^{-3}\ \text{C}$$

formula
substitution
final answer
units

$$\text{No. of electrons} = \frac{Q}{e} = \frac{0.320\times10^{-3}}{1.60\times10^{-19}} = 2.00\times10^{15}\ \text{s}^{-1}$$

(b) $Q = It$

$$t = \frac{Q}{I} = \frac{1.00}{0.320\times10^{-3}} = 3130\ \text{s}\ \ (3\ \text{sf})$$

Types of Charge Carriers

By convention, the direction of current is defined as the direction of the flow of positive charges, regardless of the actual charged particles in motion. The charged particles may differ for different conductors.

In metals, the current results from the motion of negatively charged electrons. Therefore, the direction of the current is opposite to the direction of flow of electrons.

In some cases – such as gases and electrolytes – the current is the result of both positive and negatively charged particles. It is common to refer to a moving charged particle (regardless of sign) as a mobile charge carrier.

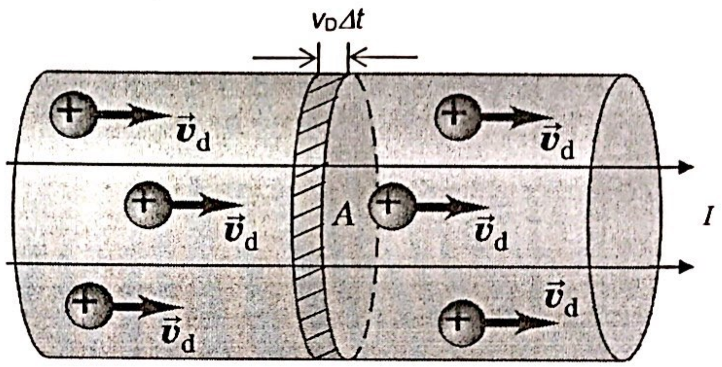
The table below shows examples of the charge carriers in different materials.

Material	*Charge carriers*	*Examples*	*Remarks*
Metals	electrons	copper, silver, aluminium	Each atom of the metal supplies one or more electrons from its outer shell to form the bond necessary to hold the positive ions together. These electrons are highly mobile and hence contribute to conduction.
Semi-conductors	electrons and holes	germanium, silicon	Although all four electrons in the outer shell are shared among neighbouring atoms to form covalent bonds, at room temperature, these electrons may have sufficient energy to leave the atoms to become "free" electrons. These electrons leave behind holes in the atoms where neighbouring electrons can move to fill them and leave holes from where they come from. In this way, the holes seem to behave like positively charged particles, moving in the opposite direction to the electron-flow.
Electrolytes	ions	sodium chloride solution	Some salts dissociate into ions when dissolve in water. These ions are either positively or negatively charged, and they are mobile. Hence, they contribute to electrical conduction.
Insulators	none	rubber, porcelain, glass	All electrons in the outer shell of the atoms of an insulator are involved in forming covalent bonds. Unlike semiconductors, these electrons cannot acquire sufficient energy to be "freed", and they are not available for electrical conduction due to large band-gaps.

Drift Velocity v_D and Electric Current I

When there is an electric current in a conductor, the free charge-carriers in the conductor move with an average velocity on the order of 10^{-4} m s^{-1}. This is called **drift velocity** (v_D).

Consider a conductor with cross-sectional area A (perpendicular to the direction of current flow) and n charge-carriers per unit volume (n is called the number density). A current I is flowing to the right.



In a time interval Δt, each charge-carrier (assumed to be positive) moves a distance $v_D\Delta t$ to the right. Thus, in Δt, charges within a distance of $v_D\Delta t$ to the left of the cross-section A (the shaded cylinder) will pass through A. The volume of the shaded cylinder is $Av_D\Delta t$. The number of charge-carriers passing through A in Δt is thus $nAv_D\Delta t$. Given that the charge carried by each charge-carrier is q, the total charge passing through A in Δt is $nAv_Dq\Delta t$. The current is then given by

$$I = \frac{\text{Total charge through } A \text{ in } \Delta t}{\text{Time interval } \Delta t} = \frac{nAv_Dq\Delta t}{\Delta t} = nAv_Dq.$$

14.3 Potential Difference V

creates electric field ~~from~~ within the wire

Potential Difference V and Its Unit

If a conductor is connected to a battery, all points on the conductor are not at the same potential. The battery sets up a ***potential difference***, and hence an electric field across the conductor. The electric field exerts a force on the free charge-carriers in the conductor, causing them to move and therefore creating a current. The direction of the current would be the same as the motion of positive charge-carriers, which move from a higher potential to a lower potential since the electric field is directed towards lower potential.

Definition

The **potential difference** between two points in a circuit is defined as the amount of electrical energy per unit charge that is converted *to* other forms of energy when charges pass from one point to the other.

+ve: high to low potential
-ve: low to high potential

Hence, the potential difference between two points **in a circuit** is given by

PD can only be used if 2 points are specified (e.g. across a resistor)

$$V = \frac{W}{Q} \qquad (5)$$

where W = electrical energy converted to other forms of energy in J
Q = net amount of charge that passes from one point to the other in C

The S.I. unit for potential difference is **the volt,** V.

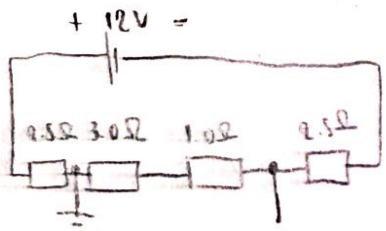
Definition

> One **volt** is the potential difference between two points in a circuit when one joule of electrical energy is converted to other forms of energy as one coulomb of charge passes from one point to the other (1 V = 1 J C^{-1}).

If a charge Q flows in a part of a circuit across which there is a potential difference V, the electrical energy converted to other forms in that part of the circuit is given by

$$W = QV \qquad (6)$$

Example 2 A lamp has a potential difference of 10 V across it. Calculate how much electrical energy is converted to light and thermal energy when
(a) a charge of 350 C passes through it,
(b) a current of 3.0 A passes through it for 30 s.



(a)
$W = QV$
$= (350)(10)$
$= 3500$ J

(b)
$W = QV$
$= (It)V$
$= (3.0 \times 30)(10)$
$= 900$ J

potential?
lower potential than earthed point (0V) must be negative.
voltage drop ∝ resistance

Example 3 An immersion heater, rated at 1000 W, is switched on for 900 s. During this time, a charge of 3700 C is supplied to the heater. What is the potential difference across the heater?

V = RI
total resistance = 9 Ω
voltage drop per ohm = 1.33 V
I : -5.3V

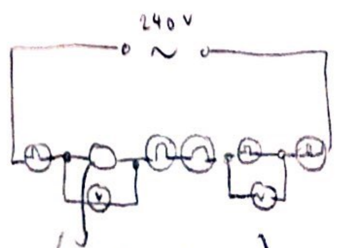
or $\frac{4}{9} \times 12$

$P = \frac{W}{t} = \frac{QV}{t}$

$V = \frac{Pt}{Q}$

$= \frac{(1000)(900)}{3700}$

$= 243$ V



filament broken

no current flowing through the circuit
X and Y are of infinite resistance
X 240V
p.d across Y: 0V
p.d across X = p.d. Y 0V

14.4 Electromotive Force (e.m.f.) *E*

E.m.f. *E* and Its Unit

When an electric current is flowing through conductors in a closed circuit, electrical energy is continuously dissipated as heat or other forms of energy. In order to sustain this constant flow of electric current, a source of electromotive force (e.m.f.) is needed. Sources of e.m.f. are any devices which can be likened to a "charge pump" in which electrical energy is converted from chemical, mechanical or other forms of energy (such as batteries, generators, solar cells and fuel cells).

Definition

The **electromotive force of a source** is defined as the amount of electrical energy per unit charge that is converted *from* other forms of energy to drive charges around a complete circuit.

In equation form,

$$E = \frac{W}{Q} \qquad (7)$$

The S.I. unit for e.m.f. is also the **volt, V.**

It is important to note that e.m.f. is **not** a force as the name may suggest.

As an example, for a battery with an e.m.f. of 1.5 V, 1.5 J of chemical energy is converted into electrical energy with every coulomb of charge the battery drives through the circuit.

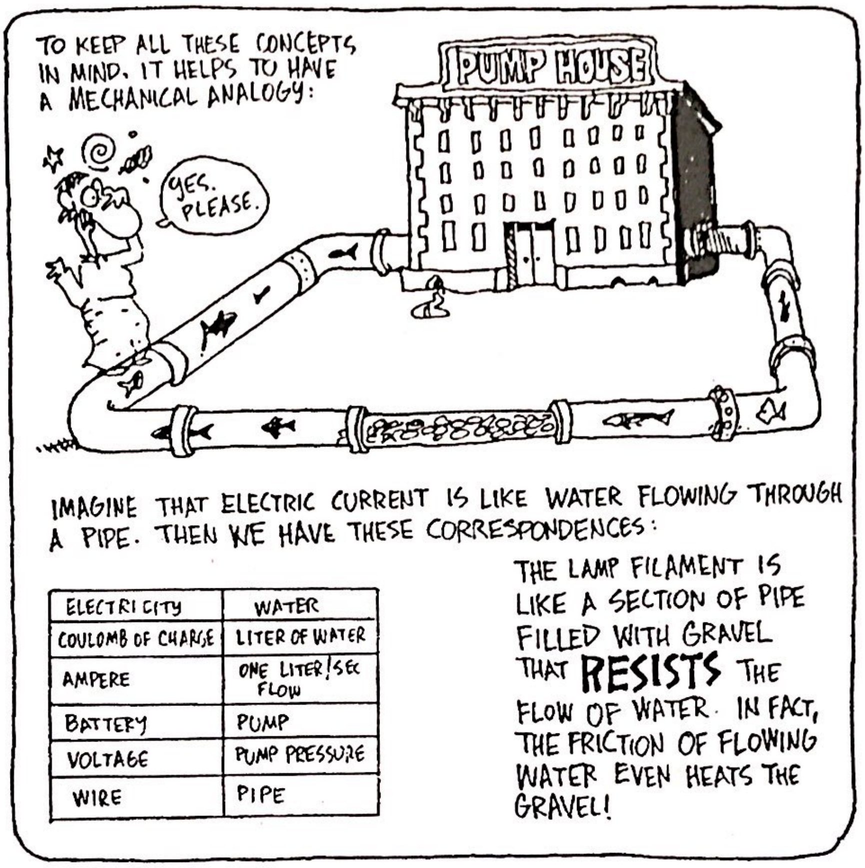
Comparing p.d. & emf

p.d. *V* across an external load	**e.m.f. *E* of a source**
$V = \frac{W}{Q}$	$E = \frac{W}{Q}$
W is the energy converted **from electrical energy** to other forms	*W* is the energy converted **from other forms** to electrical energy

14.5 Resistance R & Resistivity ρ

Resistance R and Its Unit

The electric current in a conductor depends on the potential difference applied across the conductor and its resistance.



Definition

The **resistance** of a conductor is defined as the *ratio* of the potential difference across it to the current flowing through it.

it is not constant for a given material at a fixed ~~resistance~~ temperature, but depends on shape & size.

In equation form, electrical resistance is given by

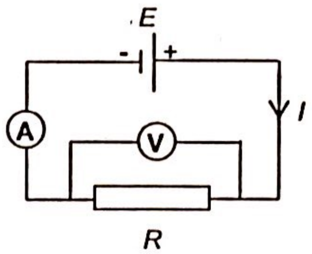
$$R = \frac{V}{I} \qquad (8)$$

The S.I. unit for resistance is the **ohm, Ω.**

Definition

One **ohm** is the resistance of a conductor when a potential difference of one volt across it causes a current of one ampere to flow through it ($1\ \Omega = 1\ \mathrm{V\ A^{-1}}$).

Determining R A simple way to determine the resistance of a conductor is to connect an ideal ammeter (assumed 0 Ω) in series with the conductor and an ideal voltmeter (assumed ∞ Ω) in parallel with the conductor as shown.



The ammeter measures the current flowing through the conductor and the voltmeter measures the p.d. across it. The resistance can thus be determined using equation (8).

Example 4 A car stereo system draws a current of 400 mA when connected to a 12.0 V battery.
(a) What is the resistance of the stereo system?
(b) The stereo is left playing from the battery for several hours while the engine is turned off. The radio can continue to operate until the current drops to 320 mA. What is the p.d. across the stereo when it stops playing?

(a)
$$R = \frac{V}{I} = \frac{12.0}{400 \times 10^{-3}} = 30.0\ \Omega$$

(b) Assume R to be constant.

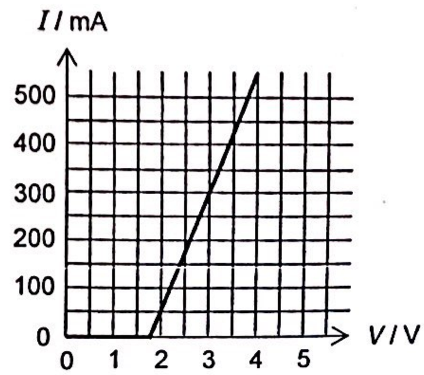
$$V_{min} = I_{min}R = (320 \times 10^{-3})(30.0) = 9.60\ \text{V}$$

Example 5 The graph below shows the relationship between the direct current I in a certain conductor and the potential difference V across it. When V < 1.8 V, the current is negligible. What is the resistance of the conductor when the p.d. is 3.0 V?

From Graph, when V = 3.0 V, I = 300 mA

$$R = \frac{V}{I} = \frac{3.0}{300 \times 10^{-3}} = 10\ \Omega$$

Note:
Realise that R is the ratio of V to I, and not the reciprocal of the gradient of the graph!



Ohm's Law

Definition

Ohm's law states that the potential difference across a conductor is proportional to the electric current passing through it, provided that its temperature remains constant.

The constant of proportionality is the resistance of the conductor.

Materials which obey this relationship are said to obey Ohm's law and they are said to be *ohmic*. Materials and devices that do not obey Ohm's law are *non-ohmic*.

Ohm's law is not a fundamental law of nature; rather it is an empirical relationship valid only for certain materials and devices, and only over a limited range of conditions.

Resistivity and Conductivity

Resistance of a uniform[1] conductor (at a given temperature) is

1) directionally proportional to its length l, and
2) inversely proportional to its cross-sectional area A,

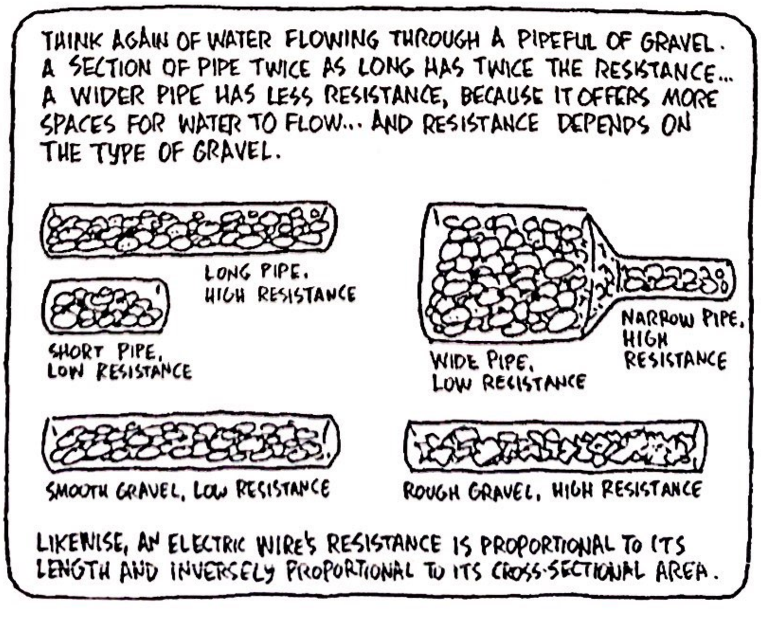
$$R \propto \frac{l}{A} \qquad (9)$$

$$\text{OR } R = \rho \frac{l}{A} \qquad (10)$$

where the constant of proportionality ρ is called the resistivity of the material.

The S.I. unit for resistivity is the **ohm metre, Ω m**.

A useful analogy:



[1] 'Uniform conductor' here refers to a conductor that is made of a single type of material.

The resistivity of a material is numerically equal to the resistance between opposite faces of a cube of the material of unit length and unit cross-sectional area. It is constant for a given area, at a fixed temperature, regardless of its shape and size.

Resistivity ρ and Its Unit

Every material has a characteristic resistivity. Good electrical conductors have low resistivities and insulators have high resistivities.

The resistance of pure semiconductors is between that of a conductor and that of an insulator. By introducing controlled amounts of impurities, their resistivities can be significantly reduced. This property makes them an ideal material for fabrication of computer chips and other electronic devices.

The following table shows a list of resistivities of some common materials at 20°C.

Type	Material	Resistivity, ρ / Ω m
Conductors	Aluminium	2.8×10^{-8}
	Copper	1.7×10^{-8}
	Gold	2.4×10^{-8}
	Iron	10×10^{-8}
	Silver	1.6×10^{-8}
	Tungsten	5.6×10^{-8}
Pure Semiconductors	Germanium	0.60
	Silicon	2300
Insulators	Glass	$10^{10} - 10^{14}$
	Rubber	$10^{13} - 10^{16}$
	Sulphur	10^{15}

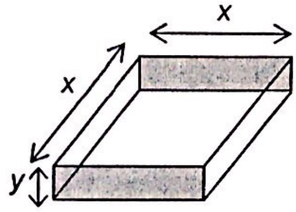
* Note: The resistivity of extrinsic semiconductors greatly depends on the amount and type of dopants used.

Example 6 Find the resistance of a tungsten filament of length 4.0 cm and diameter 0.020 mm at 20°C.

$$R = \rho \frac{l}{A}$$

$$= \left(5.6 \times 10^{-8}\right) \frac{4.0 \times 10^{-2}}{\pi \left(\frac{0.020 \times 10^{-3}}{2}\right)^2} = 7.13\ \Omega$$

Example 7 A sample of resistive material is prepared in the form of a thin square slab of side x. For a given thickness y, the resistance between opposite edge faces of the sample (shown shaded in the figure below) is:



A proportional to x^2
B proportional to x
C independent of x
D inversely proportional to x
E inversely proportional to x^2

$R = \rho \frac{l}{A} = \rho \frac{x}{xy} = \frac{\rho}{y} \Rightarrow R$ is independent of x **(C)**

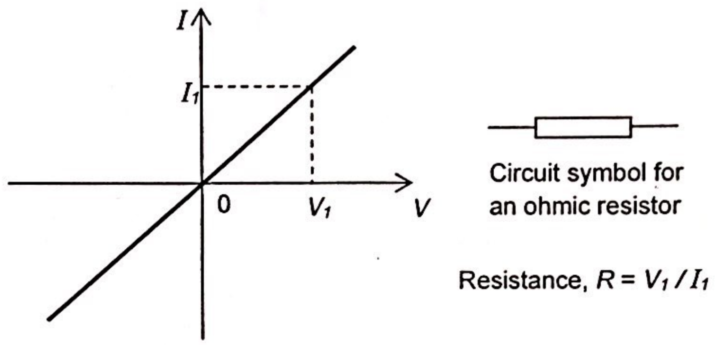
I-V Characteristic of Circuit Components

By looking at how the current through a material varies with the potential difference across it, it is possible to differentiate between ohmic and non-ohmic materials/devices.

Metallic Conductor at Constant Temperature (Ohmic)

It has been found experimentally that if the temperature of a conductor is kept constant, its resistance will remain constant.

The I-V characteristic is a **straight-line graph through the origin.**



I-V Characteristic of Metallic Conductor at Constant Temperature

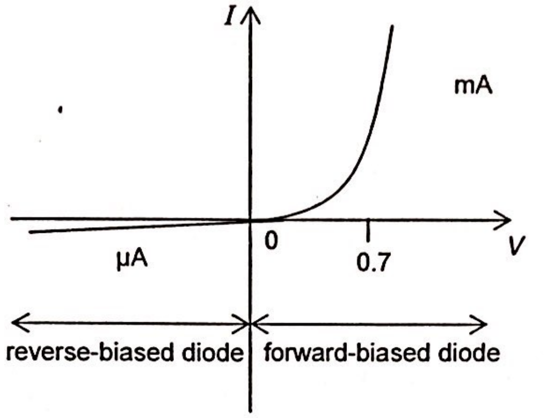
This means that
- I is proportional to V ($I \propto V$)
- ratio $\dfrac{V}{I}$ is constant
- thus, resistance R is constant

The temperature can be kept constant by immersing the conductor in a water bath.

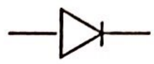
Semiconductor Diode

A diode is a semiconductor device which allows current to flow in one direction only.

Under forward-bias, the current through a diode increases very rapidly when the voltage rises above about 0.7 V (turn-on voltage), and a diode in forward-biased has a very low resistance.



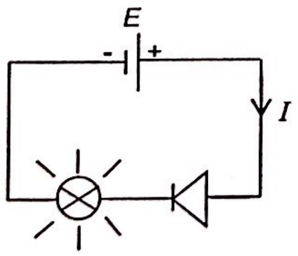
I-V Characteristic of a Semiconductor Diode



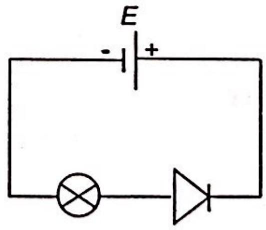
Circuit symbol for a diode

Under reverse-bias, the current through a diode is very small. A diode in reverse-bias has a very high resistance. The current is a few μA and appears to be zero on a graph, if the same scale is used for both forward and reverse-bias.

The circuits below show diodes arranged in forward-bias and reverse-bias. The diode conducts only when it is forward-biased. There is no current (or only a very small current – less than 1 mA) when it is reverse-biased.



Forward-Biased Diode



Reverse-Biased Diode

Effects associated with an increase in temperature

There are two effects that affect the resistivities of material when temperature increases.

Effect 1: increase in number of free electrons

As its temperature rises, more electrons are able to break free from the atoms. The material then becomes a better electrical conductor with more free electrons.

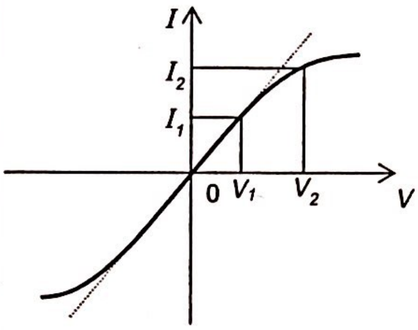
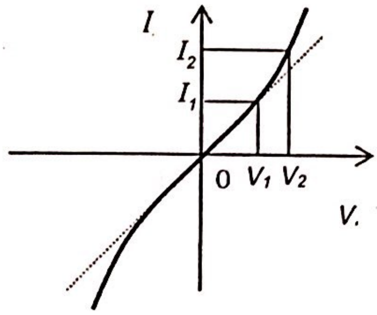
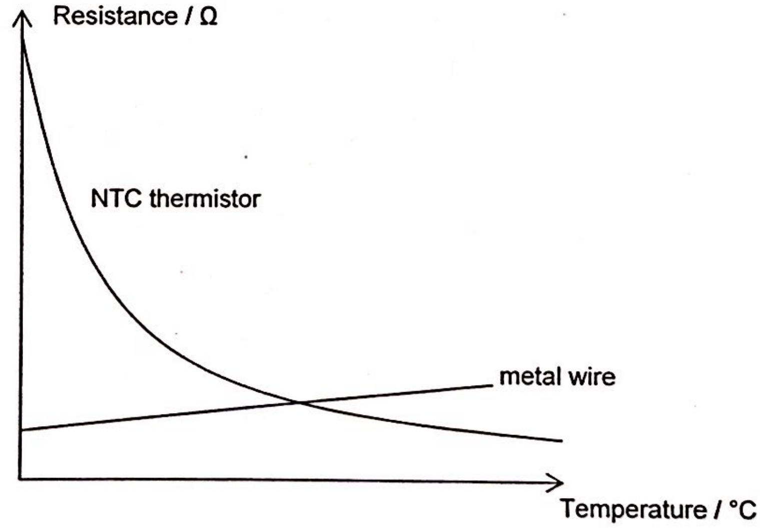
Effect 2: ions vibrate faster and with greater amplitudes

The temperature of the material increases as electrons lose their kinetic energies to the ions. As the material becomes hotter, its ions vibrate faster and with greater amplitudes, making it even more difficult for electrons to pass through the lattice.

Let us look at these effects in the metallic conductors (e.g. filament lamp) and in negative temperature coefficient (NTC) thermistors (made out of semiconductors).

Metallic conductors (e.g. filament lamp) and thermistors

Materials	*Metallic conductors (e.g. filament lamp)*	*Semiconductor NTC thermistors*
Effect 1	As the electrons are free at room temperature, any increase in temperature will not cause an appreciable increase in number of charged particles.	In semiconductors, the number of free electrons is small compared to that in metals. Hence it is a poor conductor at low temperatures. As its temperature rises, more and more electrons break free from the atoms. The semiconductor then becomes a better electrical conductor with more free electrons and holes.
Effect 2	The temperature of the filament increases as the metal acquires energy due to its collision with electrons. Its ions vibrate faster and with greater amplitudes, increasing the frequency of collision with the electrons. As a result, resistance increases.	The temperature of the semiconductor increases as electrons lose their kinetic energies to the ions. As a semiconductor becomes hotter, its ions vibrate faster and with greater amplitudes, making it even more difficult for electrons to pass through the lattice.
Change in resistance as temperature increases	As effect 1 is not significant and effect 2 is present, resistance increase as temperature increases.	As effect 1 is greater than effect 2, resistance decreases as temperature increases. A thermistor is thus termed a NTC thermistor.

<table>
<tr><th>Materials</th><th>Metallic conductors
(e.g. filament lamp)</th><th>Semiconductor
NTC thermistors</th></tr>
<tr><td>I-V characteristic</td><td>
</td><td>
</td></tr>
<tr><td></td><td>$\frac{V_2}{I_2} > \frac{V_1}{I_1}$</td><td>$\frac{V_2}{I_2} < \frac{V_1}{I_1}$</td></tr>
<tr><td>Resistance Vs temperature graph</td><td colspan="2">
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14.6 Electrical Power P

Electrical Power P and Its Unit

As shown earlier in equation (6), if a charge Q moves through a p.d. V, the amount of electrical energy converted to other forms is

$$W = QV$$

The rate at which **energy conversion** takes place is

$$P = \frac{dW}{dt} = \frac{d(QV)}{dt} = \frac{dQ}{dt}V = IV \qquad (11)$$

This equation applies for any electrical device.

The S.I. unit for power is the **watt,** W. (1 W = 1 J s^{-1})

Power Supplied by an Ideal Source

From equation (7), for an ideal source of e.m.f. E, work done by the source is

$$W = QE \qquad (12)$$

The power P supplied by source

$$P = \frac{dW}{dt} = \frac{dQ}{dt}E$$

$$\therefore \quad P = IE \qquad (13)$$

Power Dissipated by a Resistor

From equation (8), the p.d. across a resistor is

$$V = IR \qquad (14)$$

The rate at which energy is dissipated across a resistor is

$$P = IV$$

$$\text{OR} \quad P = I \times (IR) = I^2R \qquad (15)$$

$$\text{OR} \quad P = \left(\frac{V}{R}\right) \times V = \frac{V^2}{R} \qquad (16)$$

Another unit of energy - kWh

To measure the consumption of electrical energy, the unit **kilowatt-hour (kWh)**, is sometimes used instead of the joule (J). One kilowatt-hour is the electrical energy consumed in one hour by a device with a power rating of one kW.

(Note: 1 kWh = 1000 W x 3600 s = 3.6 x 10^6 J)

Example 8 A 12-V home-made electric heating element has a power of 20 W. The heating element is to be made from nichrome ribbon of width 1.0 mm and thickness 0.050 mm. Calculate
(a) the length of ribbon required, and
(b) the cost of running the heating element for 5 hours, if electricity costs $0.17 per kWh.
[Resistivity of nichrome is 1.10×10^{-6} Ω m]

(a) Current through resistor: $I = \frac{P}{V} = \frac{20}{12}$ A

Resistance of resistor: $R = \frac{V}{I} = \frac{12}{20/12} = 7.2\ \Omega$

From $R = \rho \frac{l}{A}$,

$$l = \frac{RA}{\rho} = \frac{7.2 \times \left[\left(1.0 \times 10^{-3}\right)\left(0.050 \times 10^{-3}\right)\right]}{1.10 \times 10^{-6}} = 0.327\text{ m}$$

Note: Alternatively, use $P = \frac{V^2}{R}$.

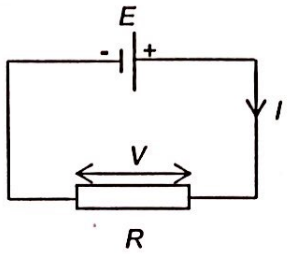
(b) $E = Pt = 20 \times 5 = 100\text{ Wh} = 0.1\text{ kWh}$

Cost $= 0.1 \times 0.17 = \$0.017$

14.7 Internal Resistance r of a Source of emf

Internal Resistance r and Its Effects

Consider a simple circuit consisting of a cell and a resistor. [The terms cell and battery are used interchangeably here.]



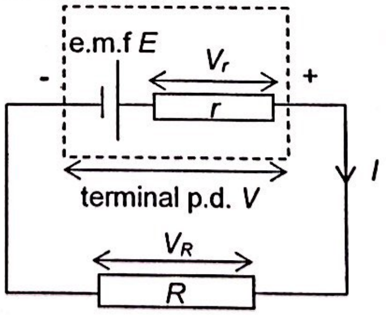
In an ideal circuit,
- the e.m.f. source would have no resistance,
- the connecting wires would have no resistance, and
- the terminal p.d. V of the cell = e.m.f. E of the cell.

However, **real** cells have internal resistance r. Hence, not all electrical energy generated is available to the external load R. Some of these energy is lost as heat within the cell due to its internal resistance. As a result, the terminal p.d. is not equal to the e.m.f. of the source.

Model of a real battery

A **real** battery can be modelled as follows
- a resistance r, which is
- connected in series to an ideal e.m.f. source

as shown in the dashed box below.



This circuit shows a **battery** of e.m.f. E and terminal p.d. V.

Within the source of e.m.f., chemical energy is converted to electrical energy,

power supplied by the source, $P_S = IE$

In the internal resistance r,

power dissipated as heat, $P_r = I^2r = IV_r$

In the external load R,

power dissipated as heat, $P_R = I^2R = IV_R$

By the principle of conservation of energy,

Power **supplied** by source = Power **dissipated** through r and R

$$P_S = P_r + P_R$$
$$IE = I^2 r + I^2 R$$

$$\boxed{E = I(r+R)} \qquad (17)$$

Also

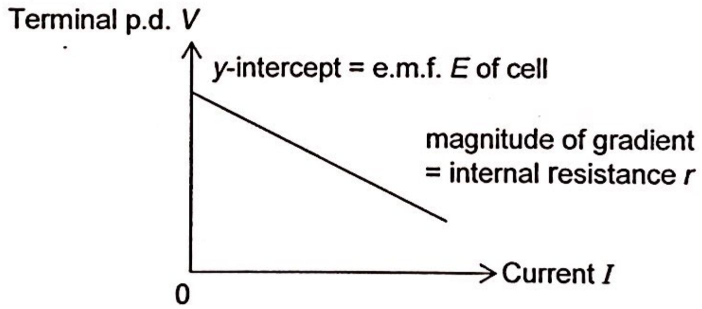
$$P_S = P_r + P_R$$
$$IE = IV_r + IV_R$$
$$E = Ir + V_R$$

$$\boxed{V_R = E - Ir} \qquad (18)$$

Effect of Internal Resistance

The terminal p.d. is $V = V_R = E - Ir$.

A plot of the terminal p.d. against current is shown below.



When **no current flows** (i.e. $R \to \infty$),
- p.d. across the internal resistance r tends to 0
- terminal p.d. = e.m.f. of source
- $V_R = E$

When a current I **flows,**
- p.d. across the internal resistance r is finite
- terminal p.d. decreases by Ir
- $V = V_R = E - Ir$

Example 9 A high-resistance voltmeter reads 1.65 V when connected across a dry-cell in an open circuit and 1.22 V when the same cell is supplying a current of 0.30 A through a lamp.

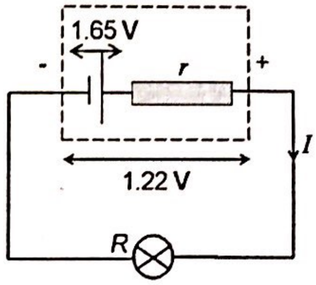
Find
(a) the e.m.f. of the cell,
(b) the internal resistance of the cell, and
(c) the resistance of the lamp.

(a) e.m.f. E = terminal p.d. when there is no current

$$= 1.65 \text{ V}$$

(b)

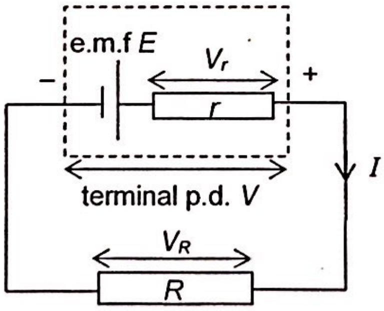
$$E = Ir + V$$
$$1.65 = (0.30)r + 1.22$$
$$r = 1.43\ \Omega$$



(c)

$$V = IR$$
$$1.22 = (0.30)R$$
$$R = 4.07\ \Omega$$

Power Output (Power Transmitted to Load) A battery with e.m.f. E and internal resistance r is connected to a load of resistance R as shown below.



One obtains equation (17):

$$E = I(r + R)$$
$$I = \frac{E}{(r+R)}$$

Hence, power transferred to external load R is

$$P_R = I^2R = \left[\frac{E}{(r+R)}\right]^2 R$$

$$P_R = \frac{E^2R}{(r+R)^2} \qquad (19)$$

Maximum Power Theorem (Maximum Power Transmitted to Load)

To find maximum power delivered to the external load R,

$$\frac{dP_R}{dR} = 0$$

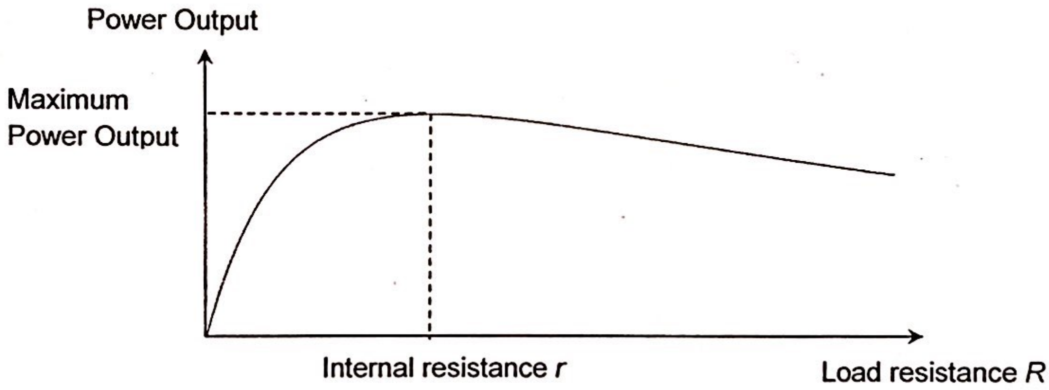
$$\frac{E^2(r+R) - 2E^2R}{(r+R)^3} = 0$$

$$E^2(r+R) - 2E^2R = 0$$

$$(r+R) - 2R = 0$$

$$R = r$$

It can be concluded that a source of e.m.f. delivers the maximum amount of power to an external load when the **resistance of the load is equal to the internal resistance** of the source. This is the maximum power theorem.



Example 10 A cell drives a current of 1.50 A through a 3.08 Ω resistor and 2.00 A through a 2.00 Ω resistor, respectively. Calculate

(a) the e.m.f. of the cell,

(b) the internal resistance of the cell, and

(c) the maximum heating effect it can develop in an external load.

(a) $E = I_1 r + I_1 R_1 = 1.50r + (1.50)(3.08) - - - - - (1)$

$E = I_2 r + I_2 R_2 = 2.00r + (2.00)(2.00) \Rightarrow r = \frac{1}{2}E - 2.00 - - - - - (2)$

Sub (2) into (1),

$E = 1.50\left(\frac{1}{2}E - 2.00\right) + 4.62 \quad \Rightarrow \quad E = 6.48 \text{ V}$

(b) $r = \frac{1}{2}E - 2.00$

$= \frac{1}{2}(6.48) - 2.00 = 1.24\ \Omega$

(c) max heating effect $\Rightarrow$ max power $\Rightarrow R = r = 1.24\ \Omega$

$E = Ir + IR$

$I = \frac{E}{r+R} = \frac{6.48}{1.24+1.24} = 2.613\ A$

$P_{max} = I^2 R = (2.613)^2(1.24) = 8.47\ W$

Annex

Colour-code System of a Resistor

A resistor is a circuit element designed to have a known resistance. Resistors are found in almost all electronic devices. The resistance of a resistor is indicated by a colour-code system on the resistor. There are resistors with 4, 5 and 6-band colour codes. For a 4-band colour code, there are usually four coloured stripes on the resistor. The first three coloured stripes indicate the value of its resistance. The first gives the first digit, the second gives the second digit and the third provides the multiplier. The multiplier is the power of ten used to multiply the two-digit number determined from the first two stripes. The table below gives the digits associated with each colour. The fourth stripe on the resistor indicates its tolerance or accuracy. The most commonly used resistors are reliable to within ± 5% of their indicated value.

To read 5 and 6-band colour codes, you may wish to explore the websites below:
http://www.williamson-labs.com/resistors.htm
http://www.the12volt.com/resistors/resistors.asp

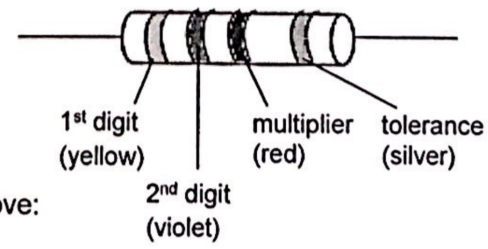
The Resistor Colour Code

Digit	Colour	Multiplier
	Silver	10^{-2} (when used as the third band)
	Gold	10^{-1} (when used as the third band)
0	Black	10^{0}
1	Brown	10^{1}
2	Red	10^{2}
3	Orange	10^{3}
4	Yellow	10^{4}
5	Green	10^{5}
6	Blue	10^{6}
7	Violet	10^{7}
8	Grey	10^{8}
9	White	10^{9}

Tolerance: (when used as the fourth band)

± 0.5% green band	± 5% gold band
± 1% brown band	± 10% silver band
± 2% red band	± 20% none

A Resistor with 4-Band Colour Code



Referring to the table above:
1st digit – 4
2nd digit – 7
multiplier - 10^2
Resistance of the above resistor: $47 \times 10^2 = 4700\ \Omega$
The figure above shows a **4700 Ω** resistor with a **10% tolerance.**

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