Solutions to Tutorial 5B: Tangents and Normals

Basic Mastery Questions

Q1(a) $y = \dfrac{\ln(x^2+1)}{x^2}$ at $(1, \ln 2)$

$$\frac{dy}{dx} = \frac{(x^2)\left(\frac{2x}{x^2+1}\right) - \ln(x^2+1)\cdot 2x}{(x^2)^2}$$

$$= \frac{2}{x(x^2+1)} - \frac{2\ln(x^2+1)}{x^3}$$

when $x=1$, $\dfrac{dy}{dx} = 1 - 2\ln 2$

$\therefore$ Eqn of tangent is

$$(y - \ln 2) = (1-2\ln 2)(x-1)$$

$$y = (1-2\ln 2)x + 3\ln 2 - 1 \quad \#$$

Eqn of normal is

$$y - \ln 2 = -\frac{1}{1-2\ln 2}(x-1)$$

$$y = \frac{1}{2\ln 2 - 1}x + \ln 2 + \frac{1}{1-2\ln 2} \quad \#$$

Q1(b) $x^3 + y^3 = 9$ at $(1, 2)$

Diff wrt x: $3x^2 + 3y^2\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-3x^2}{3y^2} = -\left(\frac{x}{y}\right)^2$$

When $x=1$, $y=2$, $\dfrac{dy}{dx} = -\left(\frac{1}{2}\right)^2 = -\frac{1}{4}$

$\therefore$ Eqn of tangent is

$$y - 2 = -\frac{1}{4}(x-1)$$

$$y = -\frac{1}{4}x + \frac{9}{4} \quad \#$$

Eqn of normal is

$$y - 2 = -\frac{1}{\left(-\frac{1}{4}\right)}(x-1)$$

$$y - 2 = 4(x-1)$$

$$y = 4x - 2 \quad \#$$

Q1(c) $x = a\left(1 + \frac{1}{t}\right)$

$$\frac{dx}{dt} = -\frac{a}{t^2}$$

$$y = a\left(t - \frac{1}{t^2}\right)$$

$$\frac{dy}{dt} = a\left(1 + \frac{2}{t^3}\right)$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{a\left(1+\frac{2}{t^3}\right)}{-\frac{a}{t^2}}$$

$$= -\left(1 + \frac{2}{t^3}\right)\cdot t^2$$

$$= -t^2 - \frac{2}{t}$$

When $t = 2$, $\dfrac{dy}{dx} = -4 - 1 = -$

When $t=2$, $x = \frac{3}{2}a$, $y = \frac{7}{4}a$

$\therefore$ Eqn of tangent is

$$y - \frac{7}{4}a = -5\left(x - \frac{3}{2}a\right)$$

$$\therefore y = -5x + \frac{37}{4}a$$

Eqn of normal is

$$y - \frac{7}{4}a = \frac{1}{5}\left(x - \frac{3}{2}a\right)$$

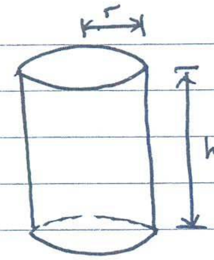
$$y = \frac{1}{5}x + \frac{29}{20}a$$

Solutions to Tutorial 5C: Rate of Change, Maxima and Minima

Basic Mastery Questions

pg 2

Q1



Given $2\pi r^2 + 2\pi r h = 100$

$2\pi r h = 100 - 2\pi r^2$

$h = \dfrac{100 - 2\pi r^2}{2\pi r}$

$= \dfrac{50 - \pi r^2}{\pi r}$.

Then volume $V = \pi r^2 h$

$= \pi r^2 \left(\dfrac{50 - \pi r^2}{\pi r} \right)$

$= 50r - \pi r^3$

$\dfrac{dV}{dr} = 50 - 3\pi r^2$

$\dfrac{d^2V}{dr^2} = -6\pi r$ = - 43.4160 < 0

Then, for stationary value of V, let $\dfrac{dV}{dr} = 0$

Then $50 - 3\pi r^2 = 0$

$r = \sqrt{\dfrac{50}{3\pi}}$

Since $\dfrac{d^2V}{dr^2} < 0$,

maximum volume $= 50r - \pi r^3$

$= \underline{77 \text{ cm}^3}$ (2 s.f.)

Q2

Given that $\dfrac{dA}{dt} = \dfrac{5\pi}{4}$ cm²/s.

$A = \pi r^2 \Rightarrow \dfrac{dA}{dr} = 2\pi r$

Then by chain rule,

$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$

$\dfrac{5\pi}{4} = 2\pi r \times \dfrac{dr}{dt}$

When $r = 20$ cm,

$\dfrac{dr}{dt} = \dfrac{\frac{5\pi}{4}}{2\pi \times 20}$

$= \dfrac{1}{32}$ cm/s.

3

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

Diff wrt t:

$-\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0$

$\Rightarrow \frac{1}{u^2}\frac{du}{dt}+\frac{1}{v^2}\frac{dv}{dt}=0$

Given $f=10$ cm, when $u=50$ cm

$\frac{1}{50}+\frac{1}{v}=\frac{1}{10}$

$\frac{1}{v}=\frac{1}{10}-\frac{1}{50}$

$=\frac{4}{50}=\frac{2}{25}$

$v=\frac{25}{2}$

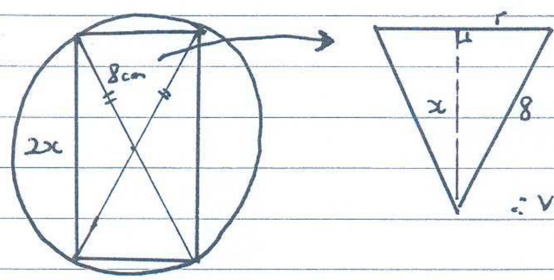
Next, given that $\frac{du}{dt}=-2$ cm/s

Then $\frac{1}{50^2}\times(-2)+\frac{1}{(\frac{25}{2})^2}\frac{dv}{dt}=0$

$\therefore \frac{dv}{dt}=\frac{2}{2500}\times\left(\frac{25}{2}\right)^2$

$=\frac{1}{8}$ cm/s.

4.



r: radius of cylinder

$r=\sqrt{8^2-x^2}$

$=\sqrt{64-x^2}$

$\therefore$ Volume of cylinder $V=\pi r^2h$

$=\pi(\sqrt{64-x^2})^2\times 2x$

$V=2\pi x(64-x^2)$ (shown)

$=128\pi x-2\pi x^3$

Next $\frac{dV}{dx}=128\pi-6\pi x^2$

$\frac{d^2V}{dx^2}=-12\pi x$

For stationary value of V, let $\frac{dV}{dx}=0$

Then $128\pi-6\pi x^2=0 \Rightarrow x^2=\frac{64}{3}$

ie $x=\frac{8}{\sqrt{3}}$.

When $x=\frac{8}{\sqrt{3}}$, $\frac{d^2V}{dx^2}<0$ (max value for V)

$\therefore$ maximum volume $=2\pi\left(\frac{8}{\sqrt{3}}\right)\left(64-\frac{64}{3}\right)$

$=\frac{2048\pi}{3\sqrt{3}}=\frac{2048\sqrt{3}}{9}\pi$ cm^3

Solutions to Tutorial 5D:

Maclaurin's Series

Basic Mastery Questions

Q1. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{r+1}x^r}{r} + \dots \quad (-1 < x \le 1)$

So,

$\ln(1-2x^2) = (-2x^2) - \frac{(-2x^2)^2}{2} + \dots$

$= -2x^2 - 2x^4 + \dots$

where $-1 < -2x^2 \le 1$

$-\frac{1}{2} \le x^2 < \frac{1}{2}$

$\Rightarrow \quad 0 < x^2 < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ #

Q2. $y = \sin(\ln(1+x))$

(i) $\frac{dy}{dx} = \cos(\ln(1+x)) \cdot \frac{1}{1+x}$

$\Rightarrow (1+x)\frac{dy}{dx} = \cos(\ln(1+x))$

(ii) Diff (i) wrt x again:

$(1+x)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)(1) = -\sin(\ln(1+x)) \cdot \frac{1}{1+x}$

$\Rightarrow (1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} = -\sin(\ln(1+x)) = -y$

$\therefore (1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 0$.

(iii) Diff (ii) wrt x again:

$(1+x)^2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}(2)(1+x) + (1+x)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)(1) + \frac{dy}{dx} = 0$

Now when $x = 0$, $y = \sin(\ln 1) = \sin 0 = 0$

Then $1\frac{dy}{dx} = \cos(\ln(1)) = 1 \Rightarrow \frac{dy}{dx} = 1$

Sub into (ii) $1\frac{d^2y}{dx^2} + 1\times 1 + 0 = 0 \Rightarrow \frac{d^2y}{dx^2} -1$

Sub into (iii) $1\frac{d^3y}{dx^3} + (-1)(2) + 1(-1) + 1 + 1 = 0 \Rightarrow \frac{d^3y}{dx^3} = 1$

$\therefore$ The Maclaurin's series for y is

$y = 0 + 1\cdot x + (-1)\frac{x^2}{2!} + (1)\frac{x^3}{3!} + \dots$

$= x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$ (up to x^3 term)

Additional Practice Questions (H2 Chapter 5 Applications of Differentiation)

1

$x = t^2+1, \quad y = t^3 - t$

$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2 - 1$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-1}{2t}$

(a) When $t = 1$, $x = 2$, $y = 0$

$\frac{dy}{dx} = \frac{3-1}{2} = 1$

$\therefore$ Eq: of tangent is

$y - 0 = 1(x-2)$

$y = x - 2.$

(b) let $\frac{dy}{dt} = \frac{dx}{dt}$

$\Rightarrow \frac{dy/dt}{dx/dt} = 1$

$\Rightarrow \frac{dy}{dx} = 1$

Let $\frac{3t^2-1}{2t} = 1$

$3t^2 - 2t - 1 = 0$

$(3t+1)(t-1) = 0 \quad \Rightarrow t = -\frac{1}{3}$ or 1

2

(a) Given $x^2 - 2xy + 2y^2 = 4$,

diff wrt x:

$2x - \left(2x\frac{dy}{dx} + y(2)\right) + 4y\frac{dy}{dx} = 0$

$\Rightarrow (4y - 2x)\frac{dy}{dx} = 2y - 2x$

$\Rightarrow \frac{dy}{dx} = \frac{2(y-x)}{2(2y-x)}$

$= \frac{x-y}{x-2y}$ *

When tangent is // to x-axis, $\frac{dy}{dx} = 0$

$\Rightarrow x - y = 0 \Rightarrow x = y$

$\therefore x^2 - 2x^2 + 2x^2 = 4$

$x^2 = 4$

$x = \pm 2$

$\therefore$ The pts are $(2, 2), (-2, -2)$

b) $x = t^2, \quad y = t^3$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t$

$\therefore$ Eq: of tangent at pt where $t = p$

$y - p^3 = \frac{3}{2}p(x - p^2)$

$2y - 2p^3 = 3px - 3p^3$

$\therefore 2y - 3px + p^3 = 0$

Next, if tangent passes thro' $(-3, -5)$

$2(-5) - 3p(-3) + p^3 = 0$

$p^3 + 9p - 10 = 0$

$(p-1)(p^2+p+10) = 0$

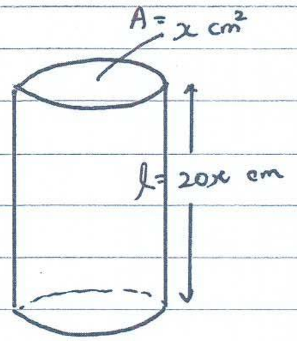
$\Rightarrow p = 1$ (Note for $p^2+p+10 = 0$

$D = 1^2 - 4(1)(10) < 0$.

When $p = 1$, $t = 1 \Rightarrow x = 1, y = 1$

$\therefore$ The only point is $(1, 1)$

3



(a) When $x = 5$, $\frac{dA}{dt} = 0.025\ \text{cm}^2/\text{s}$

$A = x \Rightarrow \frac{dA}{dx} = 1$

$\therefore \frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$0.025 = 1 \times \frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = 0.025\ \text{cm/s}$

Now $l = 20x$, So $\frac{dl}{dt} = 20\frac{dx}{dt}$

$= 20 \times 0.025$

$= 0.5\ \text{cm/s}.$

(b) Volume of cylinder $= V = (x)(20x) = 20x^2$

$\Rightarrow \frac{dV}{dx} = 40x$

So $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$= 40x\big|_{x=5} \times 0.025$

$= 5\ \text{cm}^3/\text{s}$ at the instant when $x = 5$ cm.

(c) Let r: radius of cylinder.

Then $\pi r^2 = A = x$

Diff wrt t: $2\pi r \frac{dr}{dt} = \frac{dx}{dt}$

When $x = 5$, $\pi r^2 = 5 \Rightarrow r = \sqrt{\frac{5}{\pi}}$

$\therefore \frac{dr}{dt} = \frac{\frac{dx}{dt}}{2\pi r} = \frac{0.025}{2\pi\sqrt{\frac{5}{\pi}}} = 3.15 \times 10^{-3}\ \text{cm/s}$ (3 s.f.).

4

$y = e^{\cos x} + e^{\frac{1}{2}}\sin^2 x$

Diff wrt t:

$\frac{dy}{dt} = e^{\cos x}(-\sin x)\frac{dx}{dt} + e^{\frac{1}{2}}(2\sin x)(\cos x)\frac{dx}{dt}$

Given $\frac{dx}{dt} = -4$ units/s, $x = \frac{\pi}{2}$,

$\frac{dy}{dt} = e^{\cos\frac{\pi}{2}}\left(-\sin\frac{\pi}{2}\right)(-4) + e^{\frac{1}{2}}\left(2\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2}\right)(-4)$

$= e^0(-1)(-4)$

$= 4$ units/s.

5

$y=\sqrt{1+8e^x}$ then $y^2 = 1+8e^x$

(i)
$$2y\frac{dy}{dx} = 8e^x$$
$$y\frac{dy}{dx} = 4e^x \quad \text{(shown!)}$$

Diff wrt x again: $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) = 4e^x$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 4e^x$$

Diff wrt x again: $y\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) = 4e^x$

Now, when $x=0$, $y=\sqrt{1+8e^0}=3$

Also, $\frac{dy}{dx}=\frac{4}{3}$, $\frac{d^2y}{dx^2}=\frac{20}{27}$, $\frac{d^3y}{dx^3}=\frac{28}{81}$.

∴ The Maclaurin's series for y in ascending powers of x is

$$y = 3 + \frac{4}{3}x + \frac{\frac{20}{27}}{2!}x^2 + \frac{\frac{28}{81}}{3!}x^3 + \ldots$$

$$= 3 + \frac{4}{3}x + \frac{10}{27}x^2 + \frac{14}{243}x^3 + \ldots \quad \text{(shown!)}.$$

(ii) Next, $\left(3+\frac{4}{3}x+\frac{10}{27}x^2+\frac{14}{243}x^3\right)\left(3+\frac{4}{3}x+\frac{10}{27}x^2+\frac{14}{243}x^3\right)$

$$= 9 + 4x + \frac{10}{9}x^2 + \frac{14}{81}x^3$$
$$+ 4x + \frac{16}{9}x^2 + \frac{40}{81}x^3$$
$$+ \frac{10}{9}x^2 + \frac{40}{81}x^3$$
$$+ \frac{14}{81}x^3 + \ldots\ldots$$

$$= 9 + 8x + 4x^2 + \frac{4}{3}x^3 + \ldots \quad \text{(up to } x^3 \text{ term)}.$$

(iii) Note that $y=\sqrt{1+8e^x}$.

So $y^2 = 1+8e^x$

$$= 1 + 8\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right)$$ Maclaurin's expansion series for e^x.

$$= 9 + 8x + 4x^2 + \frac{4}{3}x^3 + \ldots$$

Thus, the result in (ii) (which is y^2) can be used as a check on the correctness of the first 4 terms in the series of y.

6

Given that $y = (2 - e^{2x})^{\frac{1}{4}}$,

(i) $\frac{dy}{dx} = \frac{1}{4}(2-e^{2x})^{-\frac{3}{4}}(-2e^{2x}) = -\frac{1}{2}e^{2x}(2-e^{2x})^{-\frac{3}{4}}$ ——— ①

Then $(2-e^{2x})^{\frac{1}{4}}\frac{dy}{dx} = -\frac{1}{2}e^{2x}(2-e^{2x})^{-\frac{1}{2}}$

$y\frac{dy}{dx} = -\frac{1}{2}e^{2x}(2-e^{2x})^{-\frac{1}{2}}$ ——— ②

Diff wrt x again: $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) = -\frac{1}{2}e^{2x}(-\frac{1}{2})(2-e^{2x})^{-\frac{3}{2}}(-2e^{2x}) + (2-e^{2x})^{-\frac{1}{2}}(-e^{2x})$

Then $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -2\left(-\frac{1}{2}e^{2x}(2-e^{2x})^{-\frac{3}{4}}\right)^2 - e^{2x}(2-e^{2x})^{-\frac{1}{2}}$

$= -2\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx}$

(from ①) (from ②)

Thus, $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}$

$\therefore y\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}$. (shown!)

(ii) Next, when $x = 0$, $y = (2-e^0)^{\frac{1}{4}} = 1$.

Then $\frac{dy}{dx} = -\frac{1}{2}$, $\frac{d^2y}{dx^2} = -\frac{7}{4}$

$\therefore$ The Maclaurin's series of y is

$y = 1 + (-\frac{1}{2})x + \frac{(-\frac{7}{4})x^2}{2!} + \ldots$

$= 1 - \frac{1}{2}x - \frac{7}{8}x^2 + \ldots$ (up to term in x^2).

(iii) Now, $e^{2x}(2-e^{2x})^{-\frac{3}{4}} = \frac{e^{2x}(2-e^{2x})^{\frac{1}{4}}}{2-e^{2x}}$

$= e^{2x}(2-e^{2x})^{\frac{1}{4}}(2-e^{2x})^{-1}$

If x is sufficiently small,

$e^{2x}(2-e^{2x})^{-\frac{3}{4}} \approx (1+2x)(1-\frac{1}{2}x)(2-(1+2x))^{-1}$

$\approx (1+2x)(1-\frac{1}{2}x)(1-2x)^{-1}$

$\approx (1+2x-\frac{1}{2}x)(1+2x)$

$= (1+\frac{3}{2}x)(1+2x)$

$= 1 + \frac{3}{2}x + 2x + \ldots$

$\approx 1 + \frac{7}{2}x$.

7

$$\cos\left(\frac{\pi}{4}+x\right)\cos\left(\frac{\pi}{4}-x\right) = \left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)\left(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right)$$

$$= \frac{1}{\sqrt{2}}(\cos x - \sin x)\cdot\frac{1}{\sqrt{2}}(\cos x - \sin x)$$

$$= \frac{1}{2}(\cos^2 x - \sin^2 x)$$

$$= \frac{1}{2}(\cos 2x)$$

If x is sufficiently small for x^3 & higher powers of x to be neglecte

$$\cos\left(\frac{\pi}{4}+x\right)\cos\left(\frac{\pi}{4}-x\right) = \frac{1}{2}\cos 2x$$

$$\approx \frac{1}{2}\left(1 - \frac{(2x)^2}{2!}\right)$$

$$= \frac{1}{2}(1-2x^2).$$

Hence
$$\frac{\cos\left(\frac{\pi}{4}+x\right)\cos\left(\frac{\pi}{4}+x\right)}{2+\tan x} \approx \frac{\frac{1}{2}(1-2x^2)}{2+x}$$

$$= \frac{\frac{1}{2}(1-2x^2)}{2\left(1+\frac{x}{2}\right)}$$

$$= \frac{1}{4}(1-2x^2)\left(1+\frac{x}{2}\right)^{-1}$$

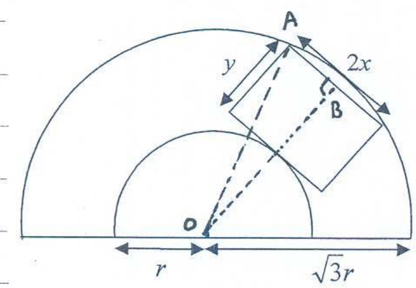
$$= \frac{1}{4}(1-2x^2)\left(1 - \frac{x}{2} + \frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2 + \ldots\right)$$

$$\approx \frac{1}{4}(1-2x^2)\left(1 - \frac{x}{2} + \frac{x^2}{4}\right)$$

$$\approx \frac{1}{4}\left(1 - 2x^2 - \frac{x}{2} + \frac{x^2}{4}\right)$$

$$= \frac{1}{4} - \frac{1}{8}x - \frac{7}{16}x^2 \quad \text{(up to } x^2 \text{ term)}.$$

8



Consider the right-angled $\triangle OAB$:

$OA = \sqrt{3}r$

$OB = r + y$

$AB = x$.

Apply pythagoras's theorem,

$$OA^2 = OB^2 + AB^2$$

$$(\sqrt{3}r)^2 = (r+y)^2 + x^2$$

Note: r: constant!

$$3r^2 = r^2 + 2ry + y^2 + x^2$$

$$\therefore x^2 = 2r^2 - 2ry - y^2 \quad (\text{shown!})$$

Next, area of rectangle $A = 2xy$

Then $A^2 = 4x^2y^2$

So $A^2 = 4(2r^2 - 2ry - y^2)y^2$

$= 8r^2y^2 - 8ry^3 - 4y^4$

For max possible area,

we have $\frac{d(A^2)}{dy} = 16r^2y - 24ry^2 - 16y^3$

$= 8y(2r^2 - 3ry - 2y^2)$

$= 8y(2r+y)(r-2y)$

For $\frac{d(A^2)}{dy} = 0$, we have $y = 0$ (NA), $y = -2r$ (NA) or $y = \frac{1}{2}r$

Next $\frac{d^2(A^2)}{dy^2} = 16r^2 - 48ry - 48y^2$

when $y = \frac{1}{2}r$, $\frac{d^2(A^2)}{dy^2} = 16r^2 - 24r^2 - 12r^2 < 0$

$\therefore$ max value of A^2 and hence max value of A occurs when $y = \frac{1}{2}r$

$\therefore$ Max possible area of rectangle $= \sqrt{8r^2\left(\frac{1}{2}r\right)^2 - 8r\left(\frac{1}{2}r\right)^3 - 4\left(\frac{1}{2}r\right)^4}$

$= \sqrt{\frac{3}{4}r^4}$

$= \frac{r^2\sqrt{3}}{2}$ (shown!).

9

Q6. $y = \dfrac{\ln(1+x)}{1+x}$

$(1+x)y = \ln(1+x)$

(i) Then $(1+x)\dfrac{dy}{dx} + y(1) = \dfrac{1}{1+x}$

Diff wrt x again:

$$(1+x)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)(1) + \frac{dy}{dx} = -(1+x)^{-2}$$

$$\Rightarrow (1+x)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \frac{1}{(1+x)^2} = 0 \quad \text{(shown!)}$$

(ii) Diff wrt x again:

$$(1+x)\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}(1) + 2\frac{d^2y}{dx^2} - \frac{2}{(1+x)^3} = 0$$

$$(1+x)\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - \frac{2}{(1+x)^3} = 0$$

Diff wrt x again:

$$(1+x)\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3}(1) + 3\frac{d^3y}{dx^3} + \frac{6}{(1+x)^4} = 0$$

$$(1+x)\frac{d^4y}{dx^4} + 4\frac{d^3y}{dx^3} + \frac{6}{(1+x)^4} = 0$$

Now, when $x = 0$, $y = 0$

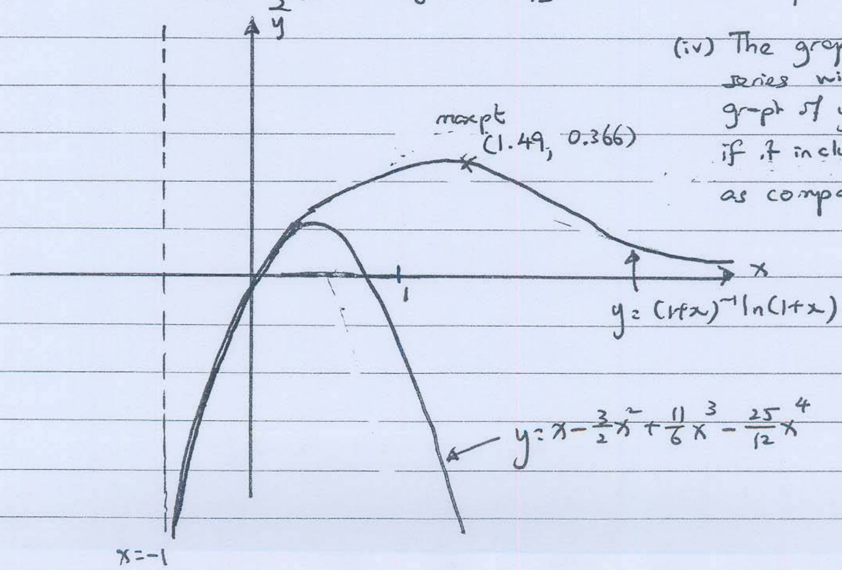
Also, $\dfrac{dy}{dx} = 1$, $\dfrac{d^2y}{dx^2} = -3$, $\dfrac{d^3y}{dx^3} = 11$, $\dfrac{d^4y}{dx^4} = -50$

$\therefore$ The Maclaurin series of y is

$$y = 0 + x + \frac{(-3)x^2}{2!} + \frac{11x^3}{3!} + \frac{(-50)x^4}{4!} + \ldots$$

$$= x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + \ldots \quad \text{(up to } x^4 \text{ term)}$$

(iii)



(iv) The graph of maclaurin's series will approach the graph of $y = (1+x)^{-1}\ln(1+x)$ if it includes more terms as compared to the one above. #