Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9740/01

Paper 1 **17 September 2015**

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Score													
Max Score	**5**	**6**	**6**	**6**	**6**	**8**	**8**	**9**	**9**	**11**	**13**	**13**	**100**

1 A graphic calculator is **not** to be used in answering this question.

(i) Find the value of $(1+4\mathrm{i})^2$, showing clearly how you obtain your answer. [1]

(ii) Given that $1+2\mathrm{i}$ is a root of the equation

$$z^2 - z + (a + b\mathrm{i}) = 0,$$

find the values of the real numbers a and b. [2]

(iii) For these values of a and b, solve the equation in part **(ii)**. [2]

2 Using partial fractions, find $\displaystyle\int \frac{6x^2+x+2}{(1-2x)(x-1)^2}\,\mathrm{d}x$. [6]

3 A curve C has parametric equations

$$x = \theta\cos\theta, \quad y = \theta\sin\theta, \qquad \text{for } 0 \le \theta \le \pi.$$

(i) Sketch the curve C. [2]

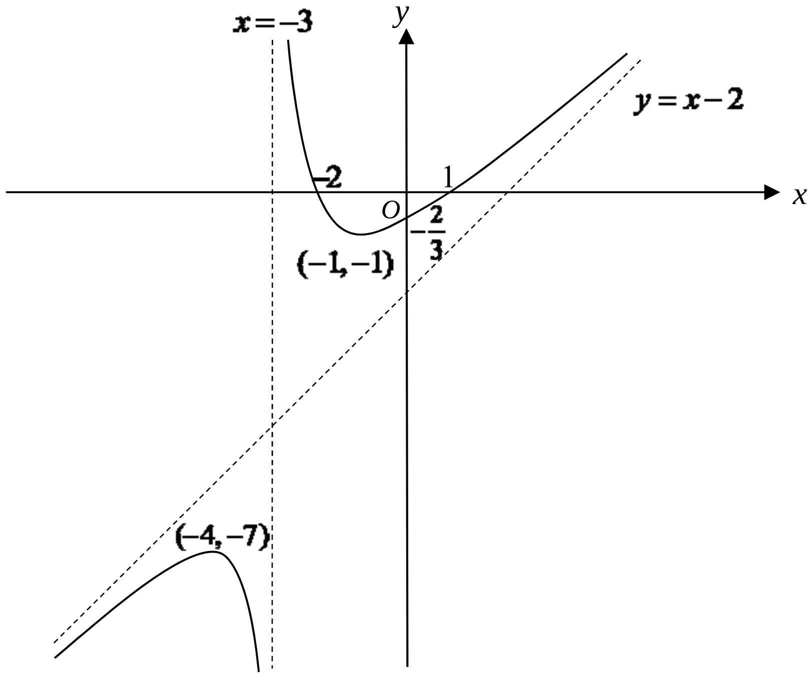
(ii) The point P on the curve C has parameter p and the point Q has coordinates $(-\pi, 0)$. The origin is denoted by O. Given that p is increasing at a constant rate of 0.1 units per second, find the rate of decrease of the area of triangle OPQ when $p = \frac{3}{4}\pi$. [4]

4 The complex number z is given by $(\sqrt{3})\mathrm{e}^{\mathrm{i}\left(\frac{1}{6}\pi\right)}$.

(i) Given that $w = (1+\mathrm{i})z$, find $|w|$ and $\arg w$ in exact form. [2]

(ii) Without using a calculator, find the smallest positive integer n such that w^n is purely imaginary. State the modulus of w^n when n takes this value. [4]

5



The diagram shows the graph of $y=\mathrm{f}(x)$. The graph has a minimum point at $(-1,-1)$ and a maximum point at $(-4,-7)$. It intersects the axes at $x=-2$, $x=1$ and $y=-\frac{2}{3}$. The equations of the asymptotes are $y=x-2$ and $x=-3$.

(i) Sketch the graph of $y=\dfrac{1}{\mathrm{f}(x)}$, giving the coordinates of any stationary points, points of intersection with the axes and the equations of any asymptotes. [3]

(ii) Solve the inequality $\mathrm{f}\left(\frac{1}{x}\right)<0$. [3]

6 **(i)** By using the Maclaurin series for e^x and $\cos x$, find the Maclaurin series for $\mathrm{g}(x)$, where $\mathrm{g}(x)=\mathrm{e}^x\cos 2x$, up to and including the term in x^2. [3]

(ii) Use your answer in part **(i)** to give an approximation for $\int_0^a \mathrm{g}(x)\,\mathrm{d}x$ in terms of a, and evaluate this approximation in the case where $a=\frac{1}{3}\mathrm{e}$, giving your answer correct to 5 significant figures. [3]

(iii) Use your calculator to find an accurate value for $\int_0^{\frac{1}{3}\mathrm{e}} \mathrm{g}(x)\,\mathrm{d}x$, up to 5 significant figures. Why is the approximation in part **(ii)** not very good? [2]

7 Relative to the origin O, two points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. It is given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = 1$ and $|3\mathbf{a} - 2\mathbf{b}| = \sqrt{(37)}$.

(i) By considering the scalar product $(3\mathbf{a} - 2\mathbf{b}) \cdot (3\mathbf{a} - 2\mathbf{b})$, show that $\mathbf{a} \cdot \mathbf{b} = \frac{1}{4}$ and give the geometrical meaning of $|\mathbf{a} \cdot \mathbf{b}|$. [4]

(ii) Give the geometrical meaning of $|(\mathbf{a} - \mathbf{b}) \times \mathbf{b}|$ and find its exact value. [3]

(iii) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, a vector equation of the line that passes through O and bisects the angle AOB. [1]

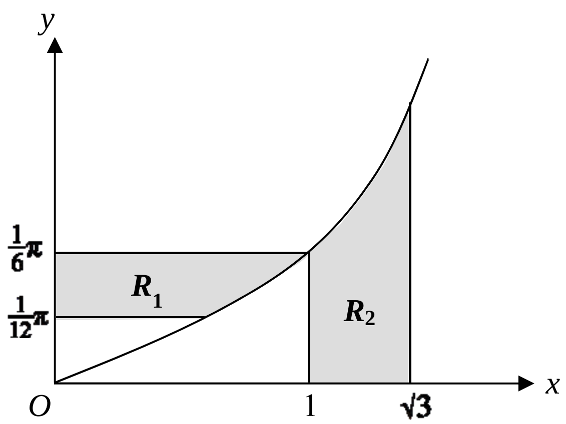
8 A curve C has equation $3x^2 + 2xy - y^2 = 80$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x + y}{y - x}$. [2]

(ii) Show that the curve C has no stationary points. [3]

(iii) The normal to the curve at the point $P(6, -2)$ meets the curve again at the point Q. Find the coordinates of Q. [4]

9



The diagram shows the curve with equation $y = \sin^{-1}\left(\frac{1}{2}x\right)$ for $0 \le x \le 2$.

(i) Find the area of the region R_1 bounded by the curve, the lines $y = \frac{1}{12}\pi$, $y = \frac{1}{6}\pi$ and the y-axis. [2]

(ii) Find the volume of revolution when the region R_1 is rotated through 2π radians about the x-axis. [3]

(iii) Without using a calculator, find the exact area of the region R_2 bounded by the curve, the lines $x = 1$, $x = \sqrt{3}$ and the x-axis. [4]

10 **(a)** Show that the substitution $w = xy^2$ reduces the differential equation

$$2xy\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^2y^4 - y^2 + 1$$

to the form

$$\frac{\mathrm{d}w}{\mathrm{d}x} = aw^2 + b,$$

where a and b are to be determined. Hence obtain the general solution in the form $y^2 = \mathrm{f}(x)$. [5]

(b) A certain species of bird with a population of size n thousand at time t months satisfies the differential equation

$$\frac{\mathrm{d}^2n}{\mathrm{d}t^2} = \mathrm{e}^{-\frac{1}{4}t}.$$

Find the general solution of this differential equation. [2]

Sketch three members of the family of solution curves, given that $n = 30$ when $t = 0$. [4]

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto -\ln(3x-1) \quad \text{for } x \in \mathbb{R},\ x > \tfrac{1}{3},$$

$$\mathrm{g} : x \mapsto \frac{4}{(x-1)(5-x)} \quad \text{for } x \in \mathbb{R},\ 1 < x \leq 4.$$

(i) Describe fully a sequence of transformations which would transform the curve $y = \ln x$ onto the curve of $y = \mathrm{f}(x)$. [3]

(ii) Sketch the graph of $y = \mathrm{g}(x)$. [2]

(iii) Find the exact range of fg. [2]

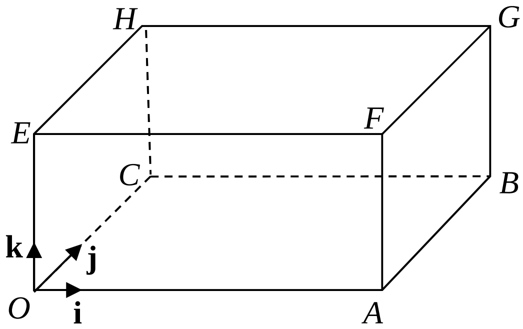
(iv) If the domain of g is further restricted to $k \leq x \leq 4$, state with a reason the least value of k for which the function g^{-1} exists. [2]

In the rest of the question, the domain of g is defined as $x \in \mathbb{R},\ k \leq x \leq 4$, where k is the value found in part **(iv)**.

(v) Find $\mathrm{g}^{-1}(x)$. [2]

(vi) If h is a function such that gh is well-defined and the point $\left(\alpha, \frac{16}{15}\right)$ lies on the graph of $y = \mathrm{gh}(x)$, find the value of $\mathrm{h}(\alpha)$. [2]

12



The diagram shows a cuboid with rectangular base *OABC* and top *EFGH*, where $OA = 4$ units, $OC = 3$ units and $OE = 2$ units. The point O is taken as the origin and unit vectors **i**, **j** and **k,** are taken along *OA*, *OC* and *OE* respectively.

(i) Find the cartesian equation of the plane p which contains the points *A*, *C* and *E*. [3]

(ii) Find the acute angle between p and the base *OABC*. [2]

The line l, passing through *O,* is perpendicular to p and intersects the plane containing *B*, *C*, *G* and *H* at the point *T*.

(iii) Find the position vector of the point *T* and deduce the perpendicular distance from *T* to p. [5]

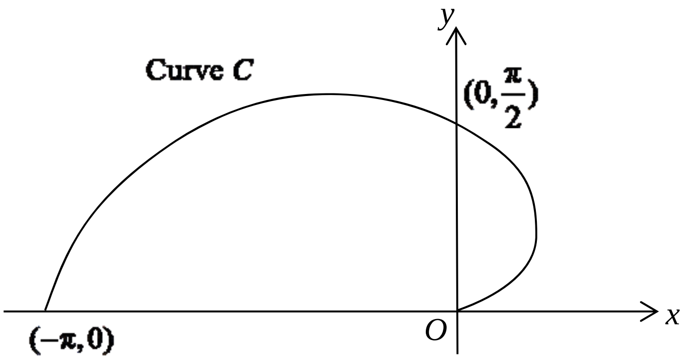
(iv) A point *Q* lies on the line passing through *C* and *T* such that its distance from p is twice that of the distance from *T* to p. Find the possible position vectors of the point *Q*. [3]

Marking Scheme H2 Mathematics Prelim P1

Qn	Suggested Solution
1(i)	$(1+4\mathrm{i})^2 = 1+8\mathrm{i}+(4\mathrm{i})^2 = -15+8\mathrm{i}$
(ii)	$(1+2\mathrm{i})^2-(1+2\mathrm{i})+(a+b\mathrm{i})=0$ $(1+4\mathrm{i}-4)-1-2\mathrm{i}+a+b\mathrm{i}=0$ $(a-4)+(2+b)\mathrm{i}=0$ Compare Re and Im parts, $a=4,\ b=-2$
(iii)	$z^2-z+(4-2\mathrm{i})=0$ $z=\dfrac{1\pm\sqrt{(-1)^2-4(1)(4-2\mathrm{i})}}{2(1)}$ $=\dfrac{1\pm\sqrt{-15+8\mathrm{i}}}{2}$ $=\dfrac{1\pm(1+4\mathrm{i})}{2}$ $=1+2\mathrm{i}$ or $-2\mathrm{i}$

Qn	Suggested Solution
2	$\dfrac{6x^2+x+2}{(1-2x)(x-1)^2}=\dfrac{A}{(1-2x)}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ $\Rightarrow 6x^2+x+2=A(x-1)^2+B(1-2x)(x-1)+C(1-2x)$ when $x=1$, $C=-9$ when $x=\dfrac{1}{2}$, $\dfrac{1}{4}A=6(\dfrac{1}{4})+\dfrac{1}{2}+2\Rightarrow A=16$ when $x=0$, $A-B+C=2$ $\Rightarrow 16-B-9=2\Rightarrow B=5$ $\int\dfrac{6x^2+x+2}{(1-2x)(x-1)^2}\ \mathrm{d}x$ $=\int\dfrac{16}{(1-2x)}+\dfrac{5}{x-1}-\dfrac{9}{(x-1)^2}\ \mathrm{d}x$ $=-8\ln\lvert 1-2x\rvert+5\ln\lvert x-1\rvert+\dfrac{9}{x-1}+d$ where d is a arbitrary constant

<table>
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>3(i)</td><td>



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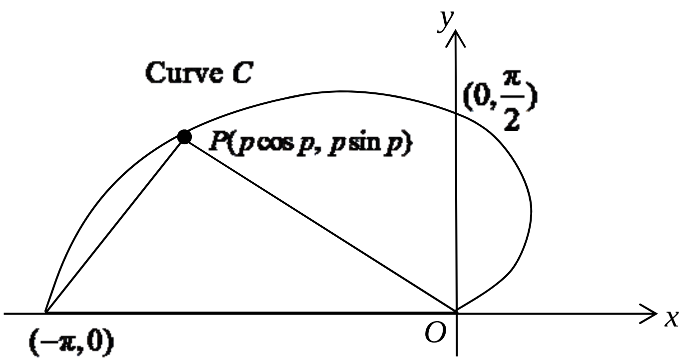
<tr><td>(ii)</td><td>



Point P has coordinates $(p\sin p,\ p\cos p)$.

A, Area of triangle OPQ
$=\frac{\pi}{2}\left(p\sin p\right)$
$\frac{\mathrm{d}A}{\mathrm{d}p}=\frac{\pi}{2}\left(p\cos p+\sin p\right)$
$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}p}\times\frac{\mathrm{d}p}{\mathrm{d}t}$
$\quad=\frac{\pi}{2}\left(p\cos p+\sin p\right)(0.1)$
When $p=\frac{3\pi}{4}$,
$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{2}\left(\frac{3\pi}{4}\cos\left(\frac{3\pi}{4}\right)+\sin\left(\frac{3\pi}{4}\right)\right)(0.1)$
$\quad=-0.151\ \text{units}^2/\text{s}\ \ (3\text{ s.f.})$

The area of the triangle OPQ is decreasing at a rate of $0.151\ \text{units}^2$ per second.

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Qn	Suggested Solution
4(i)	**<u>Method 1</u>** $\lvert w\rvert = \lvert(1+\mathrm{i})z\rvert$ $= \lvert 1+\mathrm{i}\rvert \lvert z\rvert$ $= \sqrt{2}\sqrt{3}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{6}\right)}$ $= \sqrt{6}$ $\arg(w) = \arg((1+\mathrm{i})z)$ $= \arg(1+\mathrm{i}) + \arg(z)$ $= \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$ **<u>Method 2</u>** $w = (1+\mathrm{i})z$ $= \left(\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\right)\left(\sqrt{3}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{6}\right)}\right)$ $= \left(\sqrt{6}\mathrm{e}^{\mathrm{i}\left(\frac{5\pi}{12}\right)}\right)$ $\therefore \lvert w\rvert = \sqrt{6}, \quad \arg(w) = \frac{5\pi}{12}$
(ii)	$w^n = \left(\sqrt{6}\mathrm{e}^{\mathrm{i}\left(\frac{5\pi}{12}\right)}\right)^n$ $= \left(\sqrt{6}\right)^n \mathrm{e}^{\mathrm{i}\left(\frac{5n\pi}{12}\right)}$ $= \left(\sqrt{6}\right)^n \left(\cos\left(\frac{5n\pi}{12}\right) + \mathrm{i}\sin\left(\frac{5n\pi}{12}\right)\right)$ For w^n to be purely imaginary, $\cos\left(\frac{5n\pi}{12}\right) = 0$ $\frac{5n\pi}{12} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ $n = \frac{6}{5}, \frac{18}{5}, 6, \ldots$ smallest positive integer $n = 6$ When $n = 6$, $\lvert w^6\rvert = \left(\sqrt{6}\right)^6 = 216$

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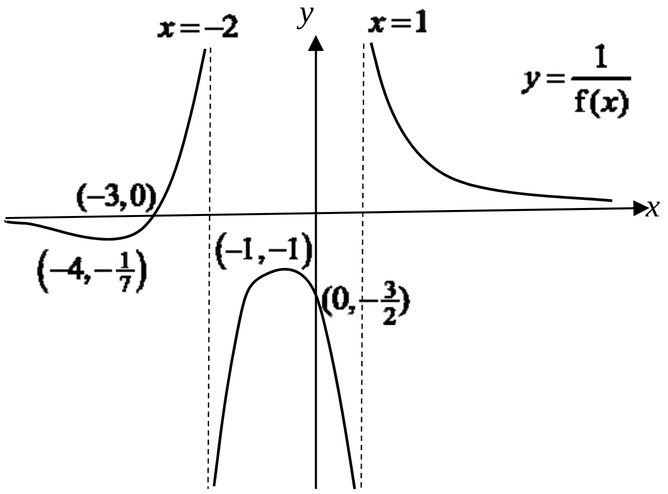
<tr><td>5(i)</td><td>



<u>Consider alternative mark scheme</u> (see specimen paper, P1/Q9)

B1 – LH portion: asymptotic to $x=-2$ and $y=0$; x-intercept at $(-3,0)$ and minimum point $\left(-4,-\frac{1}{7}\right)$

B1 – centre portion: ‘parabolic’ shape between $x=-2$ and $x=1$ with maximum point $(-1,-1)$

B1 – RH portion: asymptotic to $x=1$ and $y=0$

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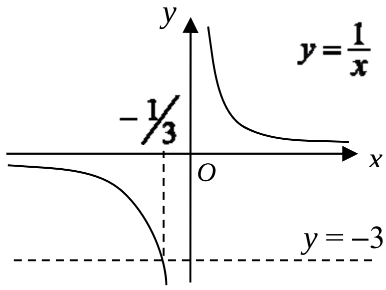
<tr><td>(ii)</td><td>

From the given graph, $\mathrm{f}(x)<0$ for $x<-3$ or $-2<x<1$

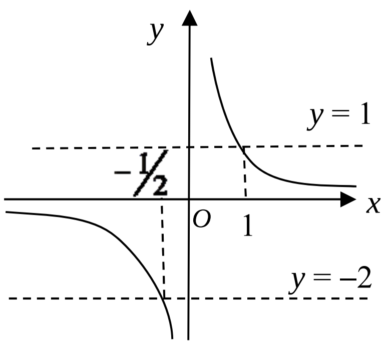
Replacing x by $\dfrac{1}{x}$,

$\dfrac{1}{x}<-3 \Rightarrow -\dfrac{1}{3}<x<0$



or

$-2<\dfrac{1}{x}<1 \Rightarrow x<-\dfrac{1}{2}$ or $x>1$



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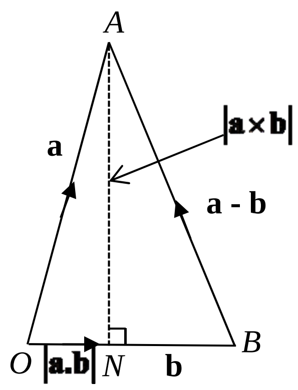
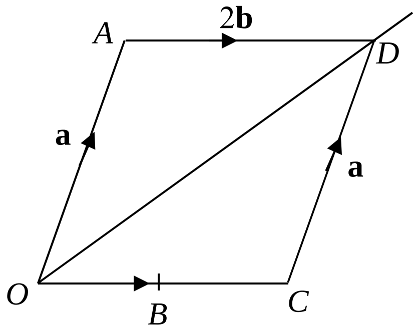
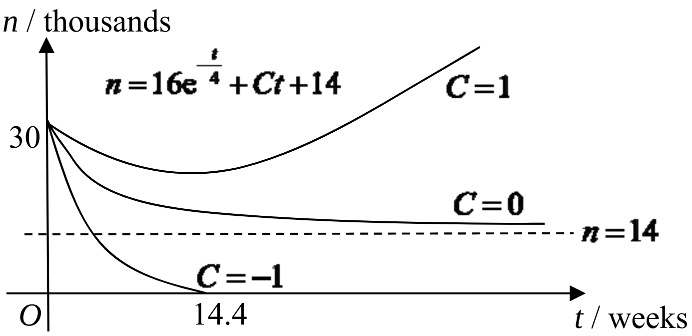
Qn	Suggested Solution
6(i)	$\cos 2x = 1 - \frac{(2x)^2}{2!} + \ldots$ $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$ $e^x \cos 2x = \left(1 + x + \frac{x^2}{2} + \ldots\right)\left(1 - 2x^2 + \ldots\right)$ $= 1 + x + \frac{x^2}{2} - 2x^2 + \ldots$ $= 1 + x - \frac{3x^2}{2} + \ldots$
(ii)	$\int_0^a \mathrm{g}(x)\ \mathrm{d}x \approx \int_0^a 1 + x - \frac{3x^2}{2}\ \mathrm{d}x$ $= \left[x + \frac{x^2}{2} - \frac{x^3}{2}\right]_0^a$ $= a + \frac{a^2}{2} - \frac{a^3}{2}$ When $a = \frac{1}{3}\mathrm{e}$, $\int_0^{\frac{1}{3}\mathrm{e}} \mathrm{g}(x)\ \mathrm{d}x \approx \left(\frac{1}{3}\mathrm{e}\right) + \frac{1}{2}\left(\frac{1}{3}\mathrm{e}\right)^2 - \frac{1}{2}\left(\frac{1}{3}\mathrm{e}\right)^3 = 0.94464$ (5 s.f.)
(iii)	From GC, $\int_0^{\frac{1}{3}\mathrm{e}} \mathrm{g}(x)\ \mathrm{d}x = 0.64284$ (5 s.f.) Insufficient terms are used in the expansion OR the value $\frac{1}{3}\mathrm{e}$ is not close to zero and hence the approximation is not good.

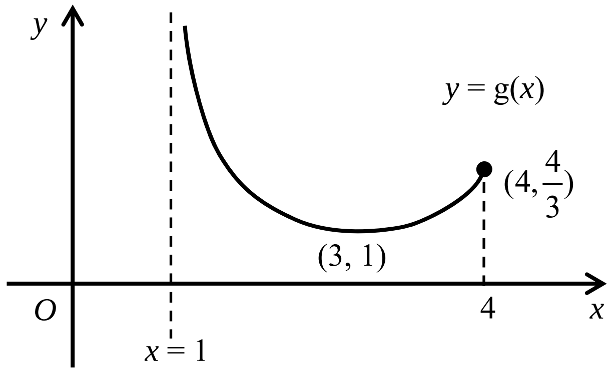
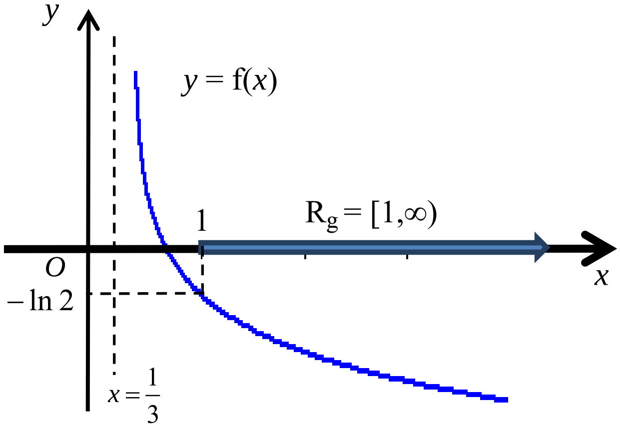
Qn	Suggested Solution
7(i)	$\lvert\mathbf{a}\rvert = 2, \lvert\mathbf{b}\rvert = 1$ and $\lvert 3\mathbf{a} - 2\mathbf{b}\rvert = \sqrt{37}$. $\lvert 3\mathbf{a} - 2\mathbf{b}\rvert = \sqrt{37}$ $\lvert 3\mathbf{a} - 2\mathbf{b}\rvert^2 = 37$ $(3\mathbf{a} - 2\mathbf{b}).(3\mathbf{a} - 2\mathbf{b}) = 37$ $9\mathbf{a}.\mathbf{a} - 12\mathbf{a}.\mathbf{b} + 4\mathbf{b}.\mathbf{b} = 37$ $9\lvert\mathbf{a}\rvert^2 - 12\mathbf{a}.\mathbf{b} + 4\lvert\mathbf{b}\rvert^2 = 37$ $9(2)^2 - 12\mathbf{a}.\mathbf{b} + 4 = 37$ $\therefore \mathbf{a}.\mathbf{b} = \frac{3}{12} = \frac{1}{4}$ (shown) $\lvert\mathbf{a}.\mathbf{b}\rvert$ is the length of projection of **a** onto **b**
(ii)	 $\lvert(\mathbf{a} - \mathbf{b}) \times \mathbf{b}\rvert$ is … 2 times the area of ΔOAB <u>OR</u> area of the parallelogram with adjacent sides *OA* and *OB* <u>OR</u> the perpendicular (or shortest) distance of the point *A* to the line through *O* and *B* $\lvert(\mathbf{a} - \mathbf{b}) \times \mathbf{b}\rvert = \lvert\mathbf{a} \times \mathbf{b}\rvert$ since $\mathbf{b} \times \mathbf{b} = \mathbf{0}$ $= 2 \times$ Area of ΔOAB $= 2 \times \frac{1}{2}(1)\sqrt{2^2 - \left(\frac{1}{4}\right)^2}$ $= \frac{3\sqrt{7}}{4}$ Alternative : $\lvert\mathbf{a} \times \mathbf{b}\rvert =$ Distance of *A* to line *OB* $= \sqrt{OA^2 - (\mathbf{a}.\mathbf{b})^2}$ $= \sqrt{2^2 - \left(\frac{1}{4}\right)^2}$ $= \frac{3\sqrt{7}}{4}$

	Alternative: Given $\mathbf{a}.\mathbf{b} = \frac{1}{4}$, $\therefore \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = \frac{1}{4}$ $\Rightarrow \cos\theta = \frac{1}{4(2)(1)} = \frac{1}{8}$ $\lvert(\mathbf{a}-\mathbf{b})\times\mathbf{b}\rvert = \lvert\mathbf{a}\times\mathbf{b}\rvert$ since $\mathbf{b}\times\mathbf{b} = \mathbf{0}$ $= \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$ $= (2)(1)\sqrt{1-\left(\frac{1}{8}\right)^2} = \frac{3}{4}\sqrt{7}$
(iii)	 Let C be the point such that $\overrightarrow{OC} = 2\mathbf{b}$. Then *OADC* is a rhombus and the line *OD* bisects the angle *AOB*. The equation of the line is $\mathbf{r} = \lambda(\mathbf{a}+2\mathbf{b}), \lambda \in \mathbf{R}$.

Qn	Suggested Solution
8(i)	$3x^2+2xy-y^2=80$ Differentiate with respect to x $6x+2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $2(3x+y)=2\left(y-x\right)\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3x+y}{y-x}$ (shown)
(ii)	At the stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{3x+y}{y-x}=0$ $3x+y=0$ $y=-3x$ Substitute $y=-3x$ into $3x^2+2xy-y^2=80$ $3x^2+2x\left(-3x\right)-\left(-3x\right)^2=80$ $-12x^2=80$ Since there is no real solutions for x, there is no stationary points on the curve.
(iii)	At $P\left(6,-2\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3(6)+(-2)}{6-(-2)}=-2$ Gradient of normal at $P\left(6,-2\right)$ is $\frac{1}{2}$, And equation of normal at $P\left(6,-2\right)$ is $y-(-2)=\frac{1}{2}\left(x-6\right)$, i.e. $y=\frac{1}{2}x-5$ Substitute $y=\frac{1}{2}x-5$ into $3x^2+2xy-y^2=80$, $3x^2+2x\left(\frac{1}{2}x-5\right)-\left(\frac{1}{2}x-5\right)^2=80$ $3x^2+x^2-10x-\frac{1}{4}x^2+5x-25=80$ $\frac{15}{4}x^2-5x-105=0$ By GC, $x=6$ (Point P) or $x=-\frac{14}{3}$ When $x=-\frac{14}{3}$, $y=\frac{1}{2}\left(-\frac{14}{3}\right)-5$ $\therefore Q\left(-\frac{14}{3},-\frac{22}{3}\right)$

Qn	Suggested Solution
9(i)	Area of R_1 $=\int_{\pi/12}^{\pi/6} 2\sin y \ \mathrm{d}y$ $\approx 0.19980 = 0.200$ (3 s.f.)
(ii)	Required volume $=\pi\left(\frac{\pi}{6}\right)^2(1)-\pi\left(\frac{\pi}{12}\right)^2\left(2\sin\frac{\pi}{12}\right)-\pi\int_{2\sin\frac{\pi}{12}}^{1}\left(\sin^{-1}\left(\frac{1}{2}x\right)\right)^2 \ \mathrm{d}x$ ≈ 0.51032 $=0.510 \text{ unit}^3$ (3 s.f.)
(iii)	<u>Method 1 (preferred)</u> Area of $R_2 = (\sqrt{3})\left(\frac{\pi}{3}\right)-(1)\left(\frac{\pi}{6}\right)-\int_{\pi/6}^{\pi/3}(2\sin y)\ \mathrm{d}y$ $=\pi\left(\frac{\sqrt{3}}{3}-\frac{1}{6}\right)+\left[2\cos y\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\frac{\pi}{6}\left(2(\sqrt{3})-1\right)+2\left[\cos\frac{\pi}{3}-\cos\frac{\pi}{6}\right]$ $=\frac{\pi}{6}\left(2(\sqrt{3})-1\right)+2\left[\frac{1}{2}-\frac{\sqrt{3}}{2}\right]$ $=\frac{\pi}{6}\left(2(\sqrt{3})-1\right)+1-\sqrt{3}$
	<u>Method 2</u> Area of $R_2 = \int_1^{\sqrt{3}} \sin^{-1}\frac{x}{2}\ \mathrm{d}x$ $=\left[x\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}-\int_1^{\sqrt{3}} x\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\ \mathrm{d}x$ $=\left[x\sin^{-1}\frac{x}{2}+\sqrt{4-x^2}\right]_1^{\sqrt{3}}$ $=(\sqrt{3})\sin^{-1}\frac{\sqrt{3}}{2}+\sqrt{4-3}-\left(\sin^{-1}\frac{1}{2}+\sqrt{4-1}\right)$ $=\sqrt{3}\left(\frac{\pi}{3}\right)+1-\left(\frac{\pi}{6}\right)-\sqrt{3}$ $=\frac{\pi}{6}\left(2(\sqrt{3})-1\right)+1-\sqrt{3}$

Qn	Suggested Solution
10(a)	$2xy\frac{\mathrm{d}y}{\mathrm{d}x}=4x^2y^4-y^2+1$ Given $w=xy^2 \Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x}=2xy\frac{\mathrm{d}y}{\mathrm{d}x}+y^2$ $2xy\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}w}{\mathrm{d}x}-y^2$ $\therefore\ \frac{\mathrm{d}w}{\mathrm{d}x}-y^2=4w^2-y^2+1$ $\frac{\mathrm{d}w}{\mathrm{d}x}=4w^2+1,$ ie $a=4,\quad b=1$ $\frac{1}{w^2+\frac{1}{4}}\frac{\mathrm{d}w}{\mathrm{d}x}=4$ $\int\frac{1}{w^2+\frac{1}{4}}\mathrm{d}w=4\int 1\,\mathrm{d}x$ $2\tan^{-1}(2w)=4x+C$ $\tan^{-1}(2w)=2x+D$ (where $D=C/2$) $2w=\tan(2x+D)$ $y^2=\frac{\tan(2x+D)}{2x}$
(b)	$\frac{\mathrm{d}^2n}{\mathrm{d}t^2}=\mathrm{e}^{-\frac{t}{4}}$ $\frac{\mathrm{d}n}{\mathrm{d}t}=-4\mathrm{e}^{-\frac{t}{4}}+C$ $n=16\mathrm{e}^{-\frac{t}{4}}+Ct+D$ Given $n=30,\ t=0,$ $30=16+D\Rightarrow D=14$  **<u>SR:</u>** If all 3 marks awarded, deduct 1 mark if – Curve(s) drawn for $n<0$ or $t<0$ **or** – value of C not clearly indicated for each graph **or** – axes not labelled (condone missing units)

Qn	Suggested solutions	Mark scheme
11(i)	<u>Method 1</u> $\ln x \xrightarrow{(a)} \ln(x-1) \xrightarrow{(b)} \ln(3x-1) \xrightarrow{(c)} -\ln(3x-1)$ (a) Translate by 1 unit in the positive x-direction. (b) Scale by a factor of $\frac{1}{3}$ parallel to the x-axis. (c) Reflection in the x-axis <u>OR</u> (c) followed by (a) & (b). <u>Method 2</u> $\ln x \xrightarrow{(a)} \ln(3x) \xrightarrow{(b)} \ln\left(3(x-\frac{1}{3})\right) \xrightarrow{(c)} -\ln(3x-1)$ (a) Scale by a factor of $\frac{1}{3}$ parallel to the x-axis. (b) Translate by $^1/_3$ unit in the positive x-direction (c) Reflection in the x-axis <u>OR</u> (c) followed by (a) & (b).	**B1** – Mtd1: correct translation followed by scaling *SR – condone error in scaling factor* Mtd2: correct scaling followed by translation *SR – condone error in direction/magnitude of translation* **B1** – Both (a) & (b) correct **B1** – (c) correct
(ii)		**G1** – Any one feature: ✓ correct shape ending at $x=4$ ✓ asymptote $x = 1$ ✓ min pint (3,1) **G1** – All 3 features present *SR* – condone if min point or y-coord of end point contain error
(iii)	 $(1,4] \xrightarrow{g} [1, \infty) \xrightarrow{f} (-\infty, -\ln(3-1)]$ Therefore, $R_{fg} = (-\infty, -\ln 2]$ or $\left(-\infty, \ln\frac{1}{2}\right]$	**M1 –** either sketch graph of $y = f(x)$ indicating restricted domain of f corresponding to range of first function g <u>OR</u> show two-stage mapping for g followed by f. **A1** *S.R.* – Award at most 1m if <u>exact</u> range given with graph of $y = fg(x)$ but no further explanation provided <u>OR</u>, correct answer given with correct method but not in exact form

11(iv)	From the graph, the minimum point is (3, 1) OR The graph of $y=\mathrm{g}(x)=-\dfrac{4}{(x-3)^2-4}$ has a line of symmetry $x=3$ Therefore, least value of $k = 3$. Reason: When the domain of g is restricted to $3\le x\le 4$, the line $y=b,\ \ b\in\mathbb{R}$, cuts the graph of $y=\mathrm{g}(x)$ **at most once**. OR The line $y=b\ \ (1\le b\le \frac{4}{3})$ cuts the graph of $y=\mathrm{g}(x)$ **exactly once**. That is, g is 1-1. Thus, g^{-1} exists.	**B1** – least $k = 3$ **B1** – justify that restricted domain gives a 1-1 function. *S.R.* – condone if “1-1” not mentioned as long as horizontal line test is clear.
(v)(a)	Let $y=-\dfrac{4}{x^2-6x+5}$ $=\dfrac{-4}{(x-3)^2-4}$ $(x-3)^2-4=-\dfrac{4}{y}$ $x-3=\sqrt{(4-\frac{4}{y})}\quad (\because 3\le x\le 4 \Rightarrow x-3\ge 0)$ $x=3+\sqrt{(4-\frac{4}{y})}$ $\therefore \mathrm{g}^{-1}(x)=3+\sqrt{(4-\frac{4}{x})}$	**M1** –attempt to make x the subject by completing the square or using quadratic formula **A1**
(v)(b)	Since the point $\left(\alpha,\dfrac{16}{15}\right)$ lies on the graph of $y=\mathrm{gh}(x)$, $\mathrm{gh}(\alpha)=\dfrac{16}{15}$ $\mathrm{g}^{-1}\mathrm{gh}(\alpha)=\mathrm{g}^{-1}\left(\dfrac{16}{15}\right)$ $\mathrm{h}(\alpha)=3+\sqrt{(4-\frac{4}{16/15})}=3+\sqrt{(\frac{1}{4})}=3\frac{1}{2}$	**M1** – $\mathrm{h}(\alpha)=\mathrm{g}^{-1}\left(\dfrac{16}{15}\right)$ **A1**
		Total marks : 13

Qn	Suggested Solution	Mark Scheme
12(i)	$\overrightarrow{AE}\times\overrightarrow{CE}$ $=\begin{pmatrix}-4\\0\\2\end{pmatrix}\times\begin{pmatrix}0\\-3\\2\end{pmatrix}$ $=\begin{pmatrix}6\\8\\12\end{pmatrix}=2\begin{pmatrix}3\\4\\6\end{pmatrix}$ Equation of p : $\mathbf{r}.\begin{pmatrix}3\\4\\6\end{pmatrix}=\begin{pmatrix}4\\0\\0\end{pmatrix}.\begin{pmatrix}3\\4\\6\end{pmatrix}=12$ Cartesian equation of p : $3x+4y+6z=12$	**M1** for finding the normal vector **M1** **A1**
(ii)	Let the acute angle be θ. $\cos\theta=\frac{\left\lvert\begin{pmatrix}3\\4\\6\end{pmatrix}.\begin{pmatrix}0\\0\\1\end{pmatrix}\right\rvert}{\sqrt{3^2+4^2+6^2}}=\frac{6}{\sqrt{61}}$ $\therefore\theta=39.8^{\circ}$	**M1** to find $\cos\theta$ with correct normal vectors **A1**
(iii)	Equation of l : $\mathbf{r}=\lambda\begin{pmatrix}3\\4\\6\end{pmatrix},\lambda\in\mathbf{R}$ Equation of plane containing B,C, G and H $\mathbf{r}.\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\3\\0\end{pmatrix}.\begin{pmatrix}0\\1\\0\end{pmatrix}=3$ At the intersection, $\lambda\begin{pmatrix}3\\4\\6\end{pmatrix}.\begin{pmatrix}0\\1\\0\end{pmatrix}=3$ $\lambda=\frac{3}{4}$ $\overrightarrow{OT}=\frac{3}{4}\begin{pmatrix}3\\4\\6\end{pmatrix}$	**M1** for equation of line and plane **M1** **A1**

Perpendicular distance from T to $p = \dfrac{\left|\overrightarrow{AT}.\begin{pmatrix}3\\4\\6\end{pmatrix}\right|}{\left|\begin{pmatrix}3\\4\\6\end{pmatrix}\right|}$ **M1**

$$= \frac{\left|\overrightarrow{OT}.\begin{pmatrix}3\\4\\6\end{pmatrix} - \overrightarrow{OA}.\begin{pmatrix}3\\4\\6\end{pmatrix}\right|}{\sqrt{61}}$$

$$= \frac{\left|\frac{3}{4}\begin{pmatrix}3\\4\\6\end{pmatrix}.\begin{pmatrix}3\\4\\6\end{pmatrix} - 12\right|}{\sqrt{61}}$$

$$= \frac{135}{244}\sqrt{61}$$ **A1**

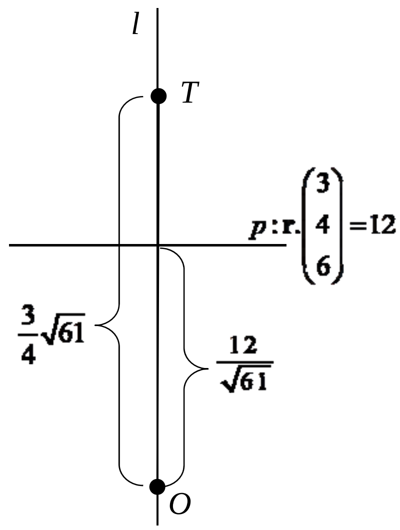
Alternative

Alternative

$\left|\overrightarrow{OT}\right| = \left|\frac{3}{4}\begin{pmatrix}3\\4\\6\end{pmatrix}\right| = \frac{3}{4}\sqrt{61}$

Perpendicular distance from O to p

$= \dfrac{12}{\left|\begin{pmatrix}3\\4\\6\end{pmatrix}\right|} = \dfrac{12}{\sqrt{61}}$

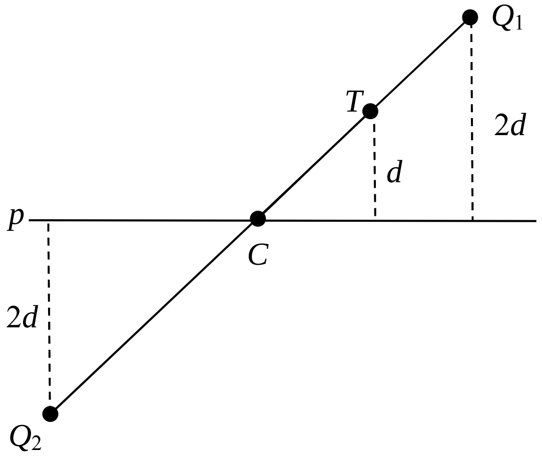


Perpendicular distance from T to p

$= \dfrac{3}{4}\sqrt{61} - \dfrac{12}{\sqrt{61}}$ **M1**

$= \dfrac{3}{4}\sqrt{61} - \dfrac{12}{61}\sqrt{61}$

$= \dfrac{135}{244}\sqrt{61}$ **A1**

12(iv)	Let the points be Q_1 and Q_2.  $\overrightarrow{CQ_1} = 2\overrightarrow{CT}$ and $\overrightarrow{CQ_2} = -2\overrightarrow{CT}$ $\overrightarrow{CQ_1} = 2\left[\left(\frac{3}{4}\right)\begin{pmatrix}3\\4\\6\end{pmatrix} - \begin{pmatrix}0\\3\\0\end{pmatrix}\right]$ $\overrightarrow{OQ_1} - \overrightarrow{OC} = \frac{1}{2}\begin{pmatrix}9\\0\\18\end{pmatrix}$ $\quad$ $\overrightarrow{OQ_2} - \overrightarrow{OC} = -\frac{1}{2}\begin{pmatrix}9\\0\\18\end{pmatrix}$ $\overrightarrow{OQ_1} = \frac{1}{2}\begin{pmatrix}9\\0\\18\end{pmatrix} + \begin{pmatrix}0\\3\\0\end{pmatrix}$ $\quad$ $\overrightarrow{OQ_2} = -\frac{1}{2}\begin{pmatrix}9\\0\\18\end{pmatrix} + \begin{pmatrix}0\\3\\0\end{pmatrix}$ $= \frac{1}{2}\begin{pmatrix}9\\6\\18\end{pmatrix}$ $\quad$ $= \frac{1}{2}\begin{pmatrix}-9\\6\\-18\end{pmatrix}$ The possible position vectors of the point Q are $\frac{1}{2}\begin{pmatrix}9\\6\\18\end{pmatrix}$ and $\frac{1}{2}\begin{pmatrix}-9\\6\\-18\end{pmatrix}$	**M1** for writing one of the 2 equations or any form that shows the relationship **A1 A1**
		Total marks : 13