CANDIDATE NAME						
CLASS		INDEX NUMBER				

Anglo-Chinese School (Independent)

/ 90

THE BEST IS YET TO BE

PRELIMINARY EXAMINATION 2024
YEAR FOUR EXPRESS
MATHEMATICS
PAPER 2

4052/02

Wednesday **7 August 2024** **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number, name and class in the spaces on top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the equation $2(x-5)=3x-1$.

Answer x = ……….………...………. [2]

(b) Solve the inequality $7-4y>3(y+2)$.

Answer ……….……….…………..……… [2]

(c) Given that $\dfrac{1}{p}-\dfrac{1}{2q}=\dfrac{1}{3r}$,

(i) find p when $q=-1$ and $r=2$,

Answer p = ……….………...……… [2]

(ii) rearrange the formula to make q the subject.

Answer q = ……….………...……… [3]

(d) Solve the equation $\frac{5}{x+2}-\frac{3x}{2x-1}=3$.

Give your solutions correct to 2 decimal places.

Answer x = ……..…. or ……..……. [5]

2 **(a)** A manufacturing company produces electronic components for various devices.
They are analyzing the production data for the past 3 months, which includes quantities of components produced and the corresponding costs.

The data is presented in the table below:

Month	Quantity Produced (in units)	Cost per unit (in dollars)
May	5.8×10^5	0.0211
June	4.3×10^6	0.0183
July	7.6×10^5	0.0203

(i) Express the total quantity produced from May to July in standard form correct to 3 significant figures.

Answer …….…….…………..……… [1]

(ii) Calculate the average cost per unit for these 3 months.
Express your answer in cents.

Answer ……….……….………… cents [2]

(iii) Given that the percentage increase in the quantity produced from July to August is 11.8%, calculate the quantity produced in August.
Leave your answer in standard form correct to 3 significant figures.

Answer ……..….………..…..… units [2]

(b) The company wants to build a prototype of a particular electronic component they are manufacturing.
The radius of the actual electronic component is 3.8×10^{-5} m.
In a scale drawing, the radius of the prototype of the electronic component is 1.9 cm.

(i) Find the scale used for the drawing.
Give your answer in the form $n : 1$.

Answer ……….……….……………. : 1 [2]

(ii) Given that the prototype has a total surface area of 1.81×10^{-8} m^2, find, in cm^2, the actual total surface area of the electronic component. Give your answer in standard form.

Answer …………..……..………. cm^2 [2]

3 In Diagram I below, $ABCDE$ is a regular pentagon, centre O. $OA = OB = 4$ cm.

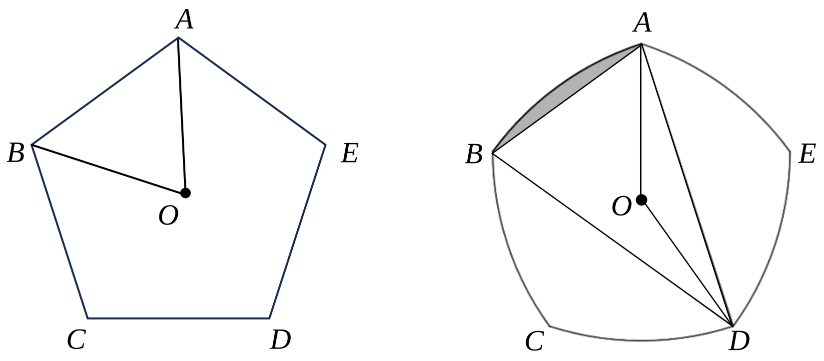


Diagram I Diagram II

(a) State the value of angle AOB.

Answer Angle AOB = ..…………..…… ° [1]

(b) Calculate the area of the pentagon $ABCDE$.

Answer ……………..……..………. cm^2 [2]

(c) Diagram II shows a design for a new badge.

The vertices of the regular pentagon $ABCDE$ are joined by circular arcs whose centres are the opposite vertices.

For example, the arc AB has centre D and radius AD.

(i) Find angle ABD.
Give reasons for each step of your working.

Answer Angle ABD = ..…………..…… ° [2]

(ii) Show that the length of BD is approximately 7.61 cm.

Answer

[2]

(iii) Calculate the area of the shaded segment in Diagram II.

Answer …………..……..………. cm^2 [3]

(iv) Calculate the area of the face $ABCDE$ of the badge.

Answer …………..……..………. cm^2 [2]

4 Here are the first four terms of a sequence.

$$2 \quad \frac{5}{3} \quad \frac{10}{5} \quad \frac{17}{7}$$

(a) Find the fifth term of the sequence.

Answer ……………………………………… [1]

(b) T_n is the nth term of the sequence.

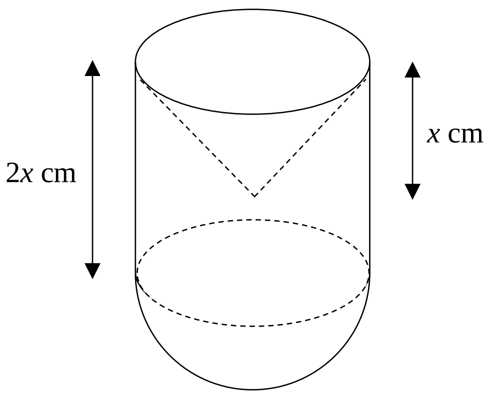
Find an expression, in terms of n, for T_n.

Answer $T_n =$ ……………………………… [2]

(c) Find the value of $T_{25} - T_{24}$.

Answer $T_{25} - T_{24} =$ ………………………… [1]

5 The diagram shows a solid ornament in the shape of a cylinder with an upright cone cut out at one end and a hemisphere attached to the other end.



The vertical heights of the cone and cylinder are x cm and $2x$ cm respectively.

(a) Find the ratio of the volume of the cone to that of the cylinder, expressing your answer as a fraction in the simplest form.

Answer ……………………………………… [1]

(b) If the volume of the cylinder is 345 cm^3 and its height is 8 cm, calculate

(i) the radius of the cylinder,

Answer ……………….....……… cm [2]

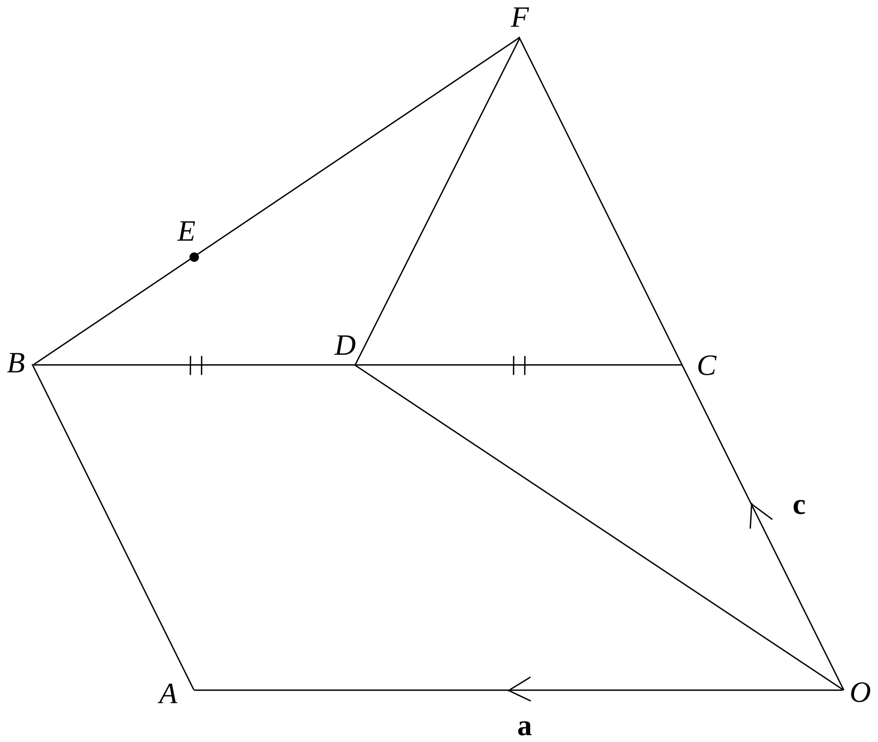
(ii) the curved surface area of the cone,

Answer …………..……..………. cm^2 [3]

(iii) the quantity of paint needed to paint the exterior of the ornament with a 0.2 mm thick coat of paint.

Answer …………..……..………. cm^3 [3]

6 In the diagram, $OABC$ is a parallelogram and D is the midpoint of BC.
BE and OC produced intersect at point F.
It is given that $BE : BF = 1 : 3$, $OC : OF = 1 : 2$, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.



(a) Express and simplify the following vectors in terms of **a** and **c**,

(i) $\overrightarrow{BF}$,

Answer $\overrightarrow{BF}$ =………………………… [1]

(ii) $\overrightarrow{AE}$,

Answer $\overrightarrow{AE}$ =………………………… [2]

(iii) $\overrightarrow{OD}$.

Answer $\overrightarrow{OD}$ =………………………… [1]

(b) Determine, with clear working shown, whether points *O*, *D* and *E* lie on a straight line.

Answer

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………[3]

(c) Find the value of $\dfrac{\text{area of triangle } CDE}{\text{area of parallelogram } OABC}$.

Answer ………………………………… [2]

7 **(a)** Complete the table of values for $y = 2x + \frac{1}{x^2} - 4$.

Values are given to two decimal places where appropriate.

x	-2	-1	-0.5	-0.3	0.3	0.5	1	2	3
y	-7.75	-5	-1	6.51	7.71		-1	0.25	2.1

[1]

(b) On the grid opposite, draw the graph of $y = 2x + \frac{1}{x^2} - 4$ for $-2 \le x \le 3$. [3]

(c) **(i)** $y = b$ cuts the graph of $y = 2x + \frac{1}{x^2} - 4$ at one point for $-2 \le x \le 3$, state the range of values of b.

Answer ……………………………………… [1]

(ii) On the same grid, draw the graph of $3y - 5x = 4$ for $-2 \le x \le 3$. [2]

(iii) Write down the x-coordinates of the points where the graph of $3y - 5x = 4$ intersects the curve for $-2 \le x \le 3$.

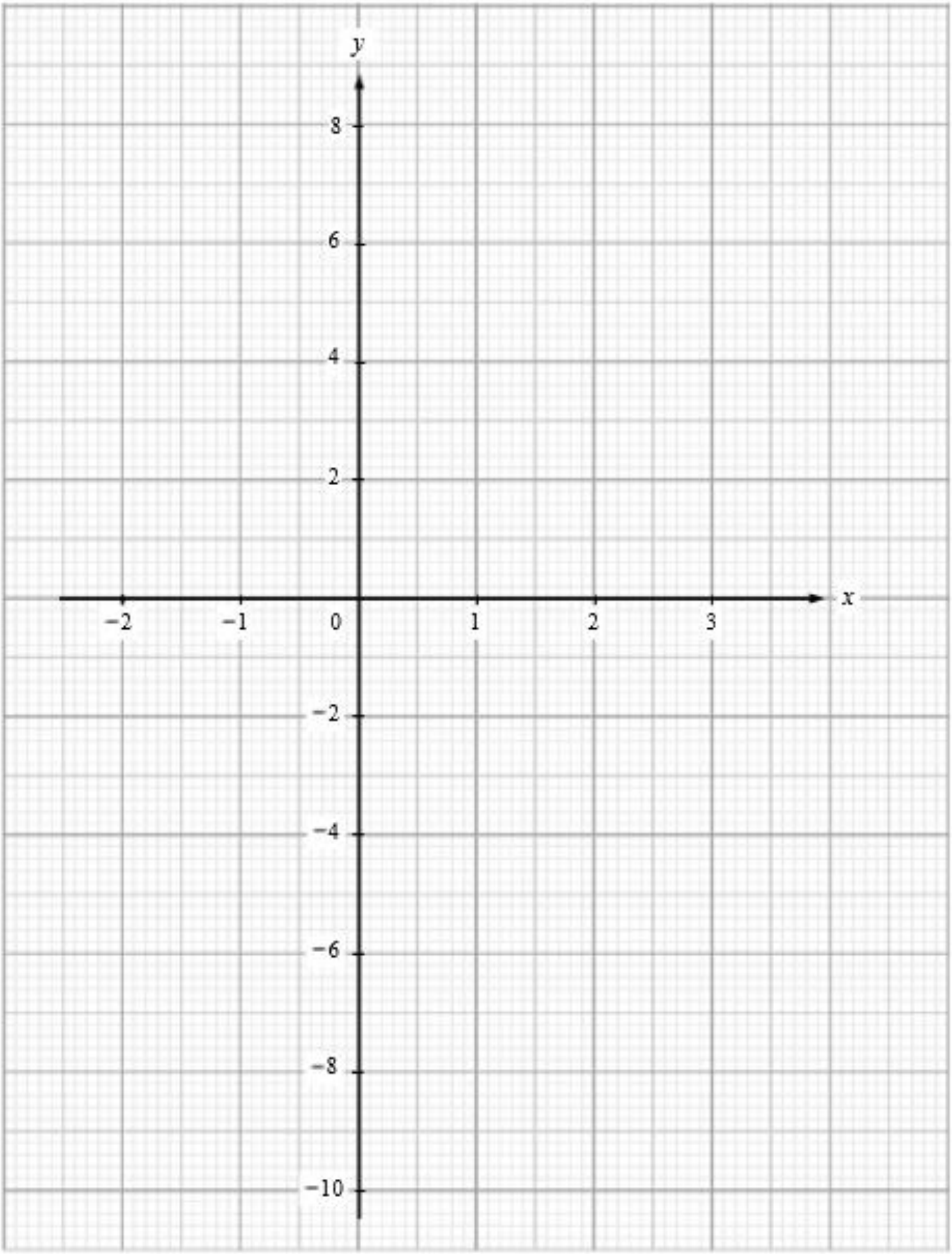
Answer $x =$ ……………and …………… [2]

(iv) These values of x are solutions of the equation $x^3 + Ax^2 + B = 0$.
Find the value of A and the value of B.

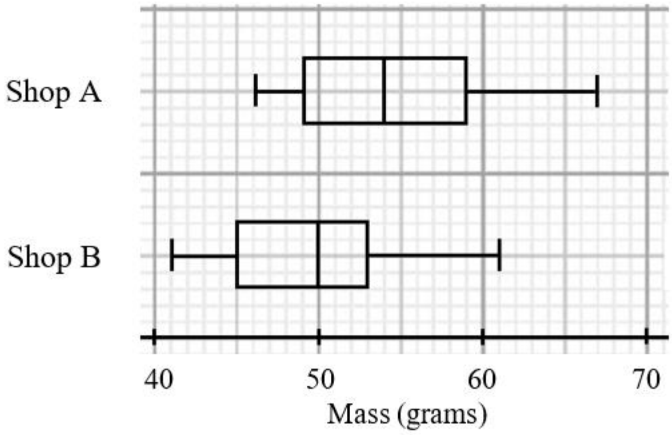
Answer $A =$ ……………………………

$B =$ ……………………………

[3]


y
8
6
4
2
x
−2
−1
0
1
2
3
−2
−4
−6
−8
−10

8 **(a)** The box-and-whisker plots show the distribution of the masses (in grams) of eggs sold in Shop A and Shop B.



(i) There are 25 eggs with masses of more than 59 g sold in Shop A.

Find the total number of eggs sold in Shop A.

Answer …………..……..……… eggs [1]

(ii) Make a comment comparing the averages and a comment comparing the distribution of the masses of the eggs sold in Shop A and Shop B.

Use figures to support your answers.

1. ……..…………………………………………………………………………..…

………...………………………………………………………………….……………

………...………………………………………………………………….……………

………...………………………………………………………………….……………

………...………………………………………………………………….……………

2. ……..…………………………………………………………………………..…

………...………………………………………………………………….……………

………...………………………………………………………………….……………

………...………………………………………………………………….……………

………...………………………………………………………………….………..[3]

(b) In a sample of 80 eggs, 4 are cracked.

(i) One egg is selected from the sample at random.

Find the probability that the egg is cracked.

Answer …………………………………… [1]

(ii) Two eggs are selected from the sample at random.

Find the probability that both eggs are cracked.

Answer …………………………………… [2]

(iii) Three eggs are selected from the sample at random.

Find the probability that at least one egg is cracked.

Answer …………………………………… [2]

9 Emily is considering signing up for a new credit card and has shortlisted three options: Card A, Card B and Card C. Each card offers distinct benefits, rewards, and imposes different annual fees.

The table below presents the essential features of each card:

Card Features	**Credit Card**		
	Card A	**Card B**	**Card C**
Annual Fee[1] (in SGD) payable at the end of 12 months	\$110	\$90	Waived off for the first year; \$250 for 2nd year onwards
Cashback[2] Rate per month	1.5% of expenditure	1.3% of expenditure	1.2% of expenditure
Sign-up Bonus (in SGD) to be used to offset the first bill payment	\$50	\$60	\$70
Number of Free Airport Lounge Access Passes per year	2	4	Unlimited

[1] Cardholder does not enjoy cashback on the annual fee.

[2] Cashback amount is the amount of money received by the cardholder based on his/her expenditure. The cashback amount will be credited to the card account and used to offset the credit card bill for that month.

(a) Calculate the net rewards (which consist of cashback and sign-up bonus) for each credit card for an expenditure of SGD 2000 within the first month of card usage.

Answer Card A: $…..……….……....…

Card B: $…..……….……....…

Card C: $…..……….……....…

[3]

(b) Emily's monthly card expenditure is SGD 2000.

Assuming Emily has signed up for credit card A, by considering only the annual fee and the net rewards, calculate the total amount she has to pay for her credit card bill after the first year of usage.

Answer $..….……….…………..……… [2]

(c) Emily enjoys travelling. As such, travel perks are important to her.

The following table provides a summary of Emily's monthly expenditure and travel needs.

Additional information

- Monthly Card Expenditure (excluding annual fee and purchases made for airport lounge access passes): SGD 2000
- Airport Lounge Access Passes[3] required per year: 5

[3] Credit card must be presented at the Airport Lounge. Any Airport Lounge access pass bought, costing $50 each, must be charged to the same credit card. Cardholder will not be able to enjoy cashback on the amount spent on Airport Lounge access passes.

Determine which credit card might be the best choice for Emily if she signs up for the card and uses it for two consecutive years.

Justify any decisions you make and show your calculations clearly.

Continuation of working space for question **9(c)**.

……………...………………………………………………………………………….……………..

……………...………………………………………………………………………….……………..

..……………...………………………………………………………………………….………...[7]

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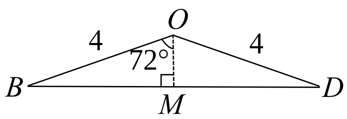
WORKED SOLUTIONS FOR ACS(I) 2024 MATHEMATICS PRELIM P2

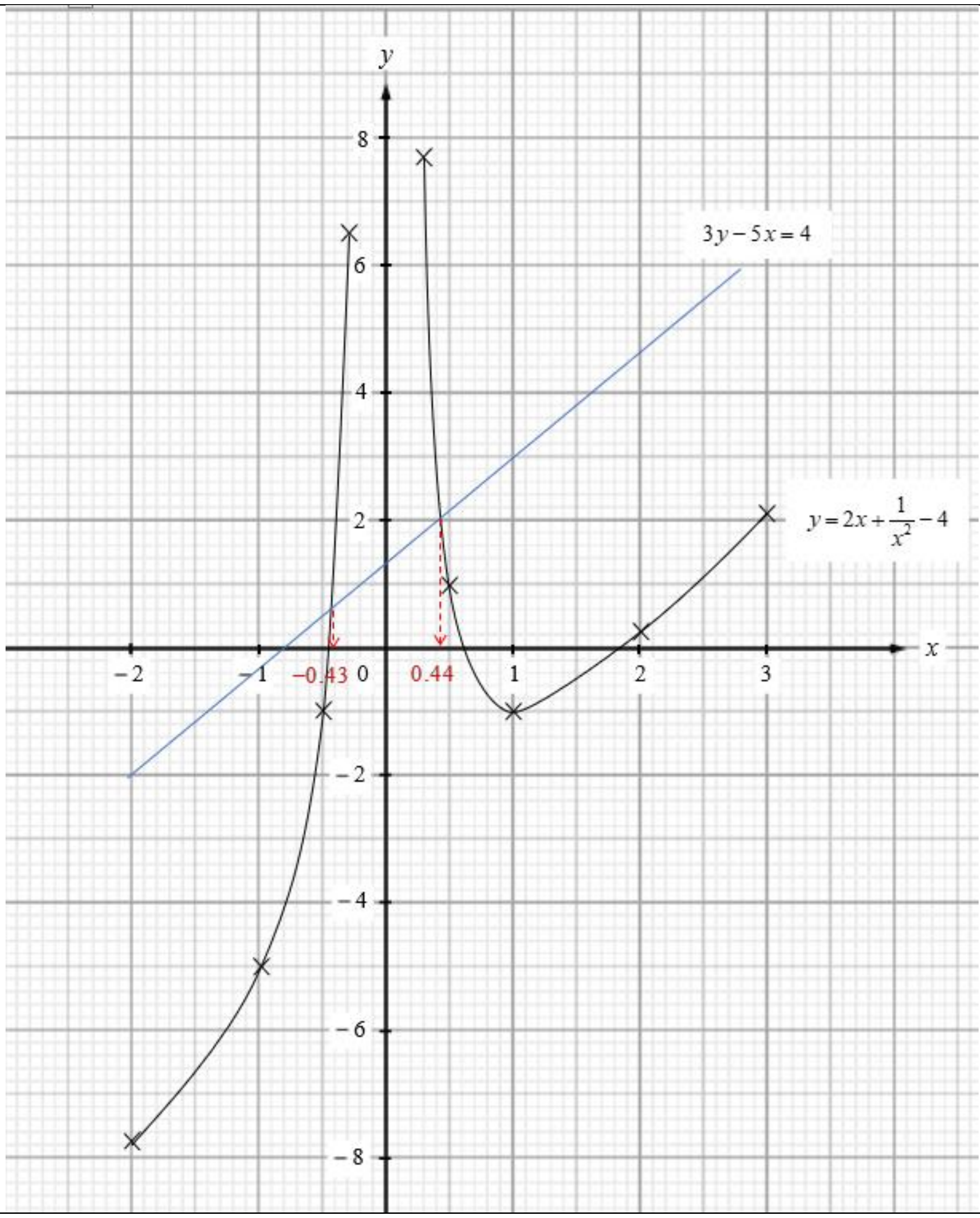
1a	$2(x-5)=3x-1$ $2x-10=3x-1$ $x=-9$
b	$7-4y>3(y+2)$ $7-4y>3y+6$ $1>7y$ $y<\frac{1}{7}$
ci	$\frac{1}{p}-\frac{1}{2q}=\frac{1}{3r}$ $\frac{1}{p}-\frac{1}{2(-1)}=\frac{1}{3(2)}$ $\frac{1}{p}=\frac{1}{6}-\frac{1}{2}$ $\frac{1}{p}=-\frac{1}{3}$ $\therefore p=-3$
cii	$\frac{1}{p}-\frac{1}{2q}=\frac{1}{3r}$ $\frac{1}{2q}=\frac{1}{p}-\frac{1}{3r}$ $\frac{1}{2q}=\frac{3r-p}{3pr}$ $q=\frac{3pr}{2(3r-p)}$
d	$\frac{5}{x+2}-\frac{3x}{2x-1}=3$ $5(2x-1)-3x(x+2)=3(x+2)(2x-1)$ $10x-5-3x^2-6x=3(2x^2+3x-2)$ $4x-5-3x^2=6x^2+9x-6$ $9x^2+5x-1=0$ $x=\frac{-5\pm\sqrt{5^2-4(9)(-1)}}{2(9)}$ $=\frac{-5\pm\sqrt{61}}{18}$ $x=-0.71\ (2\,\text{dp})\quad or\quad 0.16\ (2\,\text{dp})$
2ai	$5.8\times10^5+4.3\times10^6+7.6\times10^5$ $=5.64\times10^6$
aii	$\frac{5.8\times10^5\times0.0211+4.3\times10^6\times0.0183+7.6\times10^5\times0.0203}{5.64\times10^6}$ $=\$0.0189$ $=1.89\,\text{cents}\ (2\,\text{dp})$
aiii	$7.6\times10^5\times\frac{111.8}{100}=849680=8.50\times10^5\ (3\text{ s.f.})$

bi	3.8×10^{-5} m $= 0.0038$ cm 0.0038 cm is represented by 1.9 cm. 1 cm is represented by $\frac{1.9}{0.0038}$ cm $= 500$ cm $\therefore$ The scale used is 500 : 1.
bii	$1.81\times10^{-8}\text{ m}^2 = 1.81\times10^{-4}\text{ cm}^2$ 500 cm of the prototype represents 1 cm of the actual electronic component. $500\times500\text{ cm}^2 = 2.5\times10^5\text{ cm}^2$ of the prototype represents 1 cm^2 of the actual. $\therefore 1.81\times10^{-4}\text{ cm}^2$ of the prototype represents $\frac{1.81\times10^{-4}}{2.5\times10^5} = 7.24\times10^{-10}\text{ cm}^2$ of the actual.
3a	$\angle AOB = 360°\div5$ $= 72°$ ($\angle$s at a point)
b	Area of the pentagon $ABCDE = 5\times\frac{1}{2}(4)(4)\sin72°$ $= 38.042\text{cm}^2$ (5 sf) $= 38.0\text{ cm}^2$ (3 sf)
ci	$\angle ADB = 72°\div2$ ($\angle$ at centre = 2 $\angle$at circumference) $= 36°$ $\angle ABD = \frac{180°-36°}{2}$ (Base $\angle$s of isosceles triangle) $= 72°$
cii	 $BD = 2\times BM$ $= 2\times4\sin72°$ $= 7.61$ cm (3 sf) (Shown) **<u>OR</u>** $\frac{4}{\sin\angle ODB} = \frac{BD}{\sin\angle BOD}$ $\frac{4}{\sin18°} = \frac{BD}{\sin144°}$ $\therefore BD = 7.61$ cm (3 sf) (Shown) **<u>OR</u>** $\angle BOD = (360°-72°)\div2$ ($\angle$s at a point) $= 144°$ $BD^2 = 4^2+4^2-2(4)(4)\cos144°$ $= \sqrt{32-32\cos144°}$ $BD \approx 7.60845$ $= 7.61$ cm (3 sf) (Shown)

ciii	Area of shaded segment =Area of sector DAB – Area of ΔDAB $=\frac{36°}{360°}\times\pi\times(7.60845)^2-\frac{1}{2}\times(7.60845)^2\times\sin 36°$ $=18.186-17.013$ $=1.1732$ (3 sf)
civ	Area of the badge =Area of pentagon + Area of 5 segments $=38.042\,\text{cm}^2+5(1.1732)\text{cm}^2$ $=43.9\,\text{cm}^2$ (3 sf)
4a	$T_5=\frac{26}{9}$
b	$T_n=\frac{n^2+1}{2n-1}$
c	$T_{25}-T_{24}=\frac{25^2+1}{2(25)-1}-\frac{24^2+1}{2(24)-1}$ $=\frac{626}{49}-\frac{577}{47}$ $=\frac{1149}{2303}$
5a	$\frac{\text{Vol of Cone}}{\text{Vol of Cylinder}}=\frac{\frac{1}{3}\pi r^2 x}{\pi r^2(2x)}=\frac{1}{6}$
bi	Radius of cylinder= $\sqrt{\frac{345}{\pi(8)}}\approx 3.7050=3.71\text{cm}$ (3 sf)
bii	Slant height of Cone= $\sqrt{\left(\sqrt{\frac{345}{\pi(8)}}\right)^2+4^2}$ ≈ 5.4523 Curved surface area of Cone= $\pi(3.7050)(5.4523)$ ≈ 63.463 $=63.5\,\text{cm}^2$ (3 sf)
biii	Volume of paint $=\left[63.463+2\pi(3.7050)(8)+\frac{1}{2}\times 4\pi(3.7050)^2\right]\times\frac{0.2}{10}$ $=6.72\,\text{cm}^3$ (3 sf)
6ai	$\overrightarrow{BF}=-\mathbf{a}+\mathbf{c}$
aii	$\overrightarrow{AE}=\mathbf{c}+\frac{1}{3}(\mathbf{c}-\mathbf{a})$ $=\frac{1}{3}(4\mathbf{c}-\mathbf{a})$
aiii	$\overrightarrow{OD}=\frac{1}{2}\mathbf{a}+\mathbf{c}$

b	$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$
	$= \mathbf{a} + \frac{1}{3}(4\mathbf{c} - \mathbf{a})$
	$= \frac{2}{3}\mathbf{a} + \frac{4}{3}\mathbf{c}$
	$= \frac{4}{3}\left(\frac{1}{2}\mathbf{a} + \mathbf{c}\right)$
	$\overrightarrow{OE} = \frac{4}{3}\overrightarrow{OD} \Rightarrow \overrightarrow{OE} // \overrightarrow{OD}$
	$\overrightarrow{OE}$ and $\overrightarrow{OD}$ share a common point *O* and therefore points *O*, *D* and *E* lie on the same straight line.
	
	<u>OR</u>
	
	$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$
	$= \frac{1}{2}\mathbf{a} + \frac{1}{3}\overrightarrow{BF}$
	$= \frac{1}{2}\mathbf{a} + \frac{1}{3}(-\mathbf{a} + \mathbf{c})$
	$= \frac{1}{6}\mathbf{a} + \frac{1}{3}\mathbf{c}$
	$= \frac{1}{3}\left(\frac{1}{2}\mathbf{a} + \mathbf{c}\right)$
	$\overrightarrow{DE} = \frac{1}{3}\overrightarrow{OD} \Rightarrow \overrightarrow{DE} // \overrightarrow{OD}$
	$\overrightarrow{DE}$ and $\overrightarrow{OD}$ share a common point *D* and therefore points *O*, *D* and *E* lie on the same straight line.

c	$\frac{\text{area of triangle } CDE}{\text{area of parallelogram } OABC}$ $= \frac{\text{Area of } \Delta CDE}{\text{Area of } \Delta CDO} \times \frac{\text{Area of } \Delta CDO}{\text{Area of parallelogram } OABC}$ $= \frac{1}{3} \times \frac{1}{4}$ $= \frac{1}{12}$

7a	1
b	
ci	$-7.75 \le b < -1$ [Accept also $b < -1$]
cii	On graph.
ciii	$x = -0.43,\ 0.44$
civ	$y = 2x + \frac{1}{x^2} - 4$ $3y - 5x = 4$ Sub. (1) into (2). $3\left(2x + \frac{1}{x^2} - 4\right) - 5x = 4$ $6x + \frac{3}{x^2} - 12 - 5x = 4$ $x^3 - 16x^2 + 3 = 0$ $\therefore A = -16,\ B = 3$

8ai	$Q_3 = 59\text{g}$ Total number of eggs $= 25 \times 4$ $= 100$
aii	1. $Q_{2A} = 54\text{g};\ Q_{2B} = 50\text{g}$ Masses of eggs in shop A are heavier due to its higher median value. 2. $IQR_{2A} = 59g - 49\text{g} = 10\text{g}$; $IQR_{2B} = 53\text{g} - 45\text{g} = 8\text{g}$ Masses of eggs in shop B are more consistent due to its lower IQR.
bi	Probability that the egg is cracked $= \frac{4}{80}$ $= \frac{1}{20}$
bii	Probability that both eggs are cracked $= \frac{4}{80} \times \frac{3}{79}$ $= \frac{3}{1580}$
biii	Probability that at least one egg is cracked $= 1 - \frac{76}{80} \times \frac{75}{79} \times \frac{74}{78}$ $= \frac{593}{4108}$

9a	Net rewards for Card A $= \text{Expenditure after Cashback + Sign-up Bonus}$ $= \frac{1.5}{100} \times 2000 + 50$ $= \$80$ Net rewards for Card B $= \text{Expenditure after Cashback + Sign-up Bonus}$ $= \frac{1.3}{100} \times 2000 + 60$ $= \$86$ Net rewards for Card C $= \text{Expenditure after Cashback + Sign-up Bonus}$ $= \frac{1.2}{100} \times 2000 + 70$ $= \$94$
b	Credit card bill for Card A $= \text{Expenditure after Cashback} - \text{Sign-up Bonus} + \text{Annual Fee}$ $= \left[\left(1 - \frac{1.5}{100}\right) \times 2000 \times 12\right] - 50 + 110$ $= \$23700$
c	Credit card bill after 2 years for Card A $= \text{Expenditure after Cashback + Airport Lounge Passes} - \text{Sign-up Bonus} + \text{Annual Fee}$ $= \left[\left(1 - \frac{1.5}{100}\right) \times (2000 \times 12 \times 2)\right] + 6 \times 50 - 50 + (110 \times 2)$ $= \$47750$ Credit card bill after 2 years for Card B $= \text{Expenditure after Cashback + Airport Lounge Passes} - \text{Sign-up Bonus} + \text{Annual Fee}$ $= \left[\left(1 - \frac{1.3}{100}\right) \times (2000 \times 12 \times 2)\right] + 2 \times 50 - 60 + (90 \times 2)$ $= \$47596$ Credit card bill after 2 years for Card C $= \text{Expenditure after Cashback} - \text{Sign-up Bonus} + \text{Annual Fee}$ $= \left[\left(1 - \frac{1.2}{100}\right) \times (2000 \times 12 \times 2)\right] - 70 + 250$ $= \$47604$ Credit card B has the least bill amount. Therefore, Card B is the best choice.