MATHEMATICS
PAPER 2

4052/02

16 August 2023   Wednesday   2 hours 15 minutes

2023 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATION

MARKING SCHEME

For Examiner's Use											
Qn	1	2	3	4	5	6	7	8	9	10	*Marks Deducted*
Marks											

Category	Accuracy	Symbols	Others
Question No.			

TOTAL MARKS

90

Setter: Mr Tan Lip Sing
Vetter: Mdm Cynthia Chua

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\,\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$
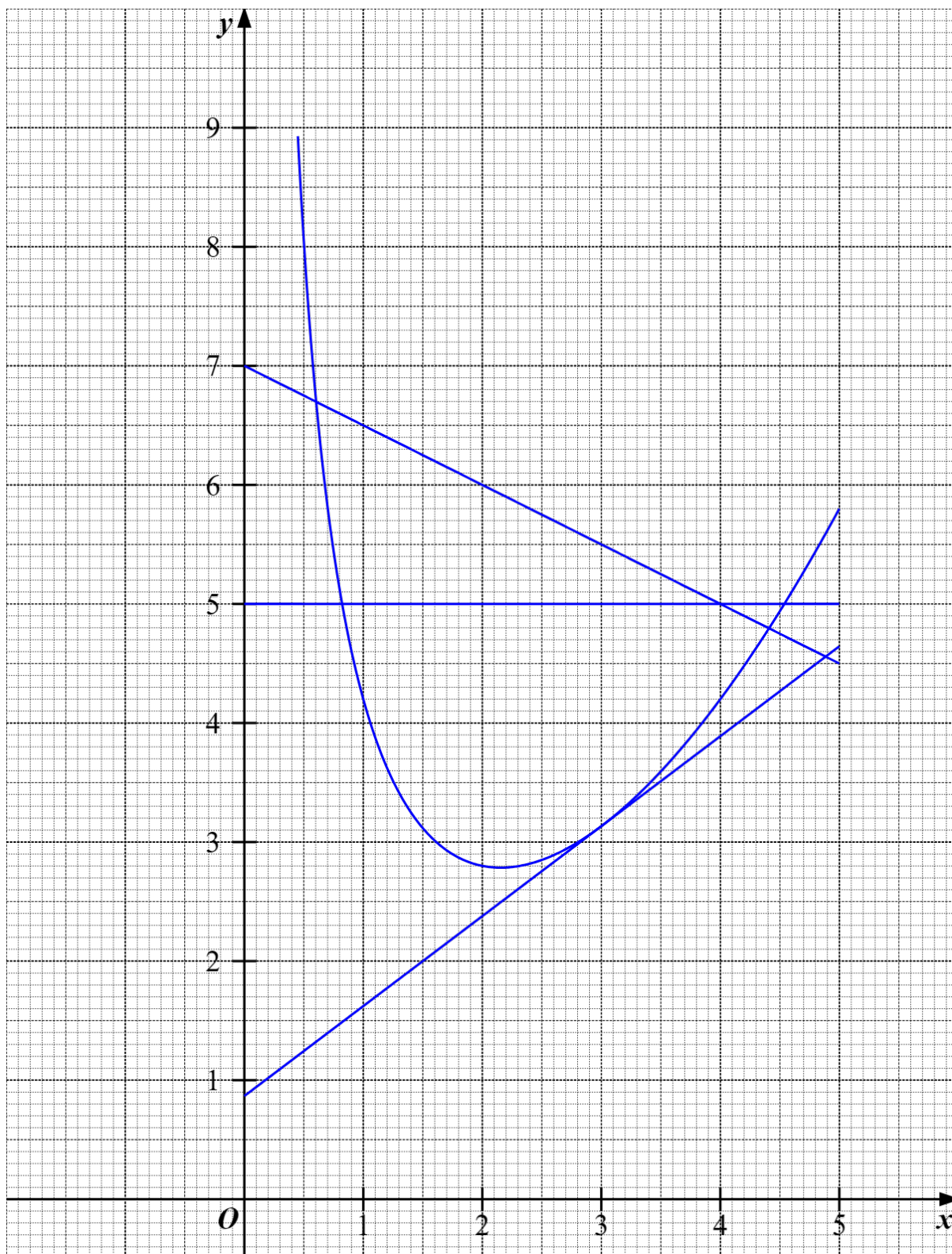
1	**(a)**	Rearrange the formula $c = \dfrac{d^2+5}{d^2-3}$ to make d the subject.
		Answer $d =$ .... [3]

		$c = \dfrac{d^2+5}{d^2-3}$ $c(d^2-3) = d^2+5$ $cd^2-3c = d^2+5$ $cd^2-d^2 = 5+3c$ $d^2(c-1) = 5+3c$ $d^2 = \dfrac{5+3c}{c-1}$ $d = \pm\sqrt{\dfrac{3c+5}{c-1}}$	M1 M1 A1

	(b)	Write as a single fraction in its simplest form $\dfrac{3}{(x-2)^2} - \dfrac{1}{2-x}$.
		Answer .... [2]

		$\dfrac{3}{(x-2)^2} - \dfrac{1}{2-x}$ $= \dfrac{3}{(x-2)^2} + \dfrac{1}{x-2}$ $= \dfrac{3+x-2}{(x-2)^2}$ $= \dfrac{x+1}{(x-2)^2}$	 M1 A1

(c)	Solve these simultaneous equations. $5x + 3y = 14$ $3x + 5y = 18$ You must show your working. *Answer* $x =$ .... or $y =$ .... [3]	
	$5x + 3y = 14$ ......(1) $3x + 5y = 18$ ......(2) (1): $y = \dfrac{14 - 5x}{3}$ ......(3) Substitute (3) into (2): $3x + 5\left(\dfrac{14 - 5x}{3}\right) = 18$ $9x + 70 - 25x = 54$ $x = 1$ $y = 3$	M1 A1 A1
(d)	Solve the equation $\dfrac{2x-1}{5x-6} = \dfrac{1}{2x-3}$. *Answer* $x =$ .... or $x =$ .... [3]	
	$\dfrac{2x-1}{5x-6} = \dfrac{1}{2x-3}$ $(2x-1)(2x-3) = 5x - 6$ $4x^2 - 6x - 2x + 3 = 5x - 6$ $4x^2 - 13x + 9 = 0$ $(4x-9)(x-1) = 0$ $x = 2\dfrac{1}{4}$ *or* $x = 1$	M1 M1 (Factorise) A1

| 2 | (a) | Before departing London for Singapore, Peter bought 3000 Singapore dollars from the bank. The exchange rate between British pounds (£) and Singapore dollars ($) was £1 = $1.71. He also had to pay the bank an additional commission fee of 1.5% for the exchange of currency.
 Calculate the total amount of pounds, inclusive of commission, he paid the bank. Give your answer correct to the nearest pound.

 Answer £ …….………………… [2] |

		Total amount before commission $= \dfrac{3000}{1.71} = £1754.385965$	M1
		Total amount inclusive of commission $= 1754.385965 \times 1.015$ $\approx £1781$	A1

| | (b) | Peter bought a laptop while he was in Singapore. He paid $664.20 inclusive of the 8% GST (Goods & Services Tax) for the laptop after getting a discount of A% on the original price. The laptop's original price is $750 before GST. |

| | | (i) | Find the GST amount paid for the laptop.

 Answer ……………………………… [2] |

			$108\% = \$664.20$ $8\% = \$664.20 \times \dfrac{8}{108} = \$49.20$	M1
			GST amount $= \$49.20$	A1

| | | (ii) | Calculate the value of A.

 Answer $A =$ …………………………% [2] |

| | | | Discounted price before GST

 $= 664.20 - 49.20 = \$615.00$

 $A = \dfrac{750 - 615}{750} \times 100\%$

 $A = \dfrac{135}{750} \times 100\% = 18\%$ | M1

 A1 |

5

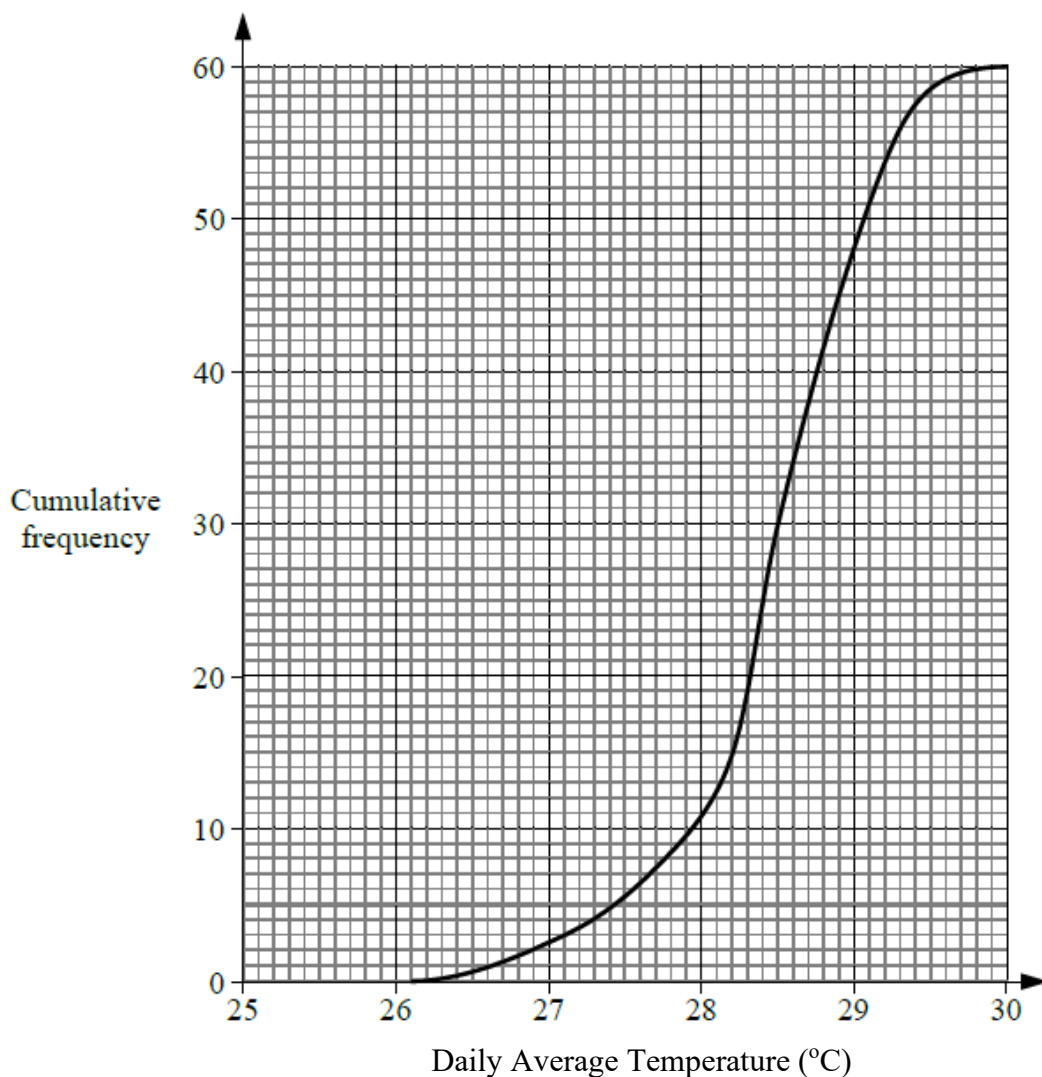|				
	(c)	Mary invests $20 000 in an endowment plan that offers 4% per year compound interest. How much interest will she receive after 10 years? Give your answer correct to the nearest cent. *Answer* $ .... [2		

Total amount after 10 years

$$= 20\,000\left(1+\frac{4}{100}\right)^{10}$$

$$= \$29604.89$$

M1

Interest received

$$= 29604.89 - 20000$$

$$= \$9604.89$$

A1

(d) A map of a province has a scale of 1 : 500 000.

(i) The length of an expressway on the map is 25 cm.
Calculate the actual length, in kilometres, of the expressway.

Answer .... km [1]

1 cm : 5 km
25 cm : 125 km
Actual length $= 125$ km

B1

(ii) The area of a reservoir is 180 km^2.
Calculate the area, in square centimetres, of the reservoir on the map.

Answer .... cm^2 [2]

(1 cm)^2 : (5 km)^2
1 cm^2 : 25 km^2

M1

Area on map $= \dfrac{180}{25} = 7.2$ cm^2

A1

3 The variables x and y are connected by the equation $y = \dfrac{x^2}{5} + \dfrac{4}{x}$.

The table below shows some values of x and the corresponding values of y correct to 2 decimal places.

x	0.5	1	1.5	2	2.5	3	4	5
y	8.05	4.20	3.12	2.80	2.85	3.13	4.20	5.80

(a) On the grid provided, draw the graph of $y = \dfrac{x^2}{5} + \dfrac{4}{x}$ for $0.5 \le x \le 5$.

Plot the points given in the table and join them with a smooth curve.   [3]

Plot all 8 points correctly.	B2 (6 or 7 points correct – B1)
Join all points with a smooth curve.	B1

(b) By drawing a tangent, find the gradient of the curve at $x = 3$.

Answer ……………………………. [2]

Draw the correct tangent line at $x = 3$. $Gradient = \dfrac{6 - 0.95}{7 - 0} \approx 0.721$ (Accept 0.7 to 0.8)	M1 A1

(c) **(i)** On the same grid, draw the line $y = 7 - \dfrac{1}{2}x$ for $0 \le x \le 5$.   [1]

Draw correct line $y = 7 - \dfrac{1}{2}x$.	B1

(ii) Write down the x-coordinates of the points where this line intersects the curve.

Answer $x =$ …………………….. or …………………….. [2]

$x = 0.6$ *or* $x = 4.4$ $(0.55 - 0.65)$ $(4.35 - 4.45)$	B1, B1

7

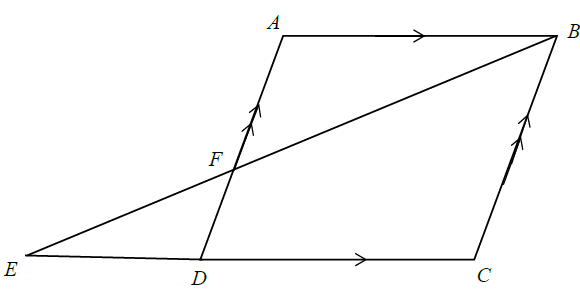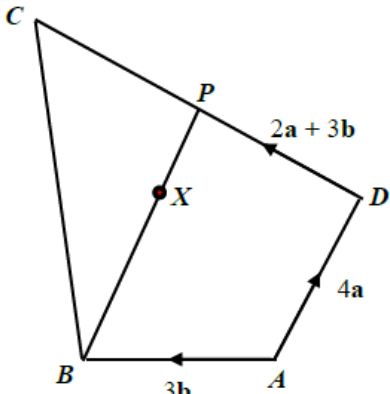|		(iii)	Find the equation, in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in **(c)(ii)**.	
			Answer ………………………………… [2]	
			$$\frac{x^2}{5} + \frac{4}{x} = 7 - \frac{x}{2}$$ $$2x^3 + 40 = 70x - 5x^2$$ $$2x^3 + 5x^2 - 70x + 40 = 0$$	M1 A1
	(d)		Use your graph to find the values of x in the range $0 \le x \le 5$ for which $$0.2x^2 + \frac{4}{x} - 2 = 3.$$ *Answer* $x = $ …………………….. or …………………….. [2]	
			$$0.2x^2 + \frac{4}{x} - 2 = 3$$ $$\frac{x^2}{5} + \frac{4}{x} = 5$$ Draw the line $y = 5$, from the graph, $x = 0.8 \quad or \quad x = 4.55$ $(0.75 - 0.85) \quad (4.5 - 4.6)$	 B1, B1

8

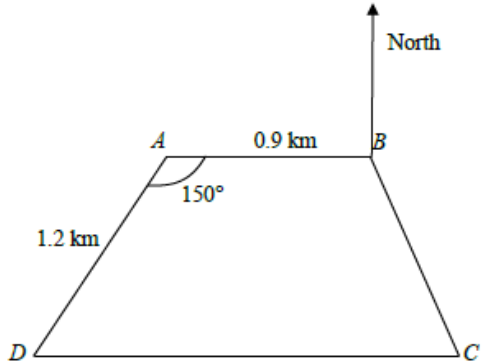| 4	(a)	The daily average temperature at Town *A* was recorded for 60 days. The cumulative frequency curve below shows the distribution of the temperatures.

Cumulative frequency

Daily Average Temperature (°C)

		(i)	Use the curve to estimate **(a)** the median temperature,
			Answer .... °C [1]
			28.5° B1
			(b) the interquartile range of the temperatures,
			Answer .... °C [2]
			$28.9° - 28.2° = 0.7°$ M1, A1
			(c) the number of days that Town *A* had temperatures above $29°C$.
			Answer .... days [1]
			$60 - 48 = 12 \ days$ B1

		(ii)	The daily average temperature at Town *B* was recorded for the same period. The interquartile range of the temperatures at Town *B* is 1.5°C. Use this information to comment on one difference between the temperature at Town *A* and at Town *B*. ……………………………………………………………………………………… [1]	
			The temperatures at Town B have a larger spread than the temperatures at Town A. OR The temperatures at Town B were less consistent than the temperatures at Town A.	B1
	(b)		Box *A* contains 6 red cards, 4 blue cards and 2 green cards. Box *B* contains 3 red cards and 5 blue cards. A card is drawn at random from Box *A* and put into Box *B*. Next, a card is drawn at random from Box *B*. Find, as a fraction in its simplest form, the probability that	
		(i)	two green cards are drawn, *Answer* …….………………… [1]	
			$P = \dfrac{2}{9} \times \dfrac{1}{9} = \dfrac{1}{54}$	B1
		(ii)	neither of the cards is green, *Answer* …………………………… [1]	
			$P = \dfrac{10}{12} \times 1 = \dfrac{5}{6}$	B1
		(iii)	the two cards are of the same colour. *Answer* ………………………… [2]	
			$P = P(RR) + P(BB) + P(GG)$ $= \dfrac{6}{12} \times \dfrac{4}{9} + \dfrac{4}{12} \times \dfrac{6}{9} + \dfrac{1}{54}$ $= \dfrac{25}{54}$	M1 A1

5	(a)	The diagram shows a parallelogram *ABCD* with *CD* produced to *E*. *F* is the point of intersection of *BE* and *AD*.

		(i)	Show that triangle *BAF* and triangle *EDF* are similar. Give a reason for each statement you make. ……………………………………………………………………… [2]

In $\triangle BAF$ and $\triangle EDF$, $\angle BAF = \angle EDF$ (Alternate angles, $AB \,/\!/\, ED$) $\angle AFB = \angle DFE$ (Vertically opposite angles) $\therefore \triangle BAF$ and $\triangle EDF$ are similar. (AA Similarity Test)	M1 (at least one correct reason) A1 (with correct reason)

		(ii)	State another triangle that is similar to *BAF* and *EDF*. *Answer* Triangle ……………………………… [1]

Triangle *ECB*.	B1

		(iii)	The ratio $ED:DC = 2:3$. Find the ratio $BC:AF$. *Answer* …………… : ……………… [1]

5 : 3	B1

		(iv)	Given that the area of triangle *EDF* is 9.5 cm², find the area of triangle *BAF*. *Answer* ……………………………… cm² [2]

$\dfrac{Area\ of\ \triangle BAF}{Area\ of\ \triangle EDF} = \left(\dfrac{3}{2}\right)^2$ $Area\ of\ \triangle BAF = \dfrac{9}{4} \times 9.5 = 21.375$ cm²	M1 A1

(b) The diagram below shows a cone of height 50 cm.

The volume of the liquid in the cone is $\dfrac{3}{4}$ of the volume of the cone.

Calculate the depth, h cm, of the liquid.

50

h

Answer ……………………………. cm [2]

$$\frac{Volume\ of\ liquid}{Volume\ of\ cone} = \left(\frac{h}{50}\right)^3$$

M1

$$\frac{3}{4} = \frac{h^3}{125000}$$

$$h^3 = 93750$$

$$h \approx 45.4 \text{ cm}$$

A1

6		In the diagram below, *P* is a point on *DC*, such that $DC = 2DP$ and *X* is a point on *BP* such that $3BX = 2BP$. It is given that $\overrightarrow{AD} = 4\mathbf{a}$, $\overrightarrow{AB} = 3\mathbf{b}$, and $\overrightarrow{DP} = 2\mathbf{a} + 3\mathbf{b}$.		
	(a)	Express, as simply as possible, in terms of **a** and/or **b**,		
		(i)	$\overrightarrow{BP}$, *Answer* ……………………………… [1]	
			$\overrightarrow{BP} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DP}$ $\qquad = -3\mathbf{b} + 4\mathbf{a} + 2\mathbf{a} + 3\mathbf{b}$ $\qquad = 6\mathbf{a}$	B1
		(ii)	$\overrightarrow{AX}$, *Answer* ……………………………… [2]	
			$\overrightarrow{BX} = \dfrac{2}{3}\overrightarrow{BP} = 4\mathbf{a}$ $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BX}$ $\qquad = 3\mathbf{b} + 4\mathbf{a}$	M1 A1
		(iii)	$\overrightarrow{AC}$. *Answer* ……………………………… [2]	
			$\overrightarrow{DC} = 2\overrightarrow{DP} = 4\mathbf{a} + 6\mathbf{b}$ $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ $\qquad = 4\mathbf{a} + 4\mathbf{a} + 6\mathbf{b}$ $\qquad = 8\mathbf{a} + 6\mathbf{b} \quad or \quad 2(4\mathbf{a} + 3\mathbf{b})$	M1 A1

	(b)	Explain whether the points *A*, *X* and *C* lie on the same straight line. .... [2]
		$\overrightarrow{AC} = 2(4\mathbf{a} + 3\mathbf{b}) = 2\overrightarrow{AX}$ M1 *AX* is parallel to *AC* and since *A* is a common point, *A, X* and *C* lie on the same straight line. A1
	(c)	Given that the area of triangle $BCP = 24\,\text{cm}^2$, find the area of triangle CXP. *Answer* ....cm^2 [2]
		Area of $\triangle CXP = \dfrac{1}{3}$ *Area of* $\triangle BCP$ $= \dfrac{1}{3}(24)$ M1 $= 8\ cm^2$ A1

7	The diagram shows four towns A, B, C and D on a piece of horizontal land.

ABCD is a trapezium. $AB = 0.9$ km, $AD = 1.2$ km and angle $BAD = 150°$.

(a) Calculate the distance between Town B and Town D.

Answer …….…………… km [3]

$(BD)^2 = (1.2)^2 + (0.9)^2 - 2(1.2)(0.9)\cos150°$	M2 (all correct)
$\qquad = 4.12061$	M1 (2 out of 3 correct)
$BD = 2.0299 \approx 2.03$ km	A1

(b) Calculate the value of angle BDC.

Answer ……………………………° [2]

$\dfrac{\sin\angle BDA}{0.9} = \dfrac{\sin150°}{2.0299}$	M1
$\sin\angle BDA = 0.22168$	
$\angle BDA = 12.808° \approx 12.8°$	
Since $ABCD$ is a trapezium,	
$\angle ADC = 180° - 150° = 30°$	
$\angle BDC = 30° - 12.808°$	
$\qquad = 17.192° \approx 17.2°$	A1

(c) Calculate the bearing of D from B.

Answer ……………………………° [2]

$\angle ABD = \angle BDC = 17.192°$	
Bearing of D from B	
$= 270 - 17.192°$	M1
$\approx 252.8°$	A1

(d) A tower is standing at Town *B*.
The greatest angle of elevation of the top of the tower, *T*, from the path *CD* is 18°. Find the height of the tower in metres.

Answer ……………………………. m [3]

Let the shortest distance from *B* to *CD* be *d* km.

$$\sin \angle BDC = \frac{d}{2.0299}$$

$$d = 2.0299 \sin 17.192°$$

$$= 0.599987 \ km$$

M1

Let *h* be the height of the tower.

$$\tan 18° = \frac{h}{0.599987}$$

$$h = 0.599987 \tan 18°$$

$$= 0.194947 \ km$$

$$\approx 195 \ m$$

M1

A1

8		James bought some essential oil for $720 at x per litre.	
	(a)	Write an expression, in terms of x, for the number of litres of essential oil he bought.	
		Answer …….…………… litres [1]	
		$\dfrac{720}{x}$ litres	B1
	(b)	Due to a leakage in the container, 5 litres of essential oil was lost. James sold the remaining essential oil at $2 per litre more than what he had paid for. Write an expression, in terms of x, for the amount of money he received from the sale of essential oil. *Answer* $ …………..………………………… [1]	
		$\$\left(\dfrac{720}{x}-5\right)(x+2)$	B1
	(c)	Given that James made a profit of $100, write down an equation in x to represent this information and show that it reduces to $x^2+22x-288=0$. [3]	
		$\left(\dfrac{720}{x}-5\right)(x+2)-720=100$	M1
		$720+\dfrac{1440}{x}-5x-10-720-100=0$	M1
		$\dfrac{1440}{x}-5x-110=0$	
		$-5x^2-110x+1440=0$	
		$x^2+22x-288=0$	AG1
	(d)	Solve the equation $x^2+22x-288=0$. *Answer* $x=$……….…………. or $x=$……………… [3]	
		$x=\dfrac{-22\pm\sqrt{22^2-4(1)(-288)}}{2(1)}$	M1
		$=\dfrac{-22\pm\sqrt{1636}}{2}$	
		$=9.2237 \quad or \quad -31.2237$	
		$\approx 9.22 \ or \ -31.2$	A1, A1

	(e)	Find, to the nearest litre, the amount of essential oil James sold.
		Answer ……………………………. litres [2]

		$\dfrac{720}{9.2237} - 5 = 73.09 \approx 73$ litres	M1, A1

| 9 | The table below shows the Income Tax Rate in Singapore. |

Table 1: Income Tax Rate

Chargeable Income	Rate (%)	Gross Tax Payable ($)
On the first $120,000	-	7,950
On the next $40,000	15	6,000
On the first $160,000	-	13,950
On the next $40,000	18	7,200
On the first $200,000	-	21,150
On the next $40,000	19	7,600
On the first $240,000	-	28,750
On the next $40,000	19.5	7,800

(a) Henry enjoyed a total tax relief of $15 000 and paid $14 130 of income tax for the year of assessment 2022. Calculate his annual income in 2022.

[Annual income = Chargeable income + tax relief]

Answer $ ……………………………. [2]

Let the chargeable income be $(160\,000 + A)$ $14130 = 13950 + (A \times 18\%)$ $A = \$1000$	M1
Annual income $= \$(160000 + 1000 + 15000)$ $\qquad = \$176\,000$	A1

Henry recently got a pay rise and his income is now $15 500 per month.
He is keen to buy a private condominium which is priced at $1 200 000. To afford this condominium, he needs to apply for a bank loan of $800 000.

(b) The maximum duration of a housing loan for private properties is up to 35 years or 65 years of age, whichever is lower.
Given that Henry is 45 years old, find the maximum number of years Henry can loan from the bank.

Answer ………………………… years [1]

Maximum number is $65 - 45 = 20$ years	B1

(c) Henry decides to apply for a loan for the maximum duration allowed for his age. The loan from the bank is subject to a simple interest of 3.5% per annum.

The government introduced the Total Debt Servicing Ratio (TDSR) to prevent individuals from over-borrowing.

Information about TDSR

- Total Debt Servicing Ratio = $\dfrac{\text{Total monthly debt repayment}}{\text{Monthly income}}$

- Total monthly debt repayment includes repayments for car loans, personal loans, credit card expenditure, home loans and other loans.

- The maximum TDSR allowed is 55%.

His current monthly debt repayment is shown in the table below:

Type	Amount ($)
Car loan	1000
Credit card Expenditure	1000
Personal loans	1000

By considering the TDSR ratio, will the bank approve his loan request?
Justify your answer and show your calculations clearly.

…………………………………………………………………………………….. [7]

		Interest on housing loan for 1 year $= \$800\,000 \times 0.035 = \$28\,000$	M1
		Interest on housing loan for 20 years $= \$28\,000 \times 20 = \$560\,000$	M1
		Total debt (loan + interest) $= \$(800\,000 + 560\,000) = \$1360\,000$	M1
		Monthly debt repayment for housing loan over 20 years $= \$1360\,000 \div (12 \times 20) = \$5666.67$	M1
		Maximum debt allowable per month under TDSR $= \$15\,500 \times 0.55 = \$8525$	M1
		Henry's total monthly debt $= \$(5666.67 + 3000) = \$8666.67$	M1
		Since Henry's monthly debt of \$8666.67 has exceeded the maximum debt allowable per month (\$8525) under the TDSR, the bank will not approve his loan.	A1