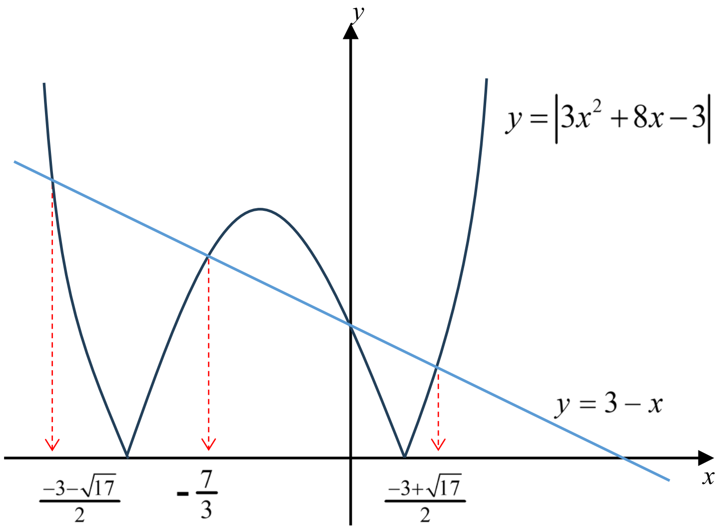
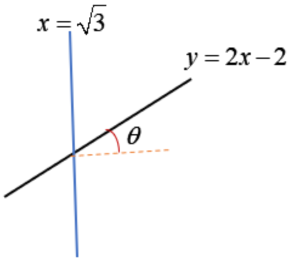
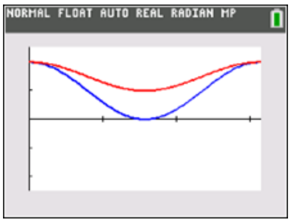
2023 JC2 H2MA Prelim Examination Paper 1 (Solutions)

1	$\mathrm{f}(x)=\ln\left(\dfrac{a}{x+3a}\right)$ $=\ln a-\ln(x+3a)$ $y=\ln x\xrightarrow{A}y=\ln(x+3a)$ $\xrightarrow{B}y=-\ln(x+3a)\xrightarrow{C}y=\ln a-\ln(x+3a)$ Sequence of transformations: A: A translation of $3a$ units in the negative x-direction B: A reflection in the x-axis C: A translation of $\ln a$ units in the positive y-direction OR $y=\ln x\xrightarrow{(1)}y=-\ln x$ $\xrightarrow{(2)}y=-\ln(x+3a)\xrightarrow{(3)}y=\ln a-\ln(x+3a)$ Sequence of transformations: (1): A reflection in the x-axis (2): A translation of $3a$ units in the negative x-direction (3): A translation of $\ln a$ units in the positive y-direction
2a	Since $2+3\mathrm{i}$ is a root and all the coefficients are real, $2-3\mathrm{i}$ is also a root. A quadratic factor is: $[z-(2+3\mathrm{i})][z-(2-3\mathrm{i})]$ $=(z-2)^2-(3\mathrm{i})^2$ $=(z^2-4z+4)+9$ $=z^2-4z+13$ $z^3-3z^2+kz+13=(z^2-4z+13)(z+1)$ Comparing coefficient of z: $k=13-4=9$ The other roots are $z=2-3\mathrm{i}$ and $z=-1$.
2b	Let $z=\mathrm{i}w$, then we get $(\mathrm{i}w)^3-3(\mathrm{i}w)^2+k(\mathrm{i}w)+13=0$ $\Rightarrow\quad -\mathrm{i}w^3+3w^2+k\mathrm{i}w+13=0$ Replace z with $\mathrm{i}w$, $\mathrm{i}w=2+3\mathrm{i}$, $\quad\mathrm{i}w=2-3\mathrm{i}\quad$ and $\quad\mathrm{i}w=-1$ $w=\dfrac{2+3\mathrm{i}}{\mathrm{i}}=3-2\mathrm{i}$, $\quad w=\dfrac{2-3\mathrm{i}}{\mathrm{i}}=-3-2\mathrm{i}\quad$ and $\quad w=-\dfrac{1}{\mathrm{i}}=\mathrm{i}$

3a	$\lvert 3x^2+8x-3\rvert = 3-x$ ------ (*) $3x^2+8x-3=3-x$ or $-\left(3x^2+8x-3\right)=3-x$ $3x^2+9x-6=0$ or $3x^2+7x=0$ $x^2+3x-2=0$ or $x(3x+7)=0$ $x=\dfrac{-3\pm\sqrt{9-4(1)(-2)}}{2}$ $x=0$ or $-\dfrac{7}{3}$ $=\dfrac{-3\pm\sqrt{17}}{2}$ (Alternative method: Squaring both sides and so on)
3b	 As seen from the graphs, for $\lvert 3x^2+8x-3\rvert \geq 3-x$ $x\leq\dfrac{-3-\sqrt{17}}{2}$ or $-\dfrac{7}{3}\leq x\leq 0$ or $x\geq\dfrac{-3+\sqrt{17}}{2}$
4a	$x=\sqrt{3}\sin 2t \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}t}=2\sqrt{3}\cos 2t$ $y=4\cos^2 t \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}t}=8\cos t(-\sin t)=-4\sin 2t$ $\therefore \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-4\sin 2t}{2\sqrt{3}\cos 2t}$ $=\dfrac{-2}{\sqrt{3}}\tan 2t$ $=-\dfrac{2\sqrt{3}}{3}\tan 2t \equiv k\sqrt{3}\tan 2t$ where $k=-\dfrac{2}{3}$.

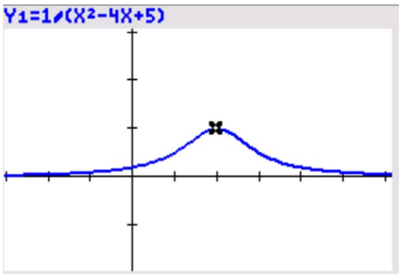
4b	When $t=\frac{\pi}{4},\ \ x=\sqrt{3}\sin 2\left(\frac{\pi}{4}\right)=\sqrt{3},\ \ y=4\cos^2\left(\frac{\pi}{4}\right)=2,$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2\sqrt{3}}{3}\tan\frac{\pi}{2}$, which is undefined. $\Rightarrow$The tangent is parallel to the y-axis. Hence the equation of the tangent at the point where $t=\frac{\pi}{4}$ is $x=\sqrt{3}$. When $t=\frac{\pi}{3},\ x=\sqrt{3}\sin 2\left(\frac{\pi}{3}\right)=\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)=\frac{3}{2},$ $y=4\cos^2\left(\frac{\pi}{3}\right)=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2\sqrt{3}}{3}\tan\frac{2\pi}{3}=-\frac{2\sqrt{3}}{3}\left(-\sqrt{3}\right)=2$ OR From GC, when $t=\frac{\pi}{3},\ x=\frac{3}{2},\ \ y=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=2.$ Equation of the tangent is: $y-1=2\left(x-\frac{3}{2}\right)$ $y=2x-2$
4c	Let θ be the angle in which the tangent $y=2x-2$ makes with the positive x-axis. Then $\tan\theta=2$. Hence, acute angle between the 2 tangents $=90°-\theta$ $=90°-\tan^{-1}2$ $\approx 26.6°$ 
5a	Sum of the all the terms after the nth term $=S_\infty-S_n=\frac{a}{1-r}-\frac{a\left(1-r^n\right)}{1-r}$ $=\frac{ar^n}{1-r}$ Given $S_\infty-S_n=2u_n$, therefore $\frac{ar^n}{1-r}=2ar^{n-1}$ $r=2(1-r)$ $r=\frac{2}{3}$

	Hence $S_\infty = \dfrac{a}{1-r} = \dfrac{a}{1-\frac{2}{3}} = 3a$ (Shown)
5bi	Total number of integers in the first (r–1)th brackets is $1+2+3+...+(r-1) = \dfrac{r-1}{2}\left(1+(r-1)\right) = \dfrac{r(r-1)}{2}$ Hence, first integer in the rth bracket $= \dfrac{r(r-1)}{2}+1 = \dfrac{r^2-r+2}{2}$ Last integer in the rth bracket $= \dfrac{r^2-r+2}{2}+(r-1)$ $= \dfrac{r^2-r+2+2r-2}{2}$ $= \dfrac{r^2+r}{2}$ <u>Alternative method</u>: Last integer in the rth bracket = First integer in the (r+1)th bracket minus 1 $= \left[\dfrac{(r+1)(r)}{2}+1\right]-1 = \dfrac{r^2+r}{2}$
5bii	There are r integers in the rth bracket. First integer in the rth bracket = $\dfrac{r^2-r+2}{2}$ Last integer in the rth bracket $= \dfrac{r^2+r}{2}$ Sum of all the integers in the rth bracket $= \dfrac{r}{2}\left(\dfrac{r^2-r+2}{2}+\dfrac{r^2+r}{2}\right) = \dfrac{r}{2}\left(\dfrac{2r^2+2}{2}\right)$ $= \dfrac{1}{2}r\left(1+r^2\right)$ (Shown)

6a	Required area $=\int_0^{\pi}\left(1+\cos^2 x\right)-\left(1+\cos 2x\right)\mathrm{d}x$  $=\int_0^{\pi}\frac{\cos 2x+1}{2}-\cos 2x\,\mathrm{d}x$ $=\int_0^{\pi}\frac{1}{2}-\frac{\cos 2x}{2}\mathrm{d}x$ $=\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]_0^{\pi}$ $=\frac{\pi}{2}\text{units}^2$
6b	$\cos^4 x=\left(\cos^2 x\right)^2$ $=\left(\frac{1+\cos 2x}{2}\right)^2$ $=\frac{1}{4}\left(1+2\cos 2x+\cos^2 2x\right)$ $=\frac{1}{4}\left(1+2\cos 2x+\frac{1+\cos 4x}{2}\right)$ $=\frac{1}{8}\left(3+4\cos 2x+\cos 4x\right)$ (shown)
6c	Required volume $=\pi\left[\int_0^{\pi}\left(1+\cos^2 x\right)^2\mathrm{d}x-\int_0^{\pi}\left(1+\cos 2x\right)^2\mathrm{d}x\right]$ $=\pi\int_0^{\pi}1+2\cos^2 x+\cos^4 x-1-2\cos 2x-\cos^2 2x\,\mathrm{d}x$ $=\pi\int_0^{\pi}\left(1+\cos 2x\right)+\frac{1}{8}\left(3+4\cos 2x+\cos 4x\right)-2\cos 2x-\frac{1}{2}\left(1+\cos 4x\right)\mathrm{d}x$ $=\pi\int_0^{\pi}-\frac{3}{8}\cos 4x-\frac{1}{2}\cos 2x+\frac{7}{8}\mathrm{d}x$ $=\pi\left[\frac{7}{8}x-\frac{1}{4}\sin 2x-\frac{3}{32}\sin 4x\right]_0^{\pi}$ $=\frac{7\pi^2}{8}\text{units}^3$

7ai	$v=4x-y\Rightarrow\dfrac{dv}{dx}=4-\dfrac{dy}{dx}$ $4-\dfrac{dv}{dx}=\left(v+2\right)^2$ $\dfrac{dv}{dx}=4-\left(v+2\right)^2$
7aii	$\dfrac{dv}{dx}=4-\left(v+2\right)^2$ $\displaystyle\int\frac{1}{4-\left(v+2\right)^2}\,dv=\int 1\,dx$ $\dfrac{1}{2(2)}\ln\left\|\dfrac{2+(v+2)}{2-(v+2)}\right\|=x+C$ $\ln\left\|\dfrac{v+4}{-v}\right\|=4x+4C$ $1+\dfrac{4}{v}=\pm e^{4x+4C}$ $1+\dfrac{4}{v}=Ae^{4x}$, where $A=\pm e^{4C}$ When $x=0,\ y=-2,\ \therefore v=0-\left(-2\right)=2$. Hence $1+\dfrac{4}{2}=Ae^{0}\Rightarrow A=3$ $1+\dfrac{4}{v}=3e^{4x}$ $v=\dfrac{4}{3e^{4x}-1}$ $4x-y=\dfrac{4}{3e^{4x}-1}$ $y=4x-\dfrac{4}{3e^{4x}-1}$
7bi	$\dfrac{d^2y}{dx^2}=e^{-2x}+\sqrt{x}$ $\dfrac{dy}{dx}=-\dfrac{1}{2}e^{-2x}+\dfrac{2}{3}x^{\frac{3}{2}}+C$ $y=\dfrac{1}{4}e^{-2x}+\dfrac{4}{15}x^{\frac{5}{2}}+Cx+D$
7bii	When $x=0$, $y=0$. $0=\dfrac{1}{4}+D\Rightarrow D=-\dfrac{1}{4}$ When $x=0$, $\dfrac{dy}{dx}=2$. $2=-\dfrac{1}{2}+C\Rightarrow C=\dfrac{5}{2}$ Particular solution is $y=\dfrac{1}{4}e^{-2x}+\dfrac{4}{15}x^{\frac{5}{2}}+\dfrac{5}{2}x-\dfrac{1}{4}$.

8a



$$f(x)=\frac{1}{x^2-4ax+5a^2}$$

$$f'(x)=\frac{2x-4a}{\left(x^2-4ax+5a^2\right)^2}$$

For maximum point,

$$\frac{2x-4a}{\left(x^2-4ax+5a^2\right)^2}=0$$

$$2x-4a=0$$

$$x=2a$$

Hence **largest** $k=2a$

Alternative method:

$$x^2-4ax+5a^2$$

$$=\left(x-2a\right)^2-\left(2a\right)^2+5a^2$$

$$=\left(x-2a\right)^2+a^2$$

Since $x=2a$ gives the minimum value of $x^2-4ax+5a^2$,

it gives the maximum value for $f(x)=\dfrac{1}{x^2-4ax+5a^2}$.

Hence **largest** $k=2a$

8b

When $x=a$,

$$f(a)=\frac{1}{a^2-4a(a)+5a^2}=\frac{1}{2a^2}$$

From the graph, $0<f(x)<\dfrac{1}{2a^2}$

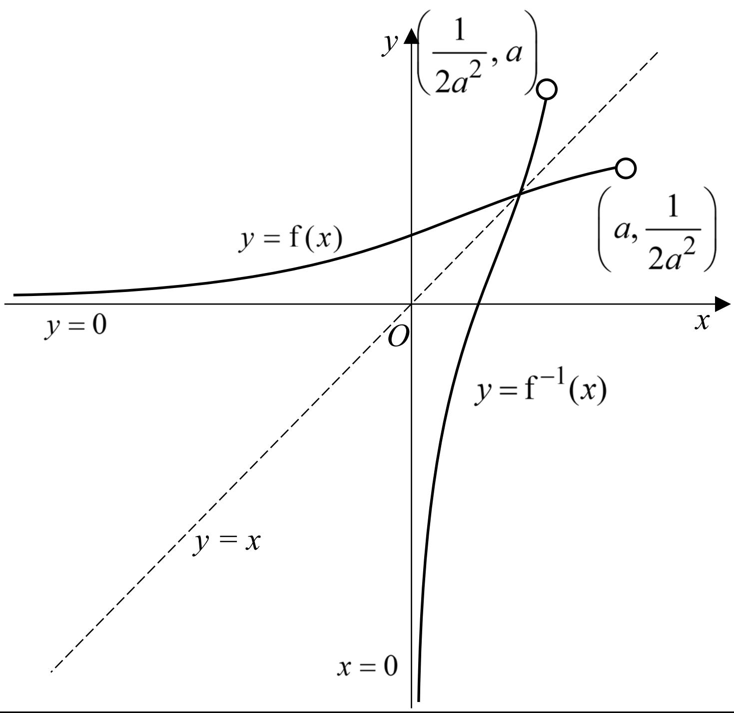
$$\therefore\ R_f=\left(0,\frac{1}{2a^2}\right)$$

To show f^2 exists, $R_f\subseteq D_f$.

Since, $a>1\Rightarrow\dfrac{1}{a}<1\Rightarrow\dfrac{1}{2a^2}<\dfrac{1}{2}\Rightarrow\dfrac{1}{2a^2}<a$,

$$\therefore R_f=\left(0,\frac{1}{2a^2}\right)\subseteq D_f=\left(-\infty,a\right).$$

Thus f^2 exists. (Shown)

8c	Let $y=\dfrac{1}{x^2-4ax+5a^2}$ $y=\dfrac{1}{\left(x-2a\right)^2+a^2}$ $\left(x-2a\right)^2+a^2=\dfrac{1}{y}$ $x-2a=\pm\sqrt{\dfrac{1}{y}-a^2}$ $x=2a\pm\sqrt{\dfrac{1}{y}-a^2}$ Since $x<a$, hence $x=2a-\sqrt{\dfrac{1}{y}-a^2}$ Hence $\mathrm{f}^{-1}(x)=2a-\sqrt{\dfrac{1}{x}-a^2}$
8d	

9ai	$\int x^2\sqrt{x^3+1}\,\mathrm{d}x=\dfrac{1}{3}\int 3x^2\left(x^3+1\right)^{\frac{1}{2}}\,\mathrm{d}x$ $=\dfrac{1}{3}\left(\dfrac{\left(x^3+1\right)^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$ $=\dfrac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}+C$

9aii	$\int_{-1}^{0} x^5\sqrt{x^3+1}\,\mathrm{d}x$ $=\int_{-1}^{0} x^3\cdot x^2\sqrt{x^3+1}\,\mathrm{d}x$ $=\left[\frac{2}{9}x^3\left(x^3+1\right)^{\frac{3}{2}}\right]_{-1}^{0}-\int_{-1}^{0}\left(3x^2\right)\cdot\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}\,\mathrm{d}x$ $=\left[0-\frac{2}{9}(-1)^3\left((-1)^3+1\right)^{\frac{3}{2}}\right]-\frac{2}{9}\int_{-1}^{0}\left(3x^2\right)\cdot\left(x^3+1\right)^{\frac{3}{2}}\,\mathrm{d}x$ $=0-\frac{2}{9}\left[\frac{\left(x^3+1\right)^{\frac{5}{2}}}{\frac{5}{2}}\right]_{-1}^{0}$ $=-\frac{4}{45}(1-0)$ $=-\frac{4}{45}$
9b	$u=1+\mathrm{e}^x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x=u-1\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u-1}$ $\int \mathrm{e}^{2x}\sqrt{\mathrm{e}^x+1}\,\mathrm{d}x=\int(u-1)^2\sqrt{u}\,\frac{1}{u-1}\,\mathrm{d}u$ $=\int(u-1)u^{\frac{1}{2}}\,\mathrm{d}u$ $=\int u^{\frac{3}{2}}-u^{\frac{1}{2}}\,\mathrm{d}u$ $=\frac{2}{5}u^{\frac{5}{2}}-\frac{2}{3}u^{\frac{3}{2}}+C$ $=\frac{2}{5}\left(\sqrt{1+\mathrm{e}^x}\right)^5-\frac{2}{3}\left(\sqrt{1+\mathrm{e}^x}\right)^3+C$
9c	$\int(2+\tan 5x)\cos 5x\sin 3x\,\mathrm{d}x\,\mathrm{d}x$ $=\int\left(2\cos 5x\sin 3x+\cos 5x\sin 3x\frac{\sin 5x}{\cos 5x}\right)\mathrm{d}x$ $=\int 2\cos 5x\sin 3x\,\mathrm{d}x+\int\sin 5x\sin 3x\,\mathrm{d}x$ $=\int(\sin 8x-\sin 2x)\,\mathrm{d}x-\frac{1}{2}\int -2\sin 5x\sin 3x\,\mathrm{d}x$ $=\left(-\frac{\cos 8x}{8}+\frac{\cos 2x}{2}\right)-\frac{1}{2}\int(\cos 8x-\cos 2x)\,\mathrm{d}x$ $=\left(-\frac{\cos 8x}{8}+\frac{\cos 2x}{2}\right)-\frac{1}{2}\left(\frac{\sin 8x}{8}-\frac{\sin 2x}{2}\right)+C$ $=\frac{1}{16}(8\cos 2x+4\sin 2x-2\cos 8x-\sin 8x)+C$

10a	Let A, B and C be the points $(6, 9, 3)$, $(-2, 13, 1)$ and $(4, 10, 0)$. $\overrightarrow{AB}=\begin{pmatrix}-2\\13\\1\end{pmatrix}-\begin{pmatrix}6\\9\\3\end{pmatrix}=\begin{pmatrix}-8\\4\\-2\end{pmatrix}=-2\begin{pmatrix}4\\-2\\1\end{pmatrix}$ $\overrightarrow{AC}=\begin{pmatrix}4\\10\\0\end{pmatrix}-\begin{pmatrix}6\\9\\3\end{pmatrix}=\begin{pmatrix}-2\\1\\-3\end{pmatrix}$ $\underset{\sim}{n}=\begin{pmatrix}4\\-2\\1\end{pmatrix}\times\begin{pmatrix}-2\\1\\-3\end{pmatrix}=\begin{pmatrix}6-1\\-(-12-(-2))\\4-4\end{pmatrix}=\begin{pmatrix}5\\10\\0\end{pmatrix}=5\begin{pmatrix}1\\2\\0\end{pmatrix}$ $\pi:\underset{\sim}{r}\bullet\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}6\\9\\3\end{pmatrix}\bullet\begin{pmatrix}1\\2\\0\end{pmatrix}=24$ $x+2y=24$ (shown)
10b	Let l represent the path of the laser beam. $\underset{\sim}{d}=\begin{pmatrix}2\\-1\\2\end{pmatrix}-\begin{pmatrix}0\\-3\\1\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}$ $l:\underset{\sim}{r}=\begin{pmatrix}0\\-3\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\1\end{pmatrix},\ \lambda\in\mathbb{R}$ Let θ be the angle between the laser beam and the reflective shield. $\theta=\sin^{-1}\dfrac{\left|\begin{pmatrix}2\\2\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\0\end{pmatrix}\right|}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+2^2}}$ $=63.4°$ (1 d.p.)
10c	Let P be the point of intersection between the laser beam and reflective shield.

	$\left[\begin{pmatrix}0\\-3\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\1\end{pmatrix}\right]\bullet\begin{pmatrix}1\\2\\0\end{pmatrix}=24$ $2\lambda-6+4\lambda=24$ $6\lambda=30$ $\lambda=5$ $\overrightarrow{OP}=\begin{pmatrix}0\\-3\\1\end{pmatrix}+5\begin{pmatrix}2\\2\\1\end{pmatrix}=\begin{pmatrix}10\\7\\6\end{pmatrix}$
10d	Let Q be the point $(0,\ -3,\ 1)$ and the foot of perpendicular from Q to the reflective shield be N. $l_{QN}:\underset{\sim}{r}=\begin{pmatrix}0\\-3\\1\end{pmatrix}+\mu\begin{pmatrix}1\\2\\0\end{pmatrix},\ \mu\in\mathbb{R}$ $\left[\begin{pmatrix}0\\-3\\1\end{pmatrix}+\mu\begin{pmatrix}1\\2\\0\end{pmatrix}\right]\bullet\begin{pmatrix}1\\2\\0\end{pmatrix}=24$ $\mu-6+4\mu=24$ $5\mu=30$ $\mu=6$ $\overrightarrow{ON}=\begin{pmatrix}0\\-3\\1\end{pmatrix}+6\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}6\\9\\1\end{pmatrix}$ Let Q' be the reflected point of Q on the shield. Using mid-point theorem, $\overrightarrow{ON}=\dfrac{\overrightarrow{OQ}+\overrightarrow{OQ'}}{2}$ $\begin{pmatrix}6\\9\\1\end{pmatrix}=\dfrac{\begin{pmatrix}0\\-3\\1\end{pmatrix}+\overrightarrow{OQ'}}{2}$ $\overrightarrow{OQ'}=\begin{pmatrix}12\\21\\1\end{pmatrix}$

$$\overrightarrow{PQ'}=\begin{pmatrix}12\\21\\1\end{pmatrix}-\begin{pmatrix}10\\7\\6\end{pmatrix}=\begin{pmatrix}2\\14\\-5\end{pmatrix}$$

$$l_{PQ'}:\underset{\sim}{r}=\begin{pmatrix}10\\7\\6\end{pmatrix}+s\begin{pmatrix}2\\14\\-5\end{pmatrix},\ s\in\mathbb{R}$$

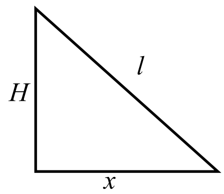
11a

$$144=4\times\frac{1}{2}(2x)(l)+(2x)^2$$

$$l=\frac{36}{x}-x$$

Let H be the height of the square pyramid

$$H=\sqrt{l^2-x^2}$$



$$V=\frac{1}{3}(2x)^2\sqrt{l^2-x^2}$$

$$=\frac{1}{3}(2x)^2\sqrt{\left(\frac{36}{x}-x\right)^2-x^2}$$

$$=\frac{1}{3}(4x^2)\sqrt{\left(\frac{36}{x}\right)\left(\frac{36}{x}-2x\right)}$$

$$=\frac{1}{3}(4)(6)\sqrt{(x^2)^2\left(\frac{36}{x^2}-2\right)}$$

$$=8\sqrt{36x^2-2x^4}$$

11b

$$\frac{\mathrm{d}V}{\mathrm{d}x}=8\left(\frac{1}{2}\right)\frac{1}{\sqrt{36x^2-2x^4}}(72x-8x^3)$$

$$=\frac{32x(9-x^2)}{\sqrt{36x^2-2x^4}}$$

$$\frac{32x(9-x^2)}{\sqrt{36x^2-2x^4}}=0$$

$$x(3-x)(3+x)=0$$

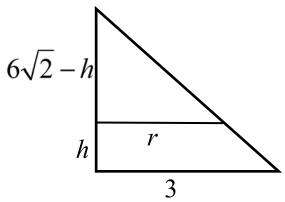
$\Rightarrow x=0$ or $x=\pm3$

From context, $x>0$. Hence $x=3$.

x	2.9	3	3.1
$\frac{\mathrm{d}V}{\mathrm{d}x}$	4.31	0	-4.77
Slope	/	—	\

Hence V is maximum when $x=3$

Alternatively,

	From GC, $\left.\frac{\mathrm{d}^2V}{\mathrm{d}x^2}\right\|_{x=3} = -45.3 < 0$ Hence V is maximum when $x = 3$ Maximum $V = 8\sqrt{36(3)^2 - 2(3^4)}$ $= 8\sqrt{162}$ $= 8\sqrt{81(2)}$ $= 8(9)\sqrt{2}$ $= 72\sqrt{2}$ (Shown)
11c	Let h, $2r$, W be the depth of liquid, side length of liquid surface, volume of liquid respectively. From (a), $l = \frac{36}{3} - 3 = 9$ $H = \sqrt{l^2 - x^2} = \sqrt{9^2 - 3^2} = 6\sqrt{2}$ By similar triangles, $\frac{r}{6\sqrt{2} - h} = \frac{3}{6\sqrt{2}}$  $r = \frac{1}{2\sqrt{2}}\left(6\sqrt{2} - h\right)$ $W = 72\sqrt{2} - \frac{1}{3}(2r)^2\left(6\sqrt{2} - h\right)$ $= 72\sqrt{2} - \frac{1}{3}\left(\frac{1}{2}\right)\left(6\sqrt{2} - h\right)^3$ $= 72\sqrt{2} - \frac{1}{6}\left(6\sqrt{2} - h\right)^3$ Given $\frac{\mathrm{d}W}{\mathrm{d}t} = 10$, $t = 6$, $10(6) = 72\sqrt{2} - \frac{1}{6}\left(6\sqrt{2} - h\right)^3$ $h = 2.1778$ (5 sf) $\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{6}(3)\left(6\sqrt{2} - h\right)^2(-1)\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{1}{2}\left(6\sqrt{2} - h\right)^2\frac{\mathrm{d}h}{\mathrm{d}t}$ When $t = 6$, $10 = \frac{1}{2}\left(6\sqrt{2} - 2.1778\right)^2\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = 0.503$ (3 sf) Depth is increasing at 0.503 cm per second