RAFFLES INSTITUTION
2022 YEAR 6 COMMON TEST

2024 Y6 H2 Math Timed Practice Revision Paper 4:

The solution will be released in Ivy on 17 June (Monday)

**** **Note that Qn 5 (Complex Numbers) will NOT be examined in the coming Timed Practice. So, you should complete this paper within 2 hrs 47 mins.**

MATHEMATICS **9758**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper. **You may use the blank pages on page 2 and 26 if necessary and you are reminded to indicate the question number(s) clearly.**
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

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Sect A: Pure Math	Q1	Q2	Q3	Q4	Q5	Q6	Q7
	/ 6	/ 8	/ 7	/ 8	/ 7	/ 12	/ 12
Sect B: Prob & Stats	Q8	Q9	Q10	Q11	Q12	TOTAL	
	/ 6	/ 6	/ 8	/ 8	/ 12	/ 100	

This document consists of **24** printed pages and **2** blank pages.

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You may continue your working on this page if necessary, indicating the question number(s) clearly.

Section A: Pure Mathematics [60 marks]

1 The curve C has equation $y=\dfrac{x^2-2x+4}{x-b}$, $x \neq b$.

(i) Given that y cannot take values between -2 and 6, show that $b=2$ using an algebraic method. [3]

(ii) Describe a sequence of 2 transformations that transform the graph of C onto the graph of $y=\dfrac{x^2+5x+4}{x}$, $x \neq 0$. [3]

2 The function g is defined by $\mathrm{g}: x \rightarrow \sin x + x$, for $x \in \mathbb{R}$, $-\pi \leq x \leq \pi$.

(i) Show that $\mathrm{g}'(x) > 0$ for $-\pi < x < \pi$. Hence, or otherwise, show that g has an inverse. [2]

(ii) Sketch on the same diagram the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$. [3]

(iii) Show that the composite function g^2 exists. [1]

(iv) Find the exact solutions of the equation $\mathrm{g}^2(x) = x$. [2]

3 **(i)** Prove by the method of differences that

$$\sum_{r=1}^{N} \frac{1}{\sqrt{r}+\sqrt{r+1}} = \sqrt{N+1}-1.$$ [3]

(ii) Find $\sum_{r=4}^{N-1} \frac{1}{\sqrt{r}+\sqrt{r+1}}$ in terms of N. [2]

(iii) Using part **(i)**, show that

$$\sum_{r=1}^{N} \frac{1}{\sqrt{r}} > 2\left(\sqrt{N+1}-1\right)$$ [2]

4 A curve C has parametric equations

$$x = e^t \cos t, \quad y = e^{-t} \sin t.$$

(i) Find the equation of the tangent to C at the point with parameter p. Give your answer in the form $y = mx + c$. [5]

(ii) The tangent meets the x-axis at point A and the y-axis at point B. Find, in terms of p, the area of the triangle OAB, where O is the origin. [3]

5 The complex number z satisfies the equation $\mathrm{f}(z)=0$, where

$$\mathrm{f}(z)=(1-a\mathrm{i})z^2-2\mathrm{i}z-10-20\mathrm{i}$$

and a is a real number. It is given that one root is of the form $1+b\mathrm{i}$, where b is an integer .

(i) Show that b satisfies the equation

$$b^4-2b^3+12b^2-42b-9=0\ .$$ [3]

(ii) Hence find the values of a and b, and the other root of $\mathrm{f}(z)=0$. [4]

6 The molecular geometry is the three-dimensional arrangement of atoms in a molecule. Fig. 1 shows the molecular geometry of a particular molecule which is of a tetrahedral form with one central atom lying inside the tetrahedron and 4 other atoms at the vertices of the tetrahedron. The points *A, B, C* and *D* represent the 4 vertices of the tetrahedron and are connected to the central atom, which is denoted by the point *E*, forming 4 pairs of bonding atoms *AE*, *BE*, *CE* and *DE*.

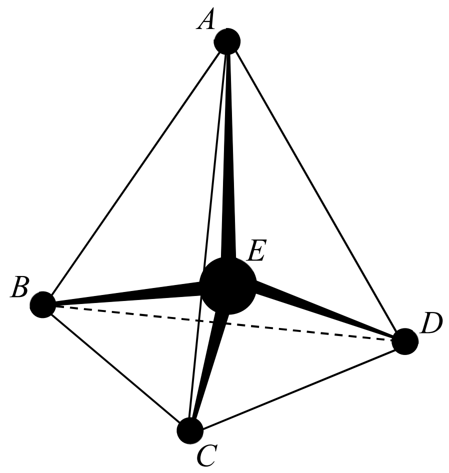

Fig.1

The coordinates of *A*, *B*, *C*, *D* and *E* are given by $(2,3,4)$, $(0,0,3)$, $(3,0,0)$, $(0,4,0)$ and $(1,1,2)$ respectively. The plane *p* passes through the points *A*, *B* and *C*. The line *l* passes through the points *D* and *E*.

(i) Find a cartesian equation of *p*. [4]

(ii) Find a vector equation of l and the coordinates of the point of intersection of l and p. [4]

(iii) Hence, find the shortest distance from point E to p. [2]

The bond angle is the angle between any two pairs of bonding atoms, measured in degrees.

(iv) Find the bond angle between bonding atoms AE and CE. [2]

7 **(a)** It is given that $x\frac{\mathrm{d}y}{\mathrm{d}x}=y+6xy$.

(i) Using the substitution $y=v^2x$, show that the differential equation can be transformed to $\frac{\mathrm{d}v}{\mathrm{d}x}=\mathrm{f}(v)$, where the function $\mathrm{f}(v)$ is to be found. [3]

(ii) Hence solve the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x}=y+6xy$ to find y in terms of x. [2]

7 [Continued]

(b) A simple mathematical model for the spread of epidemics assumes that the number of people, x, infected with an infectious disease at time t days changes at a rate proportional to the product of the number of people infected with the disease and the number of people who are not yet infected by the disease, with k being the constant of proportionality.

(i) Assuming that the population remains constant at P, write down a differential equation involving x, t, P and k. [1]

(ii) Solve the differential equation, and show that

$$x = \frac{Px_0}{x_0 + (P - x_0)\mathrm{e}^{-kPt}},$$

where x_0 is the number of infected people at time $t = 0$. [5]

(iii) What happens to the number of people infected with the disease if this situation continues after many months ? [1]

Section B: Probability and Statistics [40 marks]

8 A group of hikers is to be chosen to represent three hiking teams in an annual hiking challenge. The group is to consist of 7 hikers and is chosen at random from a set of 10 hikers consisting of 2 hikers from team A, 3 hikers from team B and 5 hikers from team C.

Find the number of ways in which the group can be chosen if

(i) there are no restrictions, [1]

(ii) there are at least 4 hikers from team C, [2]

(iii) there is at least 1 hiker from each team. [3]

9 A study[1] found that students at the National University of Singapore and the Nanyang Technological University sleep an average of 6.2 hours a day. Andy surveys 50 students from university S to estimate each person's daily sleeping hours. The sample mean is $\overline{t}$ hours and the sample standard deviation is 2.2 hours. A test is to be carried out at the 5% level of significance to draw conclusions on the average daily sleeping hours of students from university S.

[[1]Source: https://www.straitstimes.com/singapore/lack-of-sleep-is-nothing-to-yawn-about]

(i) Find the set of values of $\overline{t}$ for which he will conclude that the students from university S sleep more than 6.2 hours on average. State the appropriate hypotheses for the test, defining any symbols you use. [5]

(ii) State the set of values of $\overline{t}$ for which he will conclude that the average sleeping hours of students from university S is not 6.2 hours. [1]

10 For events A and B, it is given that $\mathrm{P}(A') = 0.3$ and $\mathrm{P}(B) = 0.55$.

(i) Given that the events A and B are independent, find $\mathrm{P}(A \cap B)$. [1]

(ii) Given that events A and B are not independent, find the range of $\mathrm{P}(A \cup B)$. [2]

(iii) Given that $\mathrm{P}(A' \mid B) = 0.4$, find

(a) $\mathrm{P}(A \cup B)$, [3]

(b) $\mathrm{P}(A' \cup B')$. [2]

11 A policeman inspects each passenger in the car that he stops during a roadblock. The number of passengers in a randomly stopped car is the random variable X. The probability distribution of X is shown in the table below.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.32	0.43	0.16	0.07	k

(i) Find the value of k and determine $\mathrm{E}(X)$. [2]

The policeman randomly stops 10 cars to inspect the passengers at a particular roadblock, and the total number of passengers inspected is the random variable Y.

(ii) Find $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$. [2]

The policeman also checks the car boot if the number of passengers in the car is not more than 2.

(iii) Find the probability that he checks more than 5 car boots at that roadblock. [2]

(iv) Find the probability that the last car is the 5^{th} car boot that he checks at that roadblock. [2]

12 In this question you should state the parameters of any normal distributions that you use.

A petrol station (Station A) finds that its daily sales, in litres, are normally distributed with mean 5400 and standard deviation 240.

(i) Sketch the distribution for the daily sales of petrol at Station A between 4500 litres and 6300 litres. [2]

(ii) Find the expected number of days of the year (365 days) the daily sales of petrol at Station A exceed 5950 litres. [2]

(iii) Find the probability that the total daily sales of petrol at Station A on 6 randomly chosen days is less than 32000 litres. [2]

The daily sales at another petrol station (Station B) are normally distributed with mean 6200 litres and standard deviation s litres.

(iv) The probability that the daily sales of petrol at Station B on a randomly chosen day are more than 5000 litres is 0.99. Find the value of s. [3]

(v) Find the probability that, on a randomly chosen day, the daily sales of petrol at Station B are more than half of that at Station A by at least 2000 litres. State an assumption needed for your calculation. [3]

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