Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2019

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 1

Friday **4 October 2019** **1 hour 30 minutes**

SOLUTIONS

ONLY

Answer all the questions.

1 $3x^2 - 11x + 6 \le 0$

$(3x-2)(x-3) \le 0$

$\frac{2}{3} \le x \le 3$

2 $2x + k = \frac{2}{1-x}$

$-2x^2 + (2-k)x + (k-2) = 0$

For line not to meet curve, D<0:

$(2-k)^2 - 4(-2)(k-2) < 0$

$(2-k)(2-k-8) < 0$

$(k-2)(k+6) < 0$

$-6 < k < 2$

Least integer $k = -5$.

3

4 **(i)** $\dfrac{2-\log_k 1+\log_m \dfrac{1}{\sqrt{m}}}{\log_m m^2+3}$

$$\frac{2-\log_k 1+\log_m \frac{1}{\sqrt{m}}}{\log_m m^2+3}$$

$$= \frac{2-0-\frac{1}{2}}{2+3}$$

$$= \frac{3}{10}$$

(ii) $\dfrac{\log_m 4\times\log_2 m^2}{\log_{25}\sqrt{m}}\times\log_5 m\,.$

$$\frac{\log_m 4\times\log_2 m^2}{\log_{25}\sqrt{m}}\times\log_5 m$$

$$= \frac{2\lg 2}{\lg m}\times\frac{2\lg m}{\lg 2}\times\frac{2\lg 5}{\frac{1}{2}\lg m}\times\frac{\lg m}{\lg 5}$$

$= 16$

5 **(a)** Line: $xy=-2y+\dfrac{1}{2}$

$$2xy=-4y+1$$

$$2=-4\left(\frac{1}{x}\right)+\frac{1}{xy}$$

$$2+4\left(\frac{1}{x}\right)=\frac{1}{xy}$$

Gradient = 4, Intercept = 2

(b) **(i)** $\ln y-7=\dfrac{-2-7}{1-4}\left(x^2-4\right)$

$$\ln y=3x^2-5$$

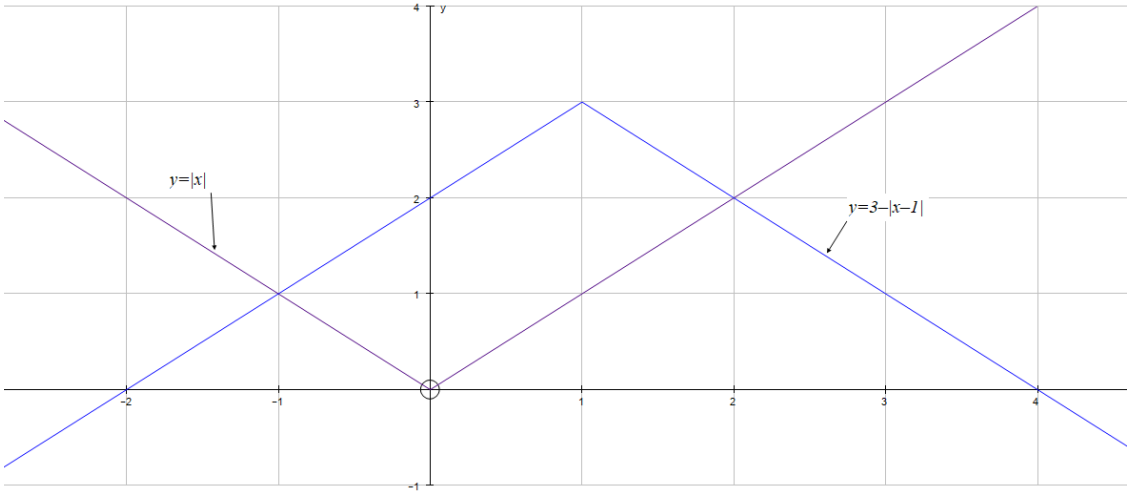
$$y=e^{3x^2-5}$$

5 **ii)** As x approaches zero, $y = e^{-5}$.

6 **(i)** $x-1=3$ or $x-1=-3$

$x=4$ or $x=-2$

(ii)



(iii) $|x|+|x-1|=3$

$x^2+x^2-2x+1+2|x(x-1)|=9$

$|2x^2-2x|=8+2x-2x^2$

$2x^2-2x=8+2x-2x^2$ or $2x^2-2x=-8-2x+2x^2$

$4x^2-4x-8=0$ No solution

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$ or $x=-1$

Alternatively,

$|x|+|x-1|=3$

$|x|=3-|x-1|$

$x=3-|x-1|$ or $x=|x-1|-3$

$|x-1|=3-x$ $|x-1|=x+3$

$x-1=3-x$ or $x-1=x-3(N.A)$ $x-1=x+3$ (N.A) or $x-1=-x-3$

$x=2\ or\ -1$

(iv) For $|x|+|x-1|\le 3$, $\quad -1\le x\le 2$

7 **(i)** $PQ = \sqrt{(9-3)^2+(11-5)^2}$

$= 6\sqrt{2}$ units

(ii) Centre = midpoint = $\left(\frac{3+9}{2},\frac{5+11}{2}\right)$

$= (6,8)$

Radius = $3\sqrt{2}$ units

Equation of circle is $(x-6)^2+(y-8)^2=18$

(iii) $y-11=-\frac{9-3}{11-5}(x-9)$

$y=-x+9+11$

Equation of tangent is $y+x=20$

(iv) When $y=0$, $x=20$ ----- $R=(20,0)$

Area of triangle PQR $= \frac{1}{2}\begin{vmatrix} 20 & 3 & 9 & 20 \\ 0 & 5 & 11 & 0 \end{vmatrix}$

= 66 units²

8 **(i)** $\log_2 x+\log_2(x+2)=3$

$\log_2(x^2+2x)=3$

$(x^2+2x)=8$

$x^2+2x-8=0$

$(x-2)(x+4)=0$

$x=2$ or $x=-4$ (NA)

(ii) $\dfrac{x}{\log_4 2}+6=\dfrac{12}{\log_{16} 2}$

$x\log_2 4+6=12\log_2 16$

$2x+6=48$

$x=21$

(iii) $\log_3 4x-2\log_9(2x-5)-\log_3(x+1)=1$

$\log_3 4x-\log_3(2x-5)-\log_3(x+1)=1$

$\log_3 \dfrac{4x}{(2x-5)(x+1)}=1$

$\dfrac{4x}{(2x-5)(x+1)}=3$

$6x^2-9x-15=4x$

$6x^2-13x-15=0$

$(x-3)(6x+5)=0$

$x=3$ or $x=-\dfrac{5}{6}$ (NA)

9 **(a)** $\dfrac{1}{\alpha}+\dfrac{1}{\beta}=-2$ $\dfrac{1}{\alpha\beta}=\dfrac{5}{2}$ ------- (1)

$\dfrac{\alpha+\beta}{\alpha\beta}=-2$ ------ (2)

(1) in (2): $(\alpha+\beta)\left(\dfrac{5}{2}\right)=-2$

New equation is $5x^2+4x+2=0$

(b) **(i)** Let α and $\dfrac{1}{\alpha}$ be the roots of the equation.

$\alpha\left(\dfrac{1}{\alpha}\right)=\dfrac{9-2k}{k}$

$9-2k=k$

$k=3$

(ii) $\alpha + \frac{1}{\alpha} = \frac{2k^2 - 1}{k}$ OR $3x^2 - 18x + 9 = 6 - x$

$\frac{\alpha^2 + 1}{\alpha} = \frac{17}{3}$ $3\alpha^2 - 17\alpha + 3 = 0$ since α is a root

$3\alpha^2 + 3 = 17\alpha$

$3\alpha^2 = 17\alpha - 3$ ------ (1) $3\alpha^2 = 17\alpha - 3$ ------ (1)

(1) x 3α : $9\alpha^3 = 17\left(3\alpha^2\right) - 9\alpha$

$= 17\left(17\alpha - 3\right) - 9\alpha$

$= 280\alpha - 51$ (shown) ------- (2)

End of Paper