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Name: ____________________________( ) Class: _________

WOODLANDS SECONDARY SCHOOL
MID YEAR EXAMINATION 2021

Level:	Sec 4Exp	Marks:	90
Subject:	Additional Mathematics	Day:	Monday
Paper:	4049/02	Date:	10th May 2021
Duration:	2 hr 15 min	Time:	1030 – 1245

READ THESE INSTRUCTIONS FIRST

Answer on the Question Paper.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.
This document consists of **15** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The equation of a curve is $y = x^2 + x - kx - k$, where k is a constant.

(i) Show that the equation $y = 0$ has real roots for all values of k. [2]

(ii) Find the values of k for which the line $y = 2x + 1$ is a tangent to the curve. [3]

2 **(a)** Given $2\log_4 y - 3 = 3\log_4(x-1)$, express *y* in terms of *x*. [3]

(b) **(i)** Given $y = 3^x$, express $9^x + 18 = 3^{x+2}$ as a quadratic equation in terms of *y*. [2]

(ii) Hence, or otherwise, find the values of *x*. [4]

3 The equation of a curve is $y = \ln\sqrt{\frac{7x-2}{x}}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x(7x-2)}$, where k is a constant to be determined. [3]

(ii) Find the equation of the normal to the curve at $x = 2$. [3]

(iii) A point (x, y) is moving on the curve.
Find the value(s) of x for which the rate of increase of y is twice the rate of increase of x. [3]

4 The curve $y = a(x-h)^2 + k$, where *a*, *h* and *k* are constants, intersects the *y*-axis at (0, –1) and the *x*-axis at (1, 0) and $\left(\frac{1}{2}, 0\right)$.

(i) Find the values of *a*, *h* and *k*. [4]

(ii) State the coordinates of the turning point. [1]

(iii) Express $2x^2 + 4x + 3$ in the form $p(x+q)^2 + r$, where *p*, *q* and *r* are constants. [2]

(iv) Explain why the two curves $y = a(x-h)^2 + k$ and $y = 2x^2 + 4x + 3$ will **not** intersect. [1]

5 The variable x and y are related by the equation $x\sqrt{y} = a(bx+1)$. By plotting a graph of $\sqrt{y}$ against $\frac{1}{x}$, a straight line graph passing through the points (2, 3) and (14, 27) is obtained.
Find the values of a and b. [4]

6 **(a)** In the binomial expansion of $\left(x^2+\frac{k}{x}\right)^{12}$, where k is a constant, the term independent of x is 126720. Find the possible values of k. [4]

(b) The expansion of $(a+x)(1-5x)^n$ in ascending powers of x is $2+px+720x^2+\ldots$, where a, n and p are constants and n is a positive integer.
Find the values of a, n and p. [6]

7 **(i)** Express $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(3x+2\right)^2\right]$ in the form $\frac{a}{3x+2}$, where a is a constant to be found. [2]

(ii) Hence evaluate $\int_0^2 \left(2-\frac{4}{3x+2}\right) \mathrm{d}x$. [4]

8 **(a)** It is given that $f'(x) = \cos\frac{1}{2}x - \sin 2x$ and $f(\pi) = 1$.

Express $2f(x) + 8f''(x) = q - p\cos 2x$, where p and q are constants to be determined. [6]

(b) The equation of a curve is given by $y = \frac{2x-1}{5x+3}$ for $x > 0$.

Determine whether the curve is an increasing or decreasing function for $x > 0$. Show your working clearly. [3]

9 **(a)** Without using a calculator, show that $\sin\left(\frac{7\pi}{12}\right)=\frac{\sqrt{p}+\sqrt{q}}{4}$, where p and q are integers to be determined. [3]

(b) Given that $\cos A=\frac{3}{5}$ and $\sin A$ is negative, calculate the value of

(i) $\sec A+\cot A$, [2]

(ii) $\cos\left(\frac{A}{2}\right)$. [2]

(c) **(i)** The function $f(x) = a \cos bx + c$, where $a > 0$, has a maximum value of 7 and a minimum value of –3. Given that the period of f is $\frac{2\pi}{3}$, find the values of a, b and c. [3]

(ii) Hence, sketch the graph of f(x) for $0 \le x \le \pi$. [2]

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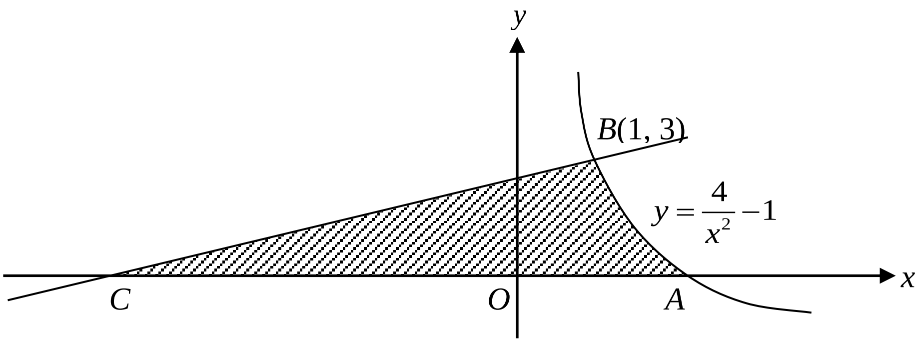

The diagram above shows part of the curve $y=\dfrac{4}{x^2}-1$, which intersects the x-axis at A.
The line BC is the normal to the curve at point B (1, 3).

(i) Find the coordinates of A. [2]

(ii) Find the equation of the line BC. [3]

(iii) The line BC cuts the x-axis at point C. Find the area of the shaded region ABC. [3]

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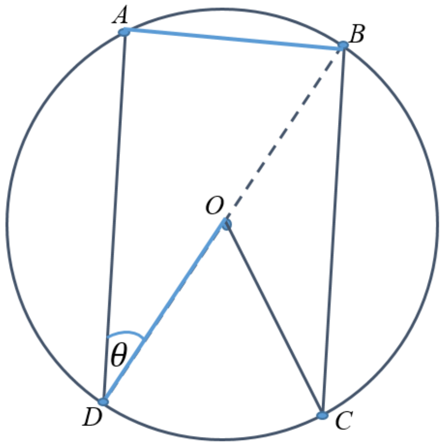

The diagram shows a circle with centre *O* and a diameter *BD* of length 12 cm.
A, *B*, *C* and *D* are points on the circle such that *AD* and *BC* are parallel. Angle *ADB* = θ.
The perimeter of *ABCOD* is *P* cm.

(i) Show that $P = 12\sin\theta + 24\cos\theta + 12$. [2]

(ii) Express *P* in the form $R\sin(\theta+\alpha)+12$, where $R > 0$ and α is acute. [3]

(iii) Find the maximum value of P and the corresponding value of θ. [2]

(iv) Find the value of θ when $P = 18$ cm. [3]

END OF PAPER

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