ANDERSON SERANGOON JUNIOR COLLEGE
JC1 Promotional Examination 2023
Higher 2

FURTHER MATHEMATICS 9649/01

Paper 1

28 September 2023 (Thursday) 8 – 11 a.m.

3 hours

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

An answer booklet will be provided with this question paper. You should follow the instructions on the front cover of the booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Write your name, class, syllabus and paper number on the answer booklet you hand in.
Write in dark blue or black pen on both sides of the answer booklet.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

Anderson Serangoon Junior College

1 ~~A conic has cartesian equation~~ $\cancel{\frac{(x-12)^2}{13^2}+\frac{y^2}{5^2}=1}$.

~~**(a)** Show that one of its foci is at the origin.~~ ~~[2]~~

~~**(b)** Obtain a polar equation of the conic.~~ ~~[3]~~

2 A dog bowl is produced by rotating the region bounded by the curve $y=\ln(x^4+1)$ and the lines $x=0$, $x=1$ and $y=-\frac{1}{5}$ fully about the y-axis. By using a substitution $u=\sqrt{\mathrm{e}^y-1}$ or otherwise, find the exact volume of the material required to produce the dog bowl. [7]

3 A polar curve has equation $r=1+\cos\theta$, where $0<\theta<\pi$. The curve has two points where the gradient of the curve is -1.

(a) Show that $\cos\theta+\cos 2\theta=\sin\theta+\sin 2\theta$ at these two points. [3]

(b) Without using a calculator, determine the value of θ for each of these two points. [3]

4 **(a)** It is given that f is a function where $\mathrm{f}'(x)<0$ for $x\geq 0$. If $U_n=\sum_{r=1}^{n}\frac{2}{n}\mathrm{f}\left(\frac{2r}{n}\right)$, where n is a positive integer and $I=\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$, explain with the aid of a sketch, why

- $U_n<I$ for all positive integers n,
- $U_\infty=I$. [3]

(b) If $V_n=\sum_{r=0}^{n-1}\frac{2}{n}\mathrm{f}\left(\frac{2r}{n}\right)$, state an inequality relating V_n and I for $n\geq 1$. [1]

(c) By choosing a suitable function f, determine the exact value of

$$\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\frac{1}{n+6}+\ldots+\frac{1}{3n}\right).$$ [3]

5 The complex number z satisfies $|z| \le |z+4+4\mathrm{i}|$ and $\left|\frac{1}{2}z+1+\mathrm{i}\right| \le \sqrt{2}$.

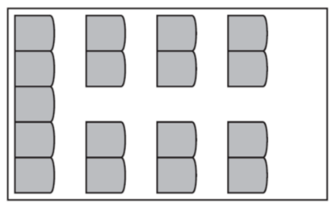
(a) Sketch the locus of z. [3]

(b) Express, in terms of k, the exact minimum value of $|\mathrm{i}z+k|$, where $k>0$. [2]

(c) Find the range of values of $\arg(z+6)$. [3]

(d) If w is a complex number such that $\left|w\mathrm{e}^{\mathrm{i}\theta}-z\right| \ne 0$ for all values of θ and z, write down the range of values of $|w|$, justifying your answer. [2]

6 ~~**(a)**~~



~~The diagram shows the seating plan for passengers in a minibus, which has 17 seats in 4 rows. The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on the minibus.~~

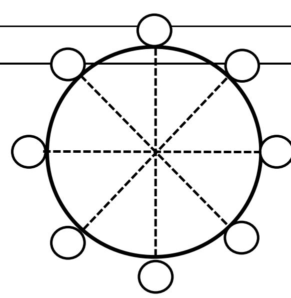
~~**(i)** How many possible seating arrangements are there for 11 passengers? [1]~~

~~**(ii)** How many possible seating arrangements are there if 5 particular people sit in the back row? [2]~~

~~Of the 11 passengers, 5 are unmarried and the other 6 consists of 3 married couples.~~

~~**(iii)** In how many ways can 5 of the 11 passengers be chosen if there must be 2 married couples and 1 other person, who may or may not be married? [2]~~

~~**(b)** There are 8 seats at a round table and they are equally spaced as shown in the diagram below.~~



~~Find the number of ways 2 women, 1 married couple and a family of 4 people can be seated at the table if~~

~~**(i)** the married couple and the family of 4 people must each be seated together, [2]~~

~~**(ii)** the married couple must be seated together and the 2 women must not be seated directly opposite each other. [3]~~

7 It is given that $\ln(y+1)=1-\sin x$.

(a) Apply Maclaurin's Theorem to find the power series for y, up to and including the term in x^2. [4]

(b) Given that x is small such that x^3 and higher powers of x can be neglected, verify the correctness of your result in part **(a)** by using standard series from the List of Formulae (MF26). [2]

(c) Use your series from part **(b)** to estimate $\int_{-1}^{1} \mathrm{e}^{1-\sin x}\,\mathrm{d}x$ in terms of e. [2]

(d) Explain with the aid of a diagram and without any further calculations, if your answer in **(c)** is an overestimation or underestimation. [2]

8 **(a)** Give a sketch of a continuous function that has a real root, but it is not possible to use linear interpolation to find approximations of this root. [1]

Consider $\mathrm{f}(x)=x^2\ln x-1$, where $x>0$. It has exactly one real root. Linear interpolation is applied to the initial interval (1, 3), and the first three approximations are $x_1=1.20228$, $x_2=1.33937$ and $x_3=1.42376$, given correct to 5 decimal places.

(b) Find the next two approximations, giving your answers correct to 2 decimal places and showing your workings clearly. Verify that the required accuracy of 2 decimal places has yet to be achieved, without comparing with the actual value of the root. [4]

For the same function, Newton-Raphson method is now to be applied to find approximations to the root.

(c) Find the exact set of values of the initial value u_1 such that the method would converge to the root. [2]

(d) Obtain a recurrence relation for u_{n+1}, the $(n+1)^{\text{th}}$ approximation of the root, in terms of u_n for positive integers n using Newton-Raphson method. [1]

(e) Use the relation in **(d)** and an initial approximation $u_1=1$ to find the root, terminating when two consecutive approximations are the same when rounded off to 2 decimal places. [2]

9 ~~A hyperbola H has equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, where a and b are fixed positive constants. One of its foci is $F(c, 0)$, where $c>0$.~~

~~**(a)** Find, in terms of a and b, the x-coordinate of the foot of perpendicular of F to the asymptote of H with positive gradient. [3]~~

~~**(b)** Prove that if a line $y=mx+k$ is tangent to H, then $k^2=a^2m^2-b^2$. [3]~~

~~The foot of perpendicular from F to a tangent line of H is denoted by N.~~

~~**(c)** Prove that the locus of N as the tangent varies is a circle with radius a and centred at the origin, excluding certain point(s) whose x-coordinates should be stated. [5]~~

10 ~~Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers on the dice.~~

~~**(a)** Calculate the probability that Tim obtains a score of at least 3. [2]~~

~~Tim plays a game with his friend Sam, who has two fair dice numbered in the same way. Both of them throw their dice simultaneously in the game.~~

~~**(b)** The random variable X represents the largest number shown on the four dice.~~

~~**(i)** Find the probability distribution of X. [3]~~

~~**(ii)** Show that $\mathrm{E}(X)=\frac{226}{81}$ and calculate the value of $\mathrm{Var}(X)$, giving your answer correct to 3 significant figures. [3]~~

~~**(c)** Given that $X=3$, find the probability that the sum of the numbers shown on the four dice is equal to 8. [4]~~

11 A bank charges $\lambda\%\left(\lambda>0\right)$ annual interest on a loan of \$$C$ taken out by a client. At the end of each year, the interest is added and then a fixed amount, \$$R$ is paid off by the client. Let the amount owed at the start of year n be L_n.

(a) Show that $L_n = aL_{n-1}+b$, giving a and b in terms of λ and R. [2]

(b) Find the solution of the recurrence relation $L_n = aL_{n-1}+b$, with $L_0 = C$, giving your solution in terms of a, b, C and n. [3]

(c) Deduce from **(a)** and **(b)** that $R>\frac{\lambda C}{100}$ for the loan to be able to be repaid. [2]

A client takes out a \$50 000 loan at 8% interest.

(d) **(i)** If $R=5000$, find the number of years it will take for the loan to be repaid. [3]

(ii) Find the total amount (to the nearest dollar) that the client repays over the lifetime of the loan. [2]

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