X JUNIOR COLLEGE
YEAR 6 PRELIMINARY EXAMINATION
Mock Arrangement
in preparation for Candidates' Examination
Higher (than) 2

CANDIDATE NAME

MATHEMATICS 9758/01

Paper 1 **Set II**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers in the space provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need of clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

ABOUT THIS PAPER

X Junior College (XJC) is an unofficial initiative aimed at preparing pre-university and/or junior college students in Singapore for school-level and/or national-level H2 Mathematics examinations through self-prepared mock papers. It has no affiliation with any existing institution in Singapore or worldwide.

This mock paper follows closely the 9758 H2 Mathematics GCE Advanced Level syllabus, most suitable for preparation towards preliminary examinations and A–Levels. The paper intends to explore the unconventional ways and/or applications in which topics within the syllabus can be tested, which may affect the difficulty of this paper to varying degrees. **While it is ideal to attempt this paper under examination constraints, prospective candidates are reminded not to use this potentially non-conforming paper as a definitive gauge for actual performance.**

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This document consists of **23** printed pages and **1** blank page.

1 Describe a sequence of transformations that transform the graph of $y = \sin x$ onto the graph of $y = \sin^2 x$. [4]

2 **(i)** Using standard series from the List of Formulae (MF26), expand $\cos(x + \sin x)$ as far as the term in x^4. [3]

(ii) By expanding $\cos(x + \sin x) + \cos(x - \sin x)$ as far as the term in x^4, or otherwise, evaluate

$$\int_0^{\frac{\pi}{6}} [\cos(x + \sin x) + \cos(x - \sin x)]\,dx,$$

giving your answer in exact form. [3]

3 An arithmetic series has an integer common difference d. The sum of the first p, $2p$ and $3p$ terms of the series are 185, 670, and 1455 respectively. Given that $0 < d < p$, find a general formula for the nth term of this series. [6]

4 Given a continuous function f, explain, with the aid of a sketch, why the expression

$$\lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

approximates to $\mathrm{f}'(x)$. [2]

Given that $\mathrm{f}(x) = \cos bx$, for some real value $b > 0$, use the expression above to show that $\mathrm{f}'(x) = -b \sin bx$. [4]

5 **(i)** Show algebraically that the inequality $\sqrt{2x+3} > 1 + \sqrt{4x-1}$ can be reduced to $\frac{1}{4} \leq x < \frac{1}{2}$. [4]

(ii) Find the range(s) of values of x, where $0 \le x \le \pi$, such that $\sqrt{\cos 2x + 4} > 1 + \sqrt{2\cos 2x + 1}$. [3]

6 A complex number $z = \cos\theta + \mathrm{i}\sin\theta$ is such that $\sin\theta \neq 0$.

(i) Show that $\displaystyle\sum_{r=1}^{n} z^{2r-1} = \frac{1 - z^{2n}}{z^{-1} - z}$. [1]

(ii) *De Moivre's* theorem states that for any complex number ω and any integer r, $\omega^r = |\omega|^r(\cos r\theta + i\sin r\theta)$. Use this theorem and the result in **(i)** to show that

$$\sum_{r=1}^{n} \sin[(2r-1)\theta] = \frac{\sin^2 n\theta}{\sin\theta}.$$ [5]

(iii) Hence, deduce that $\sum_{r=1}^{n}(2r-1)\cos\left[\frac{(2r-1)\pi}{2n}\right] = -\operatorname{cosec}\left(\frac{\pi}{2n}\right)\cot\left(\frac{\pi}{2n}\right)$. [3]

7 The integral I_n where $n = 1, 2, 3, \ldots$ is given by

$$I_n = \int_0^c \frac{1}{(1+x^2)^n}\,\mathrm{d}x, \quad c > 0.$$

(i) Find I_1 in terms of c. [1]

(ii) Show that $I_{n+1} = \dfrac{c}{2n(1+c^2)^n} + \left(\dfrac{2n-1}{2n}\right) I_n$ for $n > 1$. [5]

The region enclosed by the curve $y = \dfrac{1}{(1+x^2)^2}$ and the lines $x = 1$ and $y = 1$ is denoted by R.

(iii) Find the exact volume of the solid formed by rotating R through 2π radians about the x–axis. [4]

8 **(a)** The functions g and h are given such that

$$\mathrm{g}(x) = \frac{7 - x}{|10 - x|}, \qquad x \in \mathbb{R}, \quad x < a,$$

$$\mathrm{h}(x) = -x^2 + 2qx - 6, \qquad x \in \mathbb{R},$$

where a and q are real, and q is restricted such that $|q| < 4$.

(i) Explain whether the inverse of g exists for values $a > 10$. [2]

For the rest of part **(a)**, $a = 10$.

(ii) Find the range of h in terms of q. Hence, show that gh exists for any q in the given restriction. [3]

(iii) Write down, in terms of q, an expression for $\mathrm{gh}(x)$ including its domain and range. [3]

(b) Refer to the following table.

r	1	2	3	4	5	6	7
$\mathrm{X}(r)$	3	5	4	7	6	2	8
$\mathrm{Y}(r)$	6	7	3	6	4	5	2

Given that X and Y are functions,

(i) evaluate $\mathrm{XY}(2)$, $\mathrm{YX}^{-1}(5)$ and $\mathrm{X}^{-1}\mathrm{X}^{-1}\mathrm{Y}(4)$. [1]

(ii) do functions Y^{-1} and YX exist? Explain your reasoning briefly. [2]

9 A curve C has parametric equations

$$x = 3\alpha t^2, \quad y = \alpha t\left(3t^2 - 1\right), \quad -\frac{\sqrt{3}}{3} \leq t \leq \frac{\sqrt{3}}{3},$$

for some constant $\alpha > 0$.

(i) Sketch C, indicating the values of any axial intercepts. [2]

(ii) Find a cartesian equation for C. [3]

(iii) Do not use a calculator in answering this part.

The surface obtained by rotating a curve one full revolution around an axis of rotation is the surface of revolution. Its application ranges from design and architecture, where non-planar prototypes are concerned, to physics and engineering, where curvatures are involved in force field analysis.

The surface of revolution about the x–axis for the curve $y = \mathrm{f}(x)$, where $\beta \le x \le \gamma$, is given by the formula

$$2\pi \int_{\beta}^{\gamma} y\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x.$$

Find the surface area obtained when C is completely rotated about the x–axis. [8]

9 [Continued]

10 In Economics, economic growth can be determined using *Cobb-Douglas* production function and *Solow* model. An economy with capital K and labour L that both vary with time has production function given by $A(K^a)(L^{1-a})$, where A represents technological advancement and $0 < a < 1$. Given the rate of savings s of the economy, where $0 < s < 1$, its growth according to the model satisfies the differential equation

$$\frac{\mathrm{d}K}{\mathrm{d}t} = sA(K^a)(L^{1-a}).$$

An economy has capital-to-labour ratio k. It is known that the amount of labour in this economy increases at a rate proportional to the amount of labour, with some proportionality constant λ.

(i) Show that $\dfrac{\mathrm{d}k}{\mathrm{d}t} = sAk^a - \lambda k$. [3]

10 [Continued]

(ii) It is given that $k = k_0$ at $t = 0$. By substituting $y = Ak^{1-a}$, find k in terms of s, A, a, λ, k_0 and t. [7]

(iii) The following values are further given for the economy:

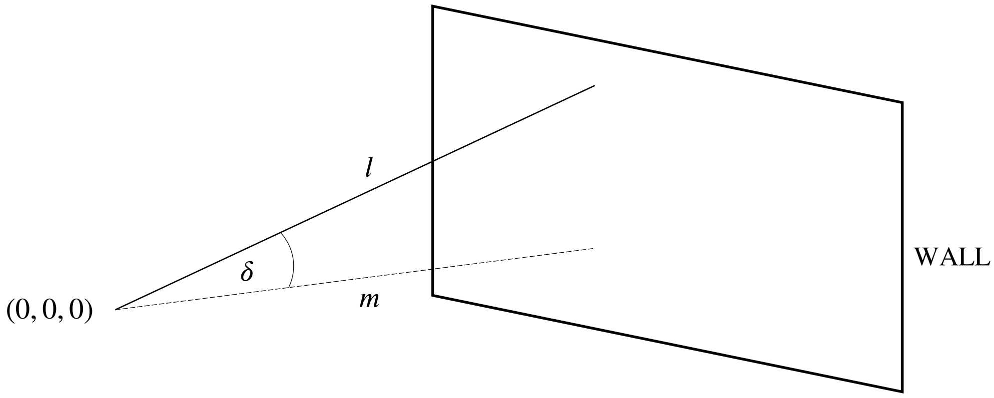
$$\frac{sA}{\lambda} = \frac{8}{3}; \qquad \lambda = \frac{3}{2}; \qquad k_0 = \frac{4}{25}; \qquad a = \frac{1}{2}.$$

Sketch the graph of k against t, indicating clearly any main features. [2]

The economy eventually experiences slower growth as it approaches its 'maximum capacity' after a long time. Economic policies may be put in place to increase the theoretical maximum capacity of the economy.

(iv) With reference to the constants in the model in **(ii)**, suggest one way to increase the maximum capacity of k. State a necessary assumption for this suggestion to be effective. [2]

11 A thermal lance is a device that produces a strong beam of fire, which is usually used in infrastructural works to cut through concretes. Due to its high operational temperature (~4500°C), it might be safer to employ such a device remotely. A design is proposed to automate thermal lances, equipped with a master laser for directional control.



The lance with line l and the master laser with line m are both fixed at a pivot with coordinates $(0, 0, 0)$. The line l has vector equation $\mathbf{r} = \lambda\mathbf{s}$, where λ is a scalar and $\mathbf{s} = \left(\cos\theta + \sqrt{3}\right)\mathbf{i} + \sqrt{2}\sin\theta\,\mathbf{j} + \left(\cos\theta - \sqrt{3}\right)\mathbf{k}$, $\theta \in \mathbb{R}$. The angle between l and m is δ, and the two lines intersect a wall at two distinct points (see diagram).

(i) An arbitrary line with equation $\mathbf{r} = \mu(a\mathbf{i} + b\mathbf{j} + c\mathbf{k})$, for some scalar μ and real constants a, b and c, makes an angle ϕ with l. Show that, for any θ,

$$\cos\phi = \frac{(a+c)\cos\theta + \sqrt{2}b\sin\theta + (a-c)\sqrt{3}}{2\sqrt{2}\left|\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right|}.$$ [3]

It is given that δ is independent of θ.

(ii) By considering the condition(s) for a, b and c in the equation in **(i)** for the case of line m, show that m has cartesian equation $x = -z$, $y = 0$. State also the value of δ in this case. [3]

(iii) If δ is instead not independent of θ, what can be said about m? [1]

11 [Continued]

The wall has cartesian equation $x - z = 4\sqrt{3}$. The lines l and m intersect the wall at P and C respectively.

(iv) Find the coordinates of P in terms of θ. [2]

(v) Find the shortest distance between C and P and show that it is independent of θ. Describe geometrically the behaviour of P as θ varies. [4]

(vi) Hence, or otherwise, find the shortest distance between the wall and the pivot. [1]

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