BENDEMEER SECONDARY SCHOOL
2024 PRELIMINARY EXAMINATION
SECONDARY FOUR NORMAL (ACADEMIC)

CANDIDATE NAME	MARKING SCHEME
CLASS	
INDEX NUMBER	

MATHEMATICS (SYLLABUS A) 4045/02

Paper 2 **5 August 2024**

2 Hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number on all the work you hand in.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
The use of an approved scientific calculator is expected, where appropriate.

Section A
Answer **all** the questions.

Section B
Answer **one** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 70.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

FOR ASSESSMENT USE
70

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved Surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Section A (62 Marks)

Answer all the questions in this section.

1 **(a)** Calculate $\left(\frac{4}{5}\right)^{-\frac{1}{2}} - \sqrt{5\frac{1}{2}}$.

-1.23

Answer ..... [1]

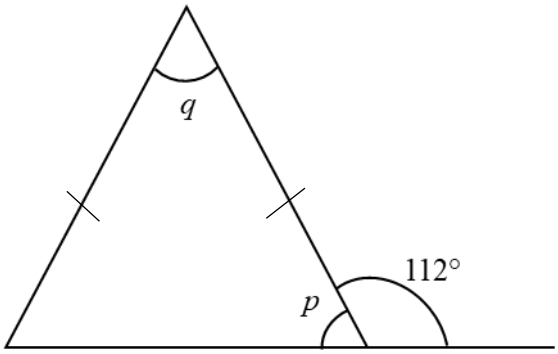
(b) By writing each number correct to 1 significant figure, estimate the value of

$$\frac{\sqrt{9.103}\times 284}{13}.$$

$\frac{\sqrt{9}\times 300}{10} = 90$

Answer ..... [1]

2 **(a)**



(i) Find *p*.

180° - 112° = 68° (Angles in a straight line)

Answer p = ...° [1]

(ii) Find *q*.

180° - 68° - 68° = 44° (Angle sum of triangle)

Answer q = ...° [1]

(b) Given that $\sin x° = 0.5$ and $x°$ is an obtuse angle, find $x°$.

$x = \sin^{-1} 0.5 = 30°$.
180° - 30° = 150°

Answer .....° [1]

3 Solve the simultaneous equations.

$$5x - 3y = 11$$
$$3x - y = 9$$

$5x - 3y = 11$ –(1)
$3x - y = 9$ -----(2)

(2) x 3
$9x - 3y = 27$ –(3) [M1]

(3) – (1)
$4x = 16 => x = 4$ [A1]

From (2)
$y = 3(4) - 9 = 3$ [A1]

Answer x = ……
y = …… [3]

4 Faris owns a furniture shop.

(a) The cost of a sofa was $1200.
He made a profit of 25% on the cost price when he sold it.

Find the selling price.

100% + 25% = 125% [M1]
$125\% \times 1200 = \$1500$ [A1]

Answer $\$\ldots\ldots$ [2]

(b) He sold a cupboard at $188 with a loss of 10%.
How much was the cost of the cupboard?

$90\% \rightarrow 188$
$100\% \rightarrow \frac{188}{90} \times 100$ [M1]
$= \$208.89$ [A1]

Answer $\$\ldots\ldots$ [2]

5 **(a)** Write these numbers in order of size, starting with the smallest.

4^3 4^0 $4^{\frac{1}{3}}$ 4^{-3}

4^{-3} , 4^0 , $4^{\frac{1}{3}}$, 4^3

Answer ……, ……., ……, …… [1]
smallest *largest*

(b) Write $\frac{a^2 \times a^1}{a^{-2}}$ as a single power of a.

$a^{2+1-(-2)} = a^5$

Answer …… [1]

(c) Simplify $7p^5 \times 2p^{-\frac{1}{2}}$.

$14p^{5-\frac{1}{2}} = 14p^{\frac{9}{2}}$ or $14p^{4.5}$ or $14p^{4\frac{1}{2}}$

Answer …… [1]

6 It is given that y is inversely proportional to the square of x.
When $x = 1$, $y = 64$.

Find the value(s) of x when $y = 25$.

$y = \frac{k}{x^2}$

$k = (64)(1) = 64$ [M1]

If state $y = \frac{k}{x}$ and solved for k [M1]

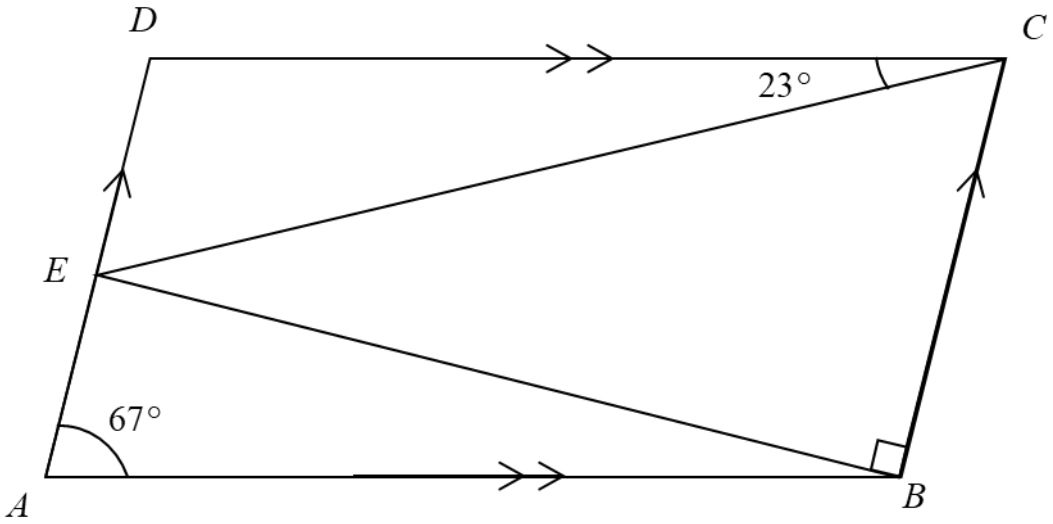
$x^2 = \frac{64}{25}$ [M1]

$x = \pm\sqrt{\frac{64}{25}} = \pm 1.6$ or $\pm\frac{8}{5}$ [A1]

Exclude $\pm$ minus 1

Answer $x =$ ..... [3

7 *ABCD* is a parallelogram in which angle *DAB* = 67° and angle *DCE* = 23°.
Given that *BE* is perpendicular to *CB*, find



(a) angle *BCE*,

67°- 23°= 44° (opp angles of parallelogram)

Answer .....° [1]

(b) angle *CEB*,

180° - 90°- 44°= 46° (angle sum of triangle)

Answer .....° [1]

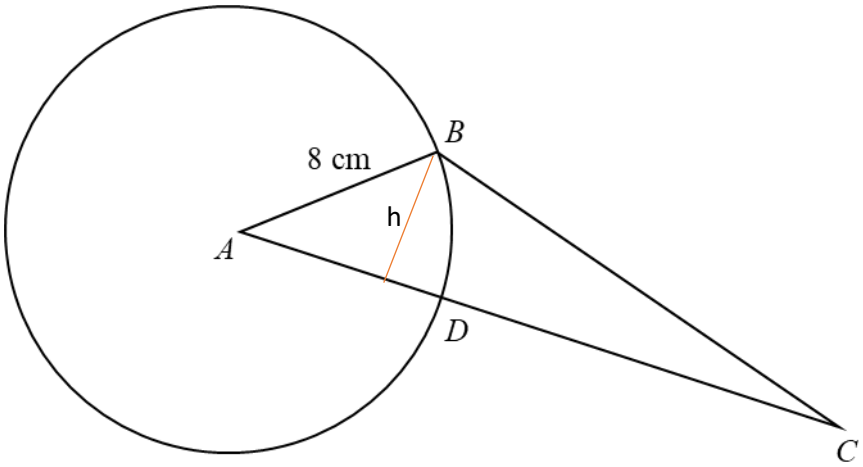
(c) angle *EDC*.

180° - 67° = 113° (Int. angles, parallel lines *AB* and *CD*

Answer .....° [1]

8 The diagram shows a triangle *ABC* and a circle with centre *A*.
The points *B* and *D* lie on the circumference of the circle.

The radius of the circle is 8 cm.
The length of the line *AC* is 17 cm.
The area of triangle *ABC* s 44 cm^2.



.

(a) Calculate the area of sector *ABD*.

Area of triangle $ABC = \frac{1}{2} \times 8 \times 17 \times \sin BAC$

$\frac{1}{2} \times 8 \times 17 \times \sin BAC = 44$ [M1]

$\sin BAC = \frac{44 \times 2}{8 \times 17} = \frac{11}{17}$
$\angle BAC = 40.320°$ [M1]

area of sector $ABD = \frac{40.320}{360} \times \pi \times 8^2$ [M1]
$= 22.5\ cm^2$ (to 3 s.f.) [A1]

Alternative method

$\frac{1}{2} \times h \times 17 = 44$
h =5.18 [M1]

$\sin BAC = \frac{5.18}{8}$
$\angle BAC = 40.320°$ [M1]

area of sector $ABD = \frac{40.320}{360} \times \pi \times 8^2$ [M1]
$= 22.5\ cm^2$ (to 3 s.f.) [A1]

Answer ……cm^2 [4]

(b) Calculate major arc length *ABD* if the reflex angle *BAD* is 5.6 rad.

$s = r\theta = 8\ (5.6) = 44.8$

Answer ……cm [1]

9 **(a)** Simplify $2x - 3(x - 4)$.

$2x - 3x + 12 = 12 - x.$

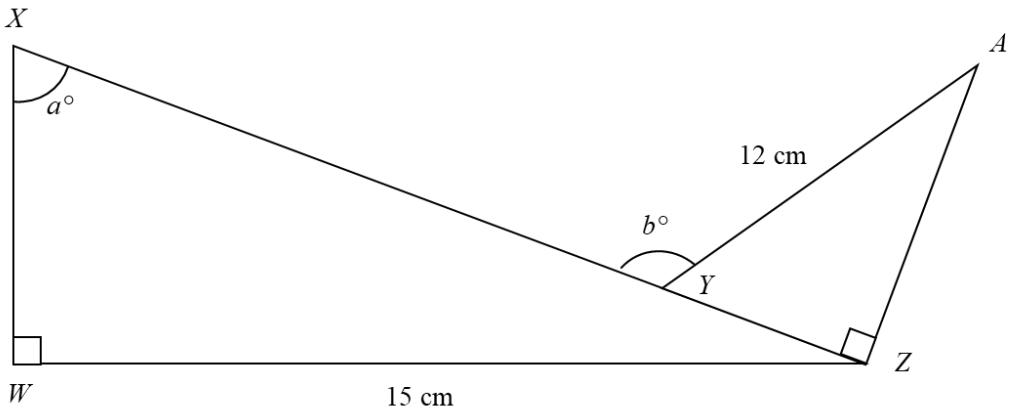
Answer …… [1]

(b) Solve $\frac{8}{x+1} = 3x - 5$, leave your answer in 2 decimal places.

$8 = (3x - 5)(x + 1)$
$8 = 3x^2 + 3x - 5x - 5$ [M1 – expansion]
$3x^2 - 2x - 13 = 0$
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-13)}}{2(3)}$ [M1 – formula]
$x = \frac{2 \pm \sqrt{160}}{6}$
$x = 2.44$ or -1.77 [A2]

Answer x = ……or …… [4]

10 In the diagram below, XYZ is a straight line. XZ is 5 times YZ.
AY = 12 cm, WZ = 15 cm and $\sin a° = \frac{3}{5}$.



(a) Find XZ.

$\sin a° = \frac{3}{5} = \frac{15}{XZ}$

$XZ = \frac{15\times5}{3} = 25$

Answer XZ = ……cm [1]

(b) Show that YZ is 5 cm. Hence find the exact value of $\cos b°$.

$YZ = \frac{25}{5}$ = 5 (shown) [B1]

$\cos b° = -\cos AYZ = \frac{YZ}{AY} = -\frac{5}{12}$ [B1]

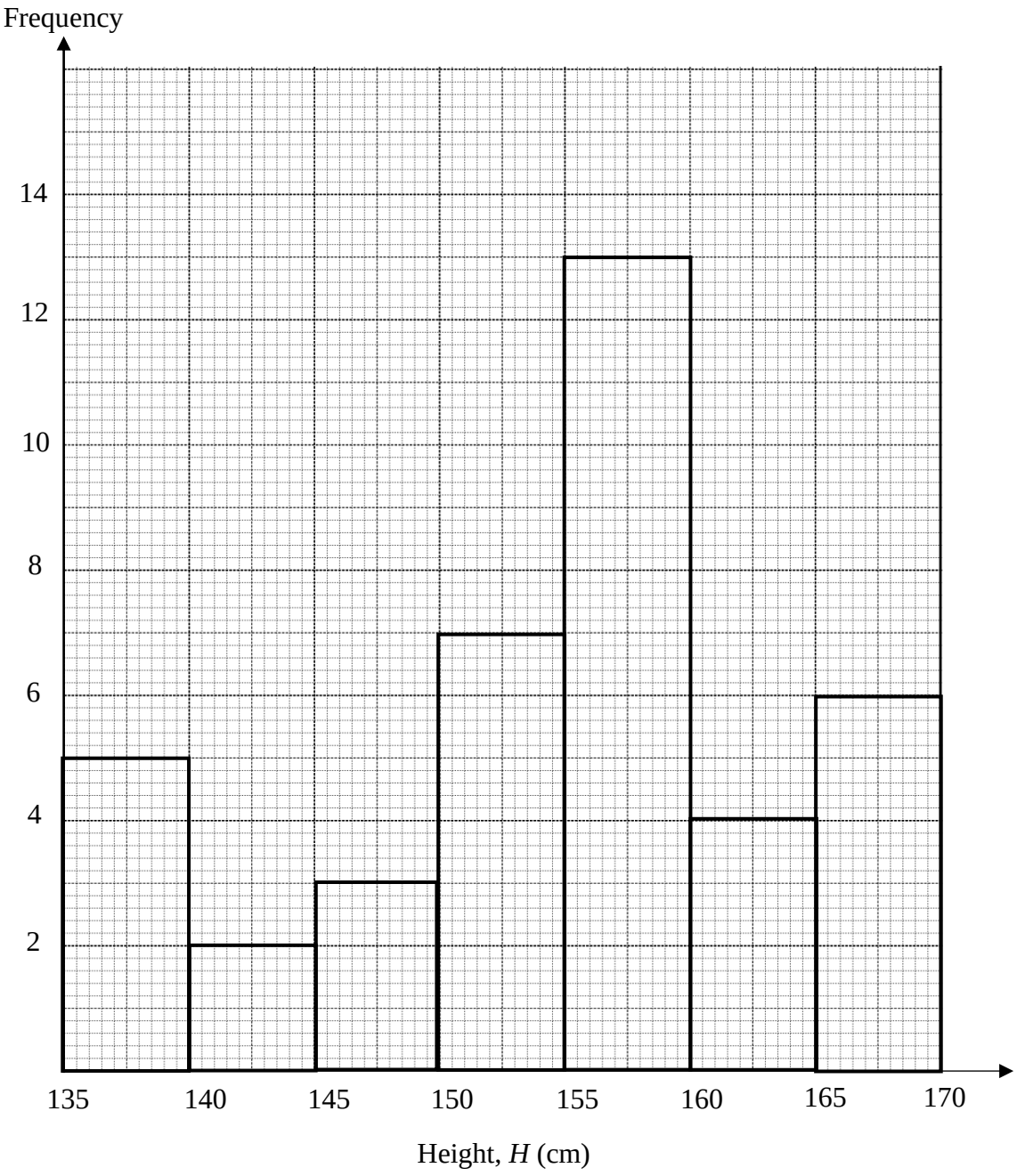
Answer …… [2]

(c) Find the length AZ.

$AZ = \sqrt{12^2 - 5^2} = 10.9$

Answer ……cm [1]

11 The following histogram shows the height of 40 students within a class.



(a) How many students had a height between 145 to 140 cm?

3

Answer …… [1]

(b) What is the modal height interval of the class?

$155 < H < 160$

Answer …… [1]

(c) The estimate of the mean height of the class is 150 cm.

Explain why this is only an **estimate** of the mean.

Since the histogram does not show the heights of the individual children, this is only an estimate of the mean.

Answer ……
…… [1]

(d) Find the probability that a student chosen at random has a height of at least 160 cm.

No. of students whose height of at least 160 cm = 4 + 6 = 10 [M1]

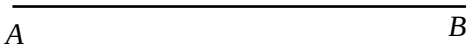
Probability = $\frac{10}{40} = \frac{1}{4}$

Answer …… [2]

12 In a triangle ABC, AB = 6 cm. AC = 5 cm and BC = 7 cm.

AB is drawn below.

(a) Construct triangle ABC. [1]



(b) Measure angle ABC.

Answer angle ABC = 44/45° [1]

(c) Construct the bisector of angle ABC. [1]

(d) Construct the perpendicular bisector of BC. [1]

(e) P is a point of intersection between the bisector of angle ABC and the perpendicular bisector of BC. Determine if P lies inside or outside triangle ABC.

Answer P lies _inside___triangle ABC [1]

13 The table below is for $y = x^3 - 2x + 3$.

x	-3	-2	-1	0	1	2	3
y	p	-1	4	3	2	q	24

(a) Calculate the value of p and the value of q.

$p = (-3)^3 - 2(-3) + 3 = -18$

$q = (2)^3 - 2(2) + 3 = 7$

Answer p = ...

q = ... [2]

(b) Draw the graph of x for $-3 \leq x \leq 3$ on the grid (page 15).

(c) Estimate the value of y when $x = 2.5$.

From graph, y = 13.5/14

Answer y = ... [1]

(d) Use the graph to find the values of x when y = 3.

x = -1.4, 0, 1.4

Answer x =..... , ... or ..... [1]

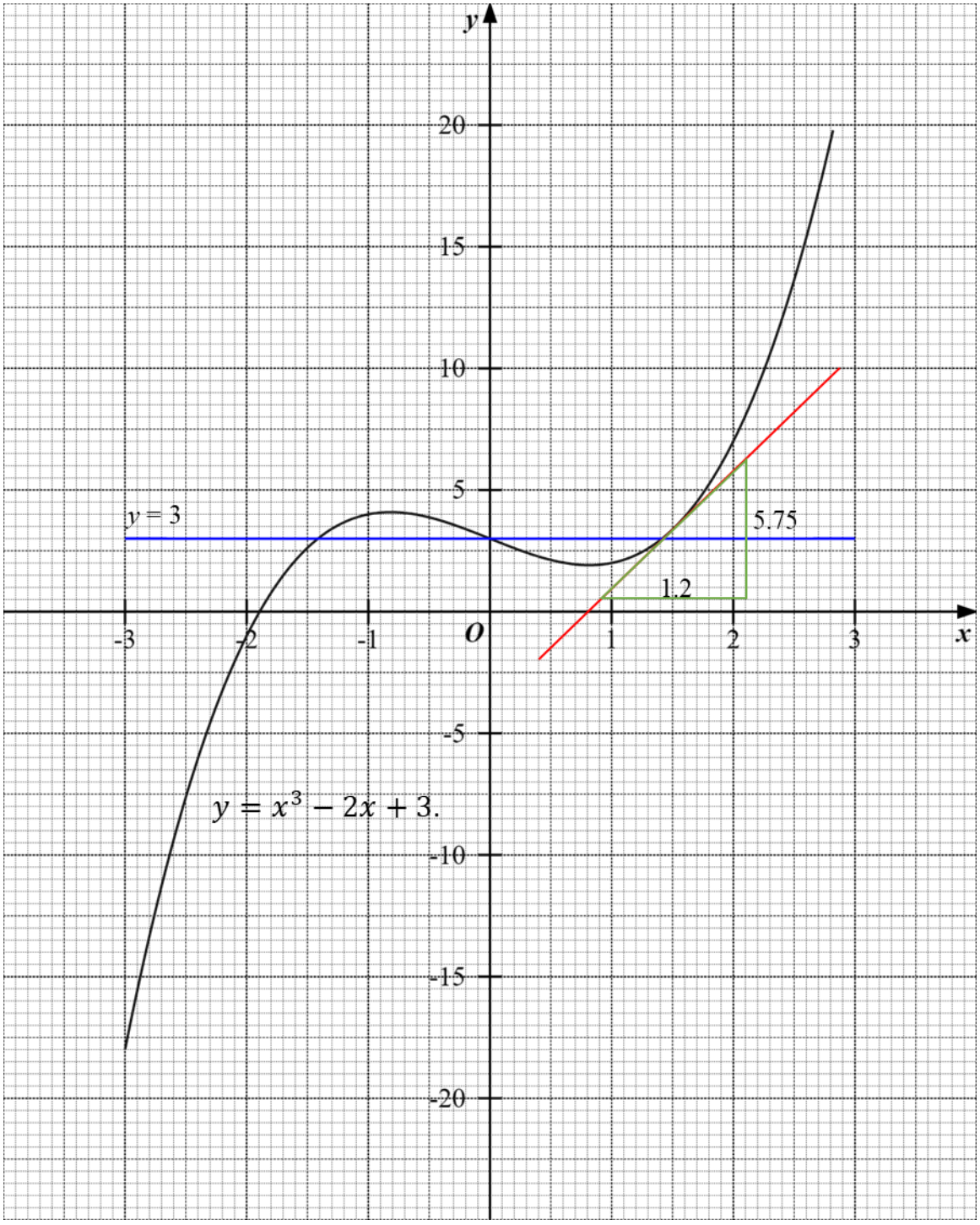
(e) By drawing a tangent, estimate the gradient of the graph of $y = x^3 - 2x + 3$ when x = 1.5.

gradient = $\frac{6.25 - 0.5}{2.1 - 0.9} = 4.79$ [M1]

Actual = 4.75

Accept ± 0.5 $(4.25, 5.25)$ [A1]

Answer ..... [2]


y
20
15
10
5
-5
-10
-15
-20
-3
-2
-1
O
1
2
3
x
y = 3
5.75
1.2
y = x³ − 2x + 3.

14 Jane wants to buy a flat in Singapore. She knows that there is Buyer's Stamp Duty (BSD) that will be added to the price of the flat.

The BSD payable is calculated in dollars and based on price of the flat.
Part of the rates are stated below:

- pay 1% on the first $180,000 of the price of the house.
- pay 2% on the price of the house that is above $180,000 and up to and including $360,000.
- pay 3% o the price of the house that is above $360, 000 and up to and including $1,000,000.
- pay 4% of the price of the house that is above $1,000,000 and up to and including $1,500,000.

A formula for calculating BSD is stated in the table below:

Property value (x)	Formula for BSD payable
$x \leq 180{,}000$	$1\% \times x$
$180{,}000 < x \leq 360{,}000$	$2\% \times x$ - $1,800
$360{,}000 < x \leq 1{,}000{,}000$	$3\% \times x$ - $5,400
$1{,}000{,}000 < x \leq 1{,}500{,}000$	$4\% \times x$ - $15,400

(a) Use the relevant information to calculate the BSD payable on the flat costing $330,000.

Method 1
2% × 330000 - $1,800 = $4800

Method 2
1% of $180,000 = $1,800
330,000 – 180,000 = 150,000
2% of next $150000 = $3,000

$1800 + $3000 = $4800

Answer $…… [2]

(b) In addition to BSD, Jane estimates that she will have to pay these extra costs:

- Home Valuation Fee $120
- Legal fee $462.60
- Renovation cost $80,000
- Fire/Home Insurance $8
- Furniture and electrical applicants $5,500

Jane plans to spend $750,000. What is the highest price of flat that Jane can afford? Give your answer to the nearest dollar.

Extra cost = $120 + $462.60 + $80,000 + $8 + $5500 = $86,090.60 [M1]

$750,000 - $86,090.60 = $663,909.40 [M1]

$663,909.40 -(0.03$x$-5400) = x [M1]

$663,909.40 +5400 = 1.03$x$

x = $649,815 [A1]

Answer $…… [4]

(c) Jane considers renting a flat instead of buying one as she will need to take a bank loan of $750 000. She listed down her cost between the option of a bank loan and rental payables.

Bank Loan (per annum)	Rental (per month)
Principal amount of $18,000 Interest of 2.6% of 750,000	$3500

Would Jane pay a lesser amount per month if she chooses to borrow from a bank than renting a flat?

Bank Loan
Method 1 – by month
Principal amount per month = $\$18000 \div 12 = \$1500$
2.6% of 750,000 = $19,500 per annum = $1625 per month
Loan payable per month = $1500 + $1625 = $3125 [M1]

Method 2 – by month
or SI = $\frac{750000(2.6)(1)}{100}$ = $19500
Bank Loan = (19500 + 18000)÷ 12 =$3125

3500 – 3125 = 375

Yes, Jane will pay $375 lesser if she chooses to take up a bank loan instead of renting a flat. [A1]

Method 3 – by year
Bank Loan
2.6% of 750,000 = $19,500
$19500+$18000 = $37500

Rental
$3500 x 12 = $42000

Answer …….. [2]

Section B (8 Marks)

Answer one question from this section. Each question carries 8 marks.

15 The frequency table below summarizes the number of students in School *A* who visited their school's online learning portal in a month. There are a total of 100 students in School *A*.

Number of visits (x)	Frequency
$0 < x \leq 5$	9
$5 < x \leq 10$	17
$10 < x \leq 15$	38
$15 < x \leq 20$	16
$20 < x \leq 25$	20

(a) Calculate an estimate of

(i) the mean monthly number of visits to the school's online learning portal by students in School *A*.

Number of visits (x)	Frequency	mid-value of x	fx	fx^2
$0 < x \ll 5$	9	2.5	22.5	56.25
$5 < x \ll 10$	17	7.5	127.5	956.25
$10 < x \ll 15$	38	12.5	475	5937.5
$15 < x \ll 20$	16	17.5	280	4900
$20 < x \ll 25$	20	22.5	450	10125
			$\sum fx = 1355$	$\sum fx^2 = 21975$

Mean $= \frac{\sum fx}{\sum f} = \frac{9\times2.5+17\times7.5+38\times12.5+16\times17.5+20\times22.5.}{100} = 13\frac{11}{20}$ or 13.55 or $\frac{271}{20}$

Answer ..... [1]

(ii) the standard deviation of the monthly number of visits from students in School *A*.

S.D = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

.= $\sqrt{\frac{21975}{100} - 13.55^2}$ [M1]

= 6.01 [A1]

Answer …… [2]

(b) Find the fraction of the students from School *A* who visited the online learning portal at most 10 times monthly.

$\frac{17+9}{100} = \frac{26}{100} = \frac{13}{50}$ [B1]

Answer …… [1]

(c) Two students are randomly selected from School *A*. Find the probability that both students visited the online learning portal more than 15 times monthly and at most 25 times monthly.

$\frac{36}{100} \times \frac{35}{99} = \frac{7}{55}$ [B1]

Answer …… [2]

(d) 7 students are randomly selected from School A and their number of visits to the online learning portal are recorded as such:

20, 13, 6, 24, 8, 5, 17

Find the interquartile range.

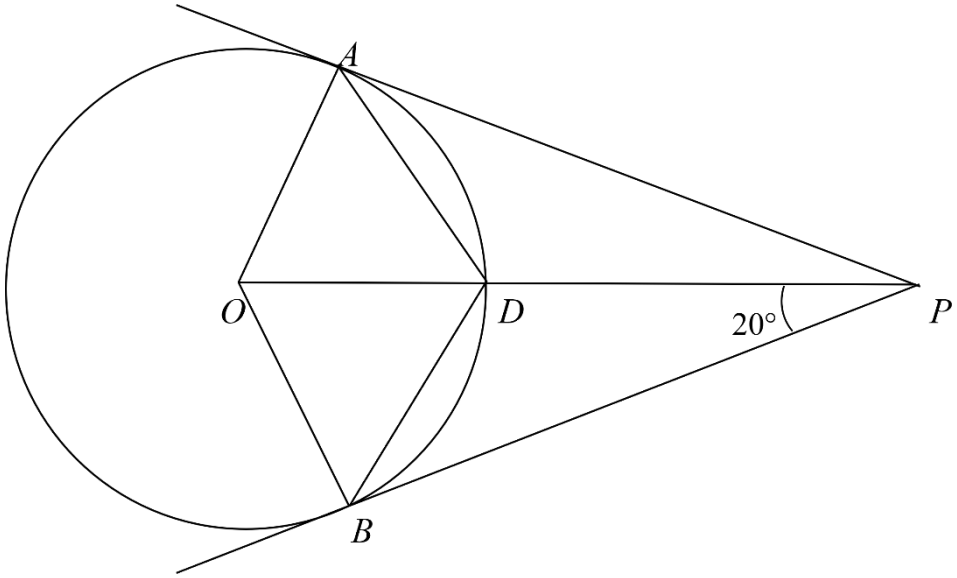
5, (6), 8, (13), 17, (20), 24

Q1 = 6, Q3 = 20 [M1]

IQR = 20 – 6 = 14 [A1]

Answer …… [2]

16 (a)



A, *B*, *D* are points on the circumference of a circle, centre *O*.
PA and *PB* are tangents to the circle.
Angle *OPB* = 20°.

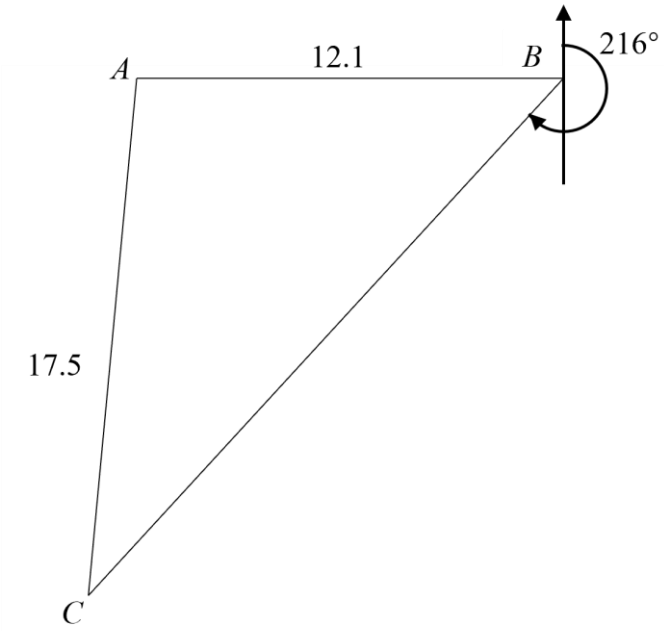
Complete these statements by calculating the size of each angle. Give a reason for each statement.

Statement	Reason
Angle *OBP* = 90°	Tangent ⊥radius [B1]
Angle *BOP* = 180° – 90° – 20° = 70°	∠ Sum of Δ [B1]
Angle *OBD* = $\frac{180° - 70°}{2}$ = 55°	base ∠ of isos. Δ [B1]
Reflect Angle AOB = 55° x 2 x 2 = 220°	∠ at center = 2 ∠ at circumference [B1]
Or 360° – 70°- 70° = 220°	Tangent from ext. pt. and ∠s at a pt.

[4]

(b) The diagram shows the location of 3 ports *A*, *B* and *C*. *A* is due west of *B*.
The bearing of *C* from *B* is 216°.
AB = 12.1 km and *AC* = 17.5 km

Find the bearing of *C* from *A*.



$\angle ABC = 270° - 216° = 54°$

Using sine rule

$$\frac{\sin 54°}{17.5} = \frac{\sin ACB}{12.1}$$ [M1]

$$\sin ACB = \frac{\sin 54° \times 12.1}{17.5}$$

$\angle ACB = \sin^{-1} 0.55938 = 34.013°$ [M1]

$\angle CAB = 180° - 54° - 34.013 = 91.987°$ [M1]

Bearing of *C* from *A* = 90° + 91.987° = 182° [A1]

Answer ……° [4]

End of Paper