

MATHEMATICS

9740/02

Paper 2

14 September 2012 3 hours

Additional Materials: Answer Paper List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in. Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

- **1** The functions f, g and h are defined as follows:
 - $f: x \to \sin(\pi x), \qquad x \in \Box, \ 0 \le x \le 2,$ g: x \to a^x, $x \in \Box, \ x \ge -1$, where a is a constant and 0 < a < 1, h: x \to f(x), $x \in \Box, \ m \le x \le n$.
 - (i) Explain why f^{-1} does not exist. Find *m* and *n* so that h^{-1} exists and the range of h is equal to the range of f. Find $h^{-1}\left(\frac{1}{2}\right)$. [3]
 - (ii) Show that the composite function gh exists, and find the range of gh in terms of a. [3]
- 2 A Hwa Chong student taking part in Project Day needs to design a rectangular poster to showcase his project. According to the rules set for the poster design, the poster should contain 1352 cm² of printing with margins of 4 cm each at the top and bottom, and 2 cm each on the left and right sides. By using differentiation, determine the dimensions of the poster to minimise the amount of paper used. [6]
- 3 (i) Show that for any complex number $z = e^{i\theta}$,

$$z^n + \frac{1}{z^n} = 2\cos n\theta .$$
 [2]

(ii) By taking n = 1, show that

$$\cos^{3}\theta = \frac{1}{8} \left(z^{3} + 3z + \frac{3}{z} + \frac{1}{z^{3}} \right).$$

Deduce that

$$\cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta .$$
 [4]

(iii) Find $\int \cos^3 3\theta \, d\theta$. [2]

4 (a) Referred to the origin *O*, the position vectors of three points *A*, *B* and *P* are **a**, **b** and $\mathbf{a} + 5\mathbf{b}$ respectively. Given that **b** is a unit vector, the angle *AOB* is 60° and *AB* is perpendicular to *OP*, find $|\mathbf{a}|$. [4]



In the triangle *OAB* where *O* is the origin, the position vectors of the points *A* and *B* are **a** and **b** respectively. The point *C* is the midpoint of *OA*, the point *E* on *BC* is such that CE:EB = 3:4, and the line *OE* meets *AB* at *D*. Find the ratio AD:AB. [4]

5 A curve G has equation $y = ax + b + \frac{c}{(x+1)^2}$, where a, b and c are positive integers.

Let *R* be the region bounded by the curve *G*, the axes and the line x = 3.

- (i) Explain why the graph of G lies above the x-axis for all x > 0. [1]
- (ii) If the area of the region R is 42 units², show that

$$\frac{9}{2}a + 3b + \frac{3}{4}c = 42.$$
 [2]

- (iii) It is also known that the curve G has a minimum turning point at (0, 5). Find the equation of the curve G. [3]
- (iv) Sketch the curve G, stating clearly the equation of any asymptote(s), turning point(s) and axial intercept(s).[3]
- (v) The region S is bounded by the curve G, the y-axis, the lines x=3 and y=8x+1. Find the volume of solid generated when region S is rotated through 2π radians about the x-axis. [3]

(b)

Section B: Statistics [60 marks]

- 6 A manager claims that the time spent by each customer at his supermarket follows a normal distribution with mean 35 minutes and standard deviation 30 minutes. A statistician comments that the distribution N(35, 30²) will not provide an adequate model.
 - (i) Do you agree with the statistician's comment? Give a reason to support your answer. [2]
 - (ii) Suppose the time spent by each customer follows a normal distribution and its standard deviation is 10 minutes instead of 30 minutes, find the probability that the total time spent by 2 randomly chosen customers in the supermarket is more than 3 times that of another randomly chosen customer. [3]
- 7 In a particular year, a company awards scholarships to 4 applicants from a group of n applicants. If the order of selection is not taken into account, the number of ways in which the 4 scholarship recipients can be chosen is 7315.
 - (i) Find the number of ways in which the 4 scholarship recipients can be chosen if the order of selection is taken into account. [1]
 - (ii) Find the value of *n*.

In the year 2012, there are 13 scholarship applicants. The 13 applicants, which include the eventual 4 scholarship recipients Ann, Belle, Connie and Don, are required to attend a briefing. The applicants are to be seated in 2 rows as shown below:



Find the number of ways in which the 13 applicants can be seated if all 4 scholarship recipients must sit together during the briefing. [3]

- 8 An eatery is giving out free burgers in a promotion to celebrate its anniversary. A total of 80 burgers, consisting of 8 different flavours, 10 from each flavour, are to be given out. The burgers are randomly chosen and given out one at a time. Of the 8 burger flavours, the 'Curry' and 'Spicy' burgers are most popular among customers. In the first four burgers to be given out, the events *A* and *B* are defined as follows:
 - A: At least 2 'Curry' burgers.
 - *B*: Exactly one 'Spicy' burger.
 - (i) Find $P(A \mid B)$. [3]
 - (ii) Justify if A' and B' are independent events.

[2]

[3]

- **9** In each large batch of SIM cards, 15% of the cards are defective. From each batch, a random sample of 10 cards is drawn for inspection.
 - (i) State the distribution of the number of defective SIM cards in the sample. Write down an assumption in order for the distribution stated to be valid. [2]
 - (ii) A batch is labeled as 'good' if the sample drawn has less than 2 defective SIM cards. If 33 batches are produced daily, find using a suitable approximation, the probability that in a day, more than 80% produced are 'good' batches. [4]
- 10 (a) The supervisor of a petrol kiosk wants to monitor the service provided by his staff. He decides to choose a sample of 30 customers to take part in a survey at the payment counter before they leave the petrol kiosk.
 - (i) Describe how a systematic sample of 30 customers can be chosen from the first 120 customers in a day. [2]
 - (ii) Give a reason whether stratified sampling is appropriate in this situation. [1]
 - (b) The waiting time, in minutes, for a customer to be served at a petrol kiosk is a random variable with mean μ and standard deviation σ . The following are observed based on a random sample of 100 customers:
 - for 95% of the time, the mean waiting time is between 7.5 minutes and 10 minutes;
 - the probability that the mean waiting time is more than 10 minutes is twothirds of the probability that the mean waiting time is less than 7.5 minutes.

Find the value of μ .

11 A machine produces a particular type of miniature wheels for model trains. The diameter in millimetres (mm) of one such miniature wheel, denoted by X, has an expected value of 10.00. In a routine check, a random sample of 8 miniature wheels is taken and their diameters (in mm) are measured as follows:

10.21 10.34 9.98 10.45 9.76 10.51 9.82 10.68

- (i) Explain what is meant by 'unbiased estimator of the population variance'. [1]
- (ii) Calculate unbiased estimates of the mean and variance of *X*. [2]
- (iii) Stating appropriate hypotheses, with any symbols used clearly defined, test at the 10% level of significance if the mean diameter of miniature wheels produced is different from expected.Write down an assumption for your test to be valid. [5]
- (iv) It is given that the standard deviation of X is 0.45 mm. If the assumption made in part (iii) is still valid, find the set of values of \overline{x} , the mean diameter of a set of 8 randomly chosen miniature wheels, so that it can be concluded at the 5% level of significance that the expected value of the diameter of the miniature wheels has increased. [3]

[4]

12 An experiment was carried out to test the effect of a new additive on the carbon emission of a petrol engine. The carbon emission E of the petrol engine, in grams, was recorded for w millilitres of additive added. The results of 9 readings are given in the table.

W	20	40	60	80	100	120	140	160	180
Ε	209	183	160	143	135	124	115	107	108

[2]

[2]

- (i) Draw a scatter diagram for these values.
- (ii) The product moment correlation coefficient of *w* and *E* based on the 9 readings is -0.958, correct to 3 decimal places. Explain why it is not appropriate to use the formula E = a + bw, where *a* and *b* are constants, to model the carbon emission *E* even though the product moment correlation coefficient is close to -1. [1]

Two additional formulae were proposed to model the carbon emission E:

$$\ln E = c + dw$$
 or $\frac{1}{E} = e + fw$

where c, d, e and f are constants.

- (iii) Determine which of the above formulae is a better model. Give a reason to support your answer. [2]
- (iv) Using the better model determined in part (iii), find a suitable least squares regression line and use it to estimate the amount of additive needed to obtain a carbon emission of 0.13 kg, correct to 2 decimal places. [2]
- 13 The average number of railway accidents occurring along the North-South (NS) line in any one month is m. Accidents occur independently of one another and the probability of a railway accident occurring due to technical fault is given by p. The probability that there is at least one railway accident occurring along the NS line in any one week is known to be 0.25. A week is taken as a fourth part of a month.
 - (i) Find the exact value of *m*.
 - (ii) Find the probability that, in any one week, there are two railway accidents occurring along the NS line with exactly one of them due to technical fault. Give your answer in terms of p. [2]

The average number of railway accidents occurring along the East-West (EW) line in a month is 2. Accidents occurring along the NS line are independent of accidents occurring along the EW line.

- (iii) Find the probability that, in any one week, no railway accidents occur along the EW line given that a total of 3 railway accidents occur along the two lines. [3]
- (iv) Find, using a suitable approximation, the probability that in 5 years, there are more than 52 months with at most 4 railway accidents occurring along the EW line.