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JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
Secondary 4 Express

STUDENT NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1

27 AUGUST 2021

2 hours 15 Minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **ALL** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

DO NOT OPEN THE BOOKLET UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use
/ 90

Setter: Mrs Adeline Pang

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 A particle moves along the curve $y = x\cos 3x$ in such a way that the y-coordinate decreases at a constant rate of 0.2 radian/sec at the instant when $x = \pi$. [3]
Find the rate of change of the x-coordinate at this instant.

2 Given that $y = x^3 + mx^2 + nx + 15$ where m and n are integers, is a decreasing function of x for $-2 < x < \frac{4}{3}$. Find the values of m and of n. [4]

3 **(a)** Simplify $\frac{7^{1-5x} \times 49^{x}}{3(343^{x})+7^{3x}}$ and express it as ab^{-6x} where *a* and *b* are non-zero constants. [2]

(b) Given that $\frac{7^{1-5x} \times 49^{x}}{3(343^{x})+7^{3x}} = 64^{x}$. Hence, find the exact value of 14^{6x}. [2]

4 A factory purchased a new machine for its operation. After t months, the value of the machine $\$A$ is given by $A = 5600e^{-kt}$, where k is a constant.

(a) Find the initial value of the machine when it is newly purchased. [1]

(b) The value of the machine after 6 months is expected to be \$4800. Find the value of k. [2]

(c) Calculate the age of the machine, to the nearest month, when the expected value is \$1200. [2]

5 Find the stationary point of the curve that is given by $y = xe^{1-2x}$ and determine the nature of the stationary point. [5]

6 **(a)** Find the range of values of x such that $2x^2 + x - 15$ is always positive. [2]

(b) Given the line $y = x + 2p$ is a tangent to the curve $y^2 = qx$, where p and q are positive constants, show that q is a multiple of p. [3]

7 Given that $\sin\theta = -\frac{3}{5}$ and $\tan\theta > 0$, find the exact value of

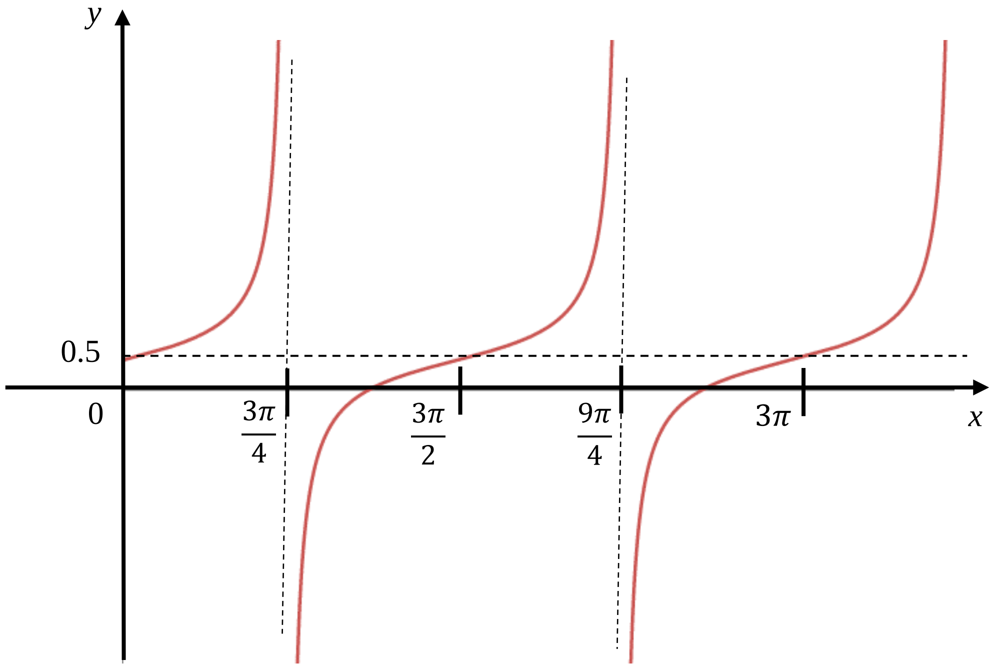
(a) $\tan 2\theta$, [2]

(b) $\cos\frac{\theta}{2}$. [3]

8 **(a)** Given that $\frac{\cos(A+B)}{\cos(A-B)} = \frac{5}{6}$, show that $\cot A = 11 \tan B$. [3]

(b) Hence, find the value of $\tan A$ when $\tan 2B = 0.18$ and B is acute. [3]

9 The diagram shows the graph of the equation $y = \tan ax + b$ where a and b are constants.

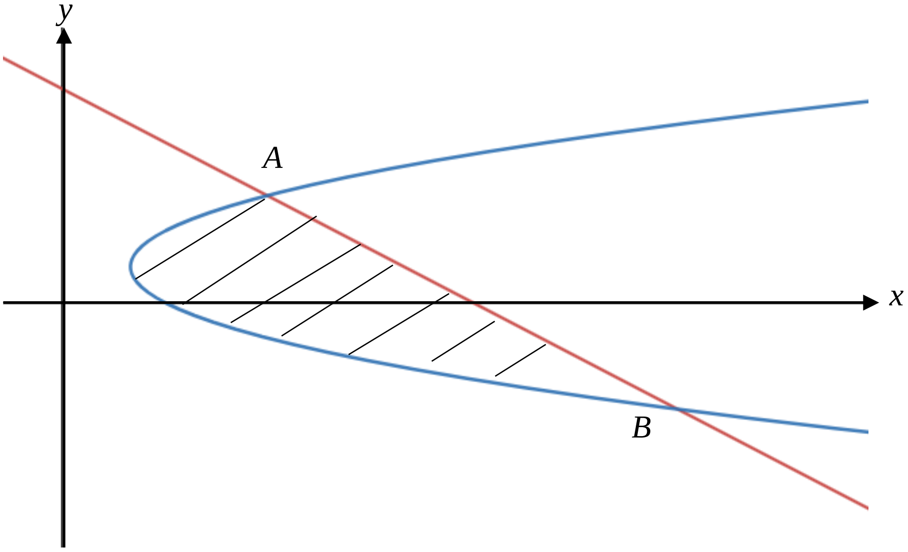


(a) Write down the value of a and of b. [2]

(b) Given that the line whose equation is $y = k$, intersects the curve $y = \tan ax + b$ at 2 points for $0 \le x \le 3\pi$,find the range of value of k. [1]

(c) Using the values of a and b, solve $\tan ax + b = 1$ where $0 \le x \le 3\pi$. [3]

10 The diagram shows part of the curve $x = (y-1)^2 + 2$ and the line $= -\frac{1}{2}x + 6$. The line intersects the curve at the points A and B. [6]



Find the area of the shaded region.

11 The expression $f(x) = x^3 + x^2 + ax + b$ where a and b are constants, is exactly divisible by $x - 2$ and leaves a remainder of 21 when divided by $x + 1$.

(a) Find the values of a and of b. [4]

(b) Using your values of a and of b from **(a)**, solve $x^3 + x^2 + ax + b = 0$. [3]

Express the non-integer roots in the form of $\frac{c \pm \sqrt{d}}{2}$, where c and d are integers.

12 **(a)** Prove $\frac{\sin 2A-1}{1+\cos 2A} = -\frac{1}{2}(\tan A - 1)^2$. [4]

(b) Hence, solve $\frac{\sin 4A-1}{1+\cos 4A} = -\frac{3}{4}$ for $0 \leq A \leq \pi$. [4]

13 **(a)** Express $\frac{9x^2-2x+10}{(2x-1)(x^2+2)}$ in partial fractions. [4]

(b) Differentiate $\ln(x^2+2)$ with respect to x. [1]

(c) Using the results from **(a)** and **(b)**, find $\int_1^2 \frac{9x^2-2x+10}{(2x-1)(x^2+2)}\, dx$. [3]

14 The points $A(1\ ,7)$ and $B(6\ ,2)$ lies on the circumference of a circle, C_1, with centre *D*. *D* lies on the line $2y = -x + 5$.

(a) Find the coordinates of *D*. [5]

(b) Find the equation of the circle, C_1. [2]

(c) Another circle, C_2 intersects the circle, C_1 at *A*, *B* and *D*. Explain why *AB* is the diameter of C_2. [2]

15 A particle travels in a straight line, such that *t* seconds after leaving a fixed point *A*, its velocity, v ms^{-1}, is given by $v = 3t^2 - 14t + 15$.

(a) Find the initial velocity. [1]

(b) Find the time(s) when the particle is at instantaneous rest at *B*. [1]

(c) Find the acceleration of the particle when $t = 1.5$. [2]

(d) Find the average speed of the particle in the interval $t = 0$ to $t = 2$. [5]

End of Paper