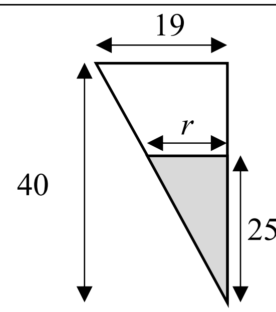
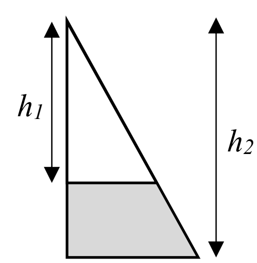
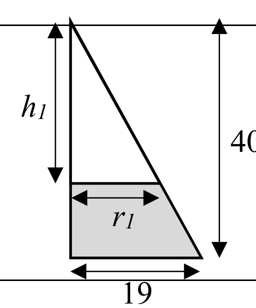
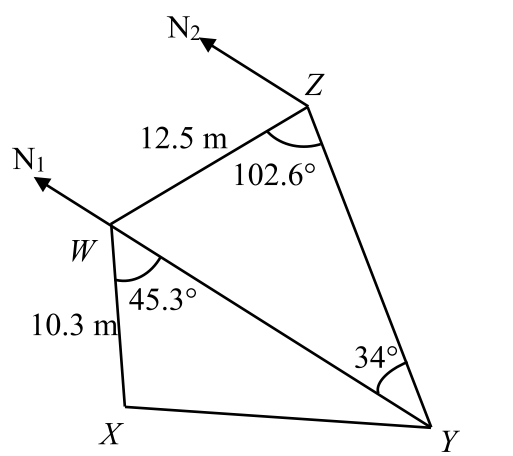
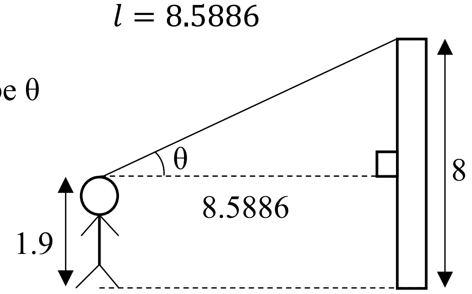
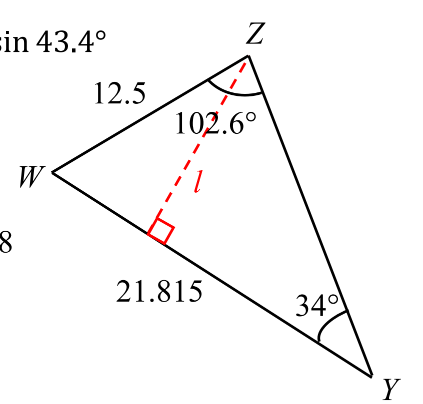
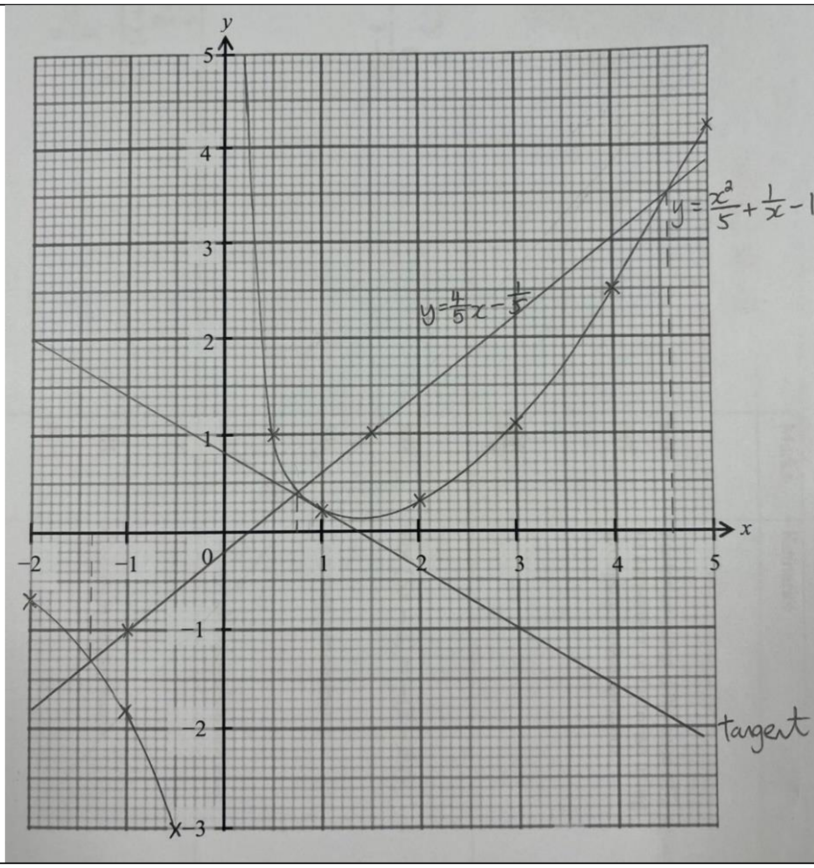
NCHS 2024 EM Paper 2 Solutions

1	(a) $\frac{9a^2-4b^2}{3ap+6aq-4bq-2bp}$ $=\frac{(3a+2b)(3a-2b)}{(3ap+6aq)-(2bp+4bq)}$ $=\frac{(3a+2b)(3a-2b)}{3a(p+2q)-2b(p+2q)}$ $=\frac{(3a+2b)(3a-2b)}{(p+2q)(3a-2b)}$ $=\frac{3a+2b}{p+2q}$	(b) $\frac{x}{6-7x-5x^2}+\frac{2}{5x-3}$ $=\frac{x}{-(5x-3)(x+2)}+\frac{2}{5x-3}$ $=\frac{-x+2(2+x)}{(5x-3)(x+2)}$ $=\frac{-x+4+2x}{(5x-3)(x+2)}$ $=\frac{x+4}{(5x-3)(x+2)}$
1(c)	$-\frac{1}{2}<\frac{2y}{5}-\frac{1+y}{3}\leq\frac{5}{6}$ $-\frac{1}{2}<\frac{3(2y)-5(1+y)}{15}$ *and* $\frac{3(2y)-5(1+y)}{15}\leq\frac{5}{6}$ $-\frac{1}{2}<\frac{6y-5-5y}{15}$ $\frac{6y-5-5y}{15}\leq\frac{5}{6}$ $-\frac{1}{2}<\frac{y-5}{15}$ $\frac{y-5}{15}\leq\frac{5}{6}$ $-15<2(y-5)$ $y-5\leq\frac{25}{2}$ $-15<2y-10$ $y\leq 17\frac{1}{2}$ $-5<2y$ $-2\frac{1}{2}<y\leq 17\frac{1}{2}$	
2(a)	$\frac{r}{19}=\frac{25}{40}$ $r=\frac{25}{40}\times 19$ $=11.875$ $V=\frac{1}{3}\pi(11.875)^2(25)$ $=3691.78\ cm^3$ $=3.69\ litres$	
(b)	**Method 1** $Volume\ of\ container=\frac{1}{3}\pi(19)^2(40)$ $\left(\frac{h_1}{h_2}\right)^3=\frac{V_1}{V_2}$ $\left(\frac{h_1}{40}\right)^3=\frac{\frac{1}{3}\pi(19)^2(40)-\frac{1}{3}\pi(11.875)^2(25)}{\frac{1}{3}\pi(19)^2(40)}$ $\frac{h_1}{40}=\sqrt[3]{\frac{(19)^2(40)-(11.875)^2(25)}{(19)^2(40)}}$ $h_1=40\times\sqrt[3]{\frac{(19)^2(40)-(11.875)^2(25)}{(19)^2(40)}}$ $=36.437$ $Depth\ of\ water=40-36.437$ $=3.56\ cm\ (3sf)$	
	Method 2 $\frac{r_1}{19}=\frac{h_1}{40}$ $r_1=\frac{19}{40}h_1$ $\frac{1}{3}\pi(19)^2(40)-\frac{1}{3}\pi(r_1)^2(h_1)=3691.78$ $\frac{1}{3}\pi(r_1)^2(h_1)=11429.75$	

	$\left(\frac{19}{40}h_1\right)^2 (h_1) = 10914.61$ ${h_1}^3 = 48375.00277$ $h_1 = 36.437$ $Depth\ of\ water = 40 - 36.437$ $= 3.56\ cm\ (3sf)$	
3(a)	$\frac{108888-100176.96}{108888} \times 100\%$ $= 8\%$	
(b)	Balance owed $= \frac{85}{100} \times 108888$ $= \$92554.80$ Interest $= \frac{92554.80 \times 2.98 \times 5}{100}$ $= \$13790.6652$ Monthly instalment $= \frac{92554.80+13790.6652}{5 \times 12}$ $= \$1772.42\ (2dp)$	
(c)	$P = 108888 - 18888$ $= \$90000$ $A = 960 \times 10 \times 12$ $= \$115200$ $A = P\left(1+\frac{R}{100}\right)^n$ $115200 = 90000\left(1+\frac{R}{100}\right)^{10}$ $\left(1+\frac{R}{100}\right)^{10} = \frac{115200}{90000}$ $1+\frac{R}{100} = \sqrt[10]{\frac{115200}{90000}}$ $R = 100\left(\sqrt[10]{\frac{115200}{90000}} - 1\right)$ $= 2.499$ $= 2.50$ (3sf)	
4(a)	$Q = \begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$	
(b)	$R = \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ $= \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$	
(c)	The elements in R represent the total cost of baking caramel, strawberry and mint cupcakes in outlet A, B and C respectively	
(d)	**Method 1** Selling Price $= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ $= \begin{pmatrix} 3.60 \\ 3.75 \\ 3.80 \end{pmatrix}$	**Method 2** Profit for each type of cupcake $= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ $= \begin{pmatrix} 2.40 \\ 2.25 \\ 2.85 \end{pmatrix}$

	Profit earned in a week $= \begin{pmatrix}60 & 68 & 55\\ 49 & 56 & 71\\ 53 & 70 & 80\end{pmatrix}\begin{pmatrix}3.60\\ 3.75\\ 3.80\end{pmatrix} - \begin{pmatrix}226.25\\ 210.25\\ 244.60\end{pmatrix}$ $= \begin{pmatrix}680\\ 656.20\\ 757.30\end{pmatrix} - \begin{pmatrix}226.25\\ 210.25\\ 244.60\end{pmatrix}$ $= \begin{pmatrix}453.75\\ 445.95\\ 512.70\end{pmatrix}$ Total Profit earned in a month $= 4\left[\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}453.75\\ 445.95\\ 512.70\end{pmatrix}\right]$ $= 4(1412.40)$ $= (5649.60)$ Total Profit = \$5649.60	Profit earned in a week $= \begin{pmatrix}60 & 68 & 55\\ 49 & 56 & 71\\ 53 & 70 & 80\end{pmatrix}\begin{pmatrix}2.40\\ 2.25\\ 2.85\end{pmatrix}$ $= \begin{pmatrix}453.75\\ 445.95\\ 512.70\end{pmatrix}$ Total Profit earned in a month $= 4\left[\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}453.75\\ 445.95\\ 512.70\end{pmatrix}\right]$ $= 4(1412.40)$ $= (5649.60)$ Total Profit = \$5649.60
5(a)	$\frac{WY}{\sin 102.6°} = \frac{12.5}{\sin 34°}$ $WY = \frac{12.5}{\sin 34°} \times \sin 102.6°$ $= 21.815$ $XY^2 = 10.3^2 + 21.815^2 - 2(10.3)(21.815)(\cos 45.3°)$ $XY = 16.306 \quad or \quad -16.306(NA)$ $= 16.3\ (3sf)$	
(b)	**Method 1** $\angle N_1WZ = 102.6° + 34°\ (ext\ \angle\ of\ \Delta)$ $= 136.6°$ $\angle WZN_2 = 180° - 136.6°\ (int\ \angle, N_1W//N_2Z)$ $= 43.4°$ $Bearing\ of\ W\ from\ Z = 360° - 43.4°\ (\angle\ at\ a\ pt)$ $= 316.6°$	
	Method 2 $\angle YWZ = 180° - (34° + 102.6°)\ (\angle\ sum\ of\ \Delta)$ $= 43.4°$ $\angle WZN_2 = 43.4°\ (alt\ \angle, N_1W//N_2Z)$ $Bearing\ of\ W\ from\ Z = 360° - 43.4°\ (\angle\ at\ a\ pt)$ $= 316.6°$	
(c)	$\angle YWZ = 43.4°\ (alt\ \angle, N_1Y//N_2Z)$ $l = 12.5 \sin 43.4°$ OR $\frac{1}{2} \times 21.815 \times l = \frac{1}{2} \times 12.5 \times 21.815 \times \sin 43.4°$ $= 8.5886$ $\quad l = 8.5886$ Let the greatest angle of elevation be θ $\tan\theta = \frac{8-1.9}{12.5 \sin 43.4°}$ $\theta = 35.38°$ $= 35.4°$ 	

6(a)	$m = 4.2$	
(b)		
(c)	Draw tangent at (1, 0.2) Gradient = −0.5 to −0.7	
(d)	$x^2 + \frac{5}{x} - 4x - 4 = 0$ $\frac{x^2}{5} + \frac{1}{x} - \frac{4}{5}x - \frac{4}{5} = 0$ $\frac{x^2}{5} + \frac{1}{x} - 1 = \frac{4}{5}x - \frac{1}{5}$ $y = \frac{4}{5}x - \frac{1}{5}$ Plot graph of $y = \frac{4}{5}x - \frac{1}{5}$ $x = -1.35 \pm 0.05 \quad or \quad 0.75 \pm 0.05 \quad or \quad 4.6 \pm 0.05$	
7(a)	**Method 1** $\angle BRC = \angle PRQ \ \ (common\ \angle)$ $\frac{CR}{QR} = \frac{1}{2} \ \ (C\ is\ midpt\ of\ QR)$ $\frac{BR}{PR} = \frac{1}{2} \ \ (diagonals\ of\ parallelogram\ bisect\ each\ other)$ $\Delta RBC\ and\ \Delta RPQ\ are\ similar\ (SAS\ Similarity)$ **Method 2** Since diagonals of a parallelogram bisect each other, *B* is the midpoint of *PR*. In addition *C* is given as the midpoint of *QR*, using midpoint theorem, *BC* is parallel to *PQ*. $\angle BRC = \angle PRQ \ \ (common\ \angle)$ $\angle BCR = \angle PQR \ \ (corresponding\ \angle, BC//PQ)$ $\Delta RBC\ and\ \Delta RPQ\ are\ similar\ (AA\ Similarity)$	

(b)

Method 1

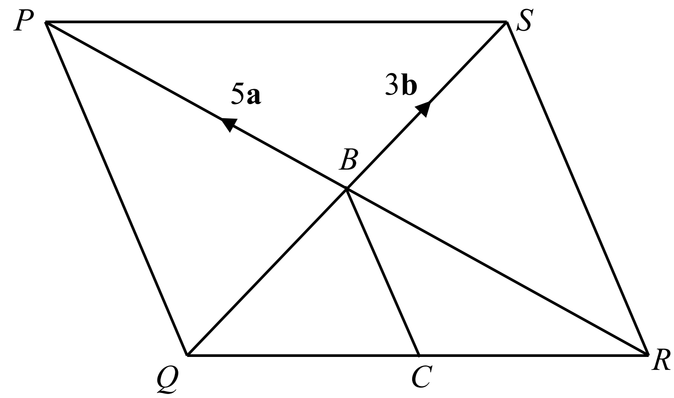
$\overrightarrow{PC} = \overrightarrow{PQ} + \overrightarrow{QC}$

$= \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) + \frac{1}{2}\overrightarrow{QR}$

$= -5\boldsymbol{a} - 3\boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b})$

$= -\frac{15}{2}\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$

$= -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ (*Shown*)



Method 2

$\overrightarrow{PC} = \overrightarrow{PB} + \overrightarrow{BC}$

$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{PQ}$

$= -5\boldsymbol{a} + \frac{1}{2}(-5\boldsymbol{a} - 3\boldsymbol{b})$

$= -\frac{15}{2}\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$

$= -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ (*Shown*)

(c)

Method 1

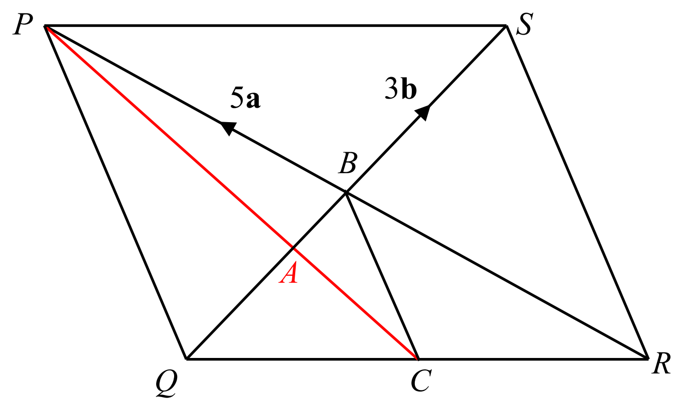
$\overrightarrow{AQ} = \frac{2}{3}(-3\boldsymbol{b})$

$= -2\boldsymbol{b}$

$\overrightarrow{AC} = \overrightarrow{AQ} + \overrightarrow{QC}$

$= -2\boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b})$

$= -\frac{5}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$ ***or*** $-\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$



Method 2

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} - 3\boldsymbol{b})$

$= -\frac{5}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$ ***or*** $-\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$

(d)

$\overrightarrow{PC} = -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ & $\overrightarrow{AC} = -\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$

$\overrightarrow{PC} = 3\overrightarrow{AC}$

∴ *C* is a common point, *P*, *A* and *C* are collinear.

(e)

Method 1

$BC // PQ$ (ΔRBC and ΔRPQ are similar)

$\text{Area } \Delta BPQ = \frac{1}{2} \times PQ \times h$ or $\boldsymbol{\text{Area } \Delta PBC = \frac{1}{2} \times BC \times h}$

$= \text{Area } \Delta CPQ$ $\boldsymbol{= \text{Area } \Delta QBC}$

Since ΔAPQ *is common area* or ***ΔABC is common area***

∴ *Area* ΔPAB *and Area* ΔQAC *are the same*

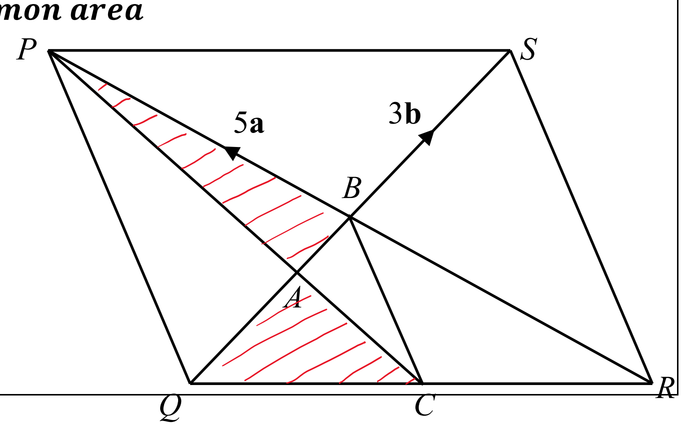
Method 2

$\angle PAB = \angle QAC$ (*vert opp* $\angle$)

$\frac{AB}{QA} = \frac{1}{2}$ (*from* (*c*)) & $\frac{PA}{AC} = \frac{2}{1}$ (*from* (*d*))

$\frac{\text{Area } \Delta PAB}{\text{Area } \Delta QAC} = \frac{\frac{1}{2} \times PA \times AB \times \sin \angle PAB}{\frac{1}{2} \times QA \times AC \times \sin \angle QAC}$

$= \frac{PA}{AC} \times \frac{AB}{QA}$



	$= \frac{2}{1} \times \frac{1}{2}$ $= \frac{1}{1}$ $\therefore Area\ \Delta PAB\ and\ Area\ \Delta QAC\ are\ the\ same$ **Method 3** $\frac{Area\ of\ \Delta PAB}{Area\ of\ \Delta QAC} = \frac{Area\ of\ \Delta PAB}{Area\ of\ \Delta ABC} \times \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QAC}$ $= \frac{\frac{1}{2}\times PA\times h_1}{\frac{1}{2}\times AC\times h_1} \times \frac{\frac{1}{2}\times AB\times h_2}{\frac{1}{2}\times QA\times h_2}$ $= \frac{PA}{AC} \times \frac{AB}{QA}$ $= \frac{2}{1} \times \frac{1}{2}$ $= \frac{1}{1}$ $\therefore Area\ \Delta PAB\ and\ Area\ \Delta QAC\ are\ the\ same$	
(f) (i)	$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QBC} = \frac{1}{3}$	
(ii)	$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta RPQ} = \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QBC} \times \frac{Area\ of\ \Delta QBC}{Area\ of\ \Delta RBC} \times \frac{Area\ of\ \Delta RBC}{Area\ of\ \Delta RPQ}$ $= \frac{1}{3} \times \frac{1}{1} \times \left(\frac{1}{2}\right)^2$ $= \frac{1}{12}$	
8(a) (i)	Middle position = 400^{th} Median Length = 10.6 cm	
(ii)	Q1 position = 200^{th} Q3 position = 600^{th} Q1 = 9.95 Q3 = 11.25 or 11.2 Interquartile Range = 11.25 – 9.95 or 11.2 – 9.95 = 1.3 cm or 1.25 cm	
(b)	$Percentage = \frac{15+25}{800} \times 100\%$ or $\frac{20+25}{800} \times 100\%$ $= 5\%$ or 5.625%	
(c)	Lobsters caught in June are **shorter** in length as the median length of **9.3 cm** is **lower** as compared to **10.6 cm** in May. The lengths of the lobsters caught in June has a **wider spread** as its ITR of **3.7 cm** is **higher** as compared to **1.3 cm** in May.	
(d)	I disagree with Jill as **25%** of lobsters needed to be released in June because they are shorter than 8.3 cm which is **higher** as compared to the total percentage released of **5.625%** in May. Or I disagree with Jill as **75%** of the lobsters caught in June are within the legal length which is **lower** as compared to **94.375%** of the lobster caught in May are within the legal length.	

9(a)

(i) $\angle MLO = 47°\ (base\ \angle\ in\ isos\ \Delta)$

$\angle MNH = 180° - 47°\ (\angle\ in\ opp\ segment)$

$= 133°$

(ii) $\angle MOH = 47° \times 2\ (\angle\ at\ center = 2\ \angle\ at\ circumference)$

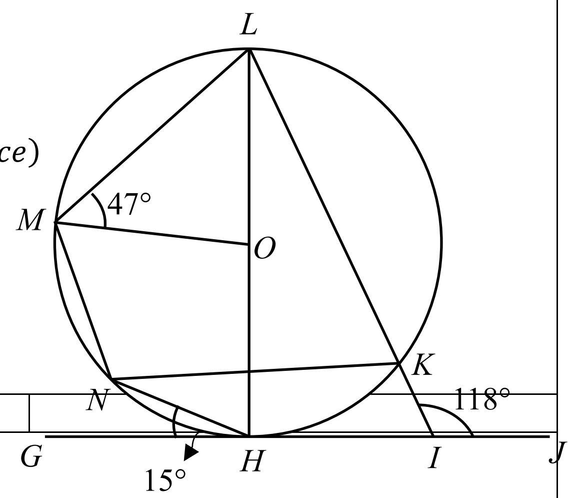
$= 94°$

(iii) $\angle LHJ = 90°\ (tangent\ \perp radius)$

$\angle HLI = 118° - 90°\ (ext\ \angle\ of\ \Delta)$

$= 28°$

$\angle KNH = 28°\ (\angle\ in\ same\ segment)$



(b)

$\angle LHG = 90°\ (tangent\ \perp radius)$

$\angle OHN = 90° - 15°$

$= 75°$

$\angle OMN = 360° - (133° + 75° + 94°)\ (\angle\ sum\ of\ quadrilateral)$

$= 58°$

$\angle NML + \angle MLI = (58° + 47°) + (47° + 28°)$

$= 180°$

Using property of interior angles, $\angle NML$ and $\angle MLI$ add up to 180°, *MN* and *LI* are parallel lines.

10 (a)

Method 1

$\sin 60° = \frac{h_1}{10}$ $\quad\quad$ $\tan\left(\frac{\frac{3\times180}{5}}{2}\right)° = \frac{h_2}{5}$

$h_1 = 10 \sin 60°$ $\quad\quad$ $h_2 = 5 \tan 54°$

$= 8.6603$ $\quad\quad$ $= 6.8819$

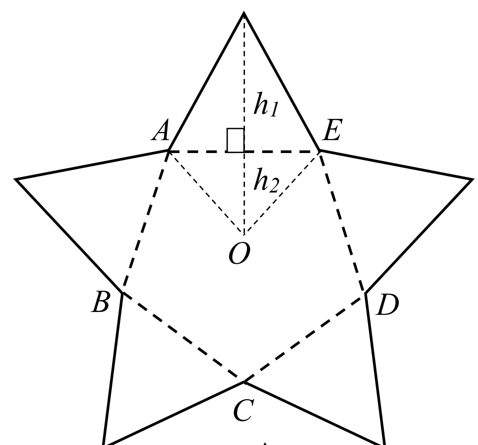
$h_1 + h_2 = 15.5422$

$Area\ PAO = \frac{1}{2} \times 15.5422 \times 5$

$Area\ of\ 5\ pointed\ star = 10 \times \frac{1}{2} \times 15.5422 \times 5$

$= 388.555$

$= 389\ cm^2\ (3sf)$



Method 2

Area of 1 equilateral Δ $\quad\quad$ Area of 1 isosceles Δ

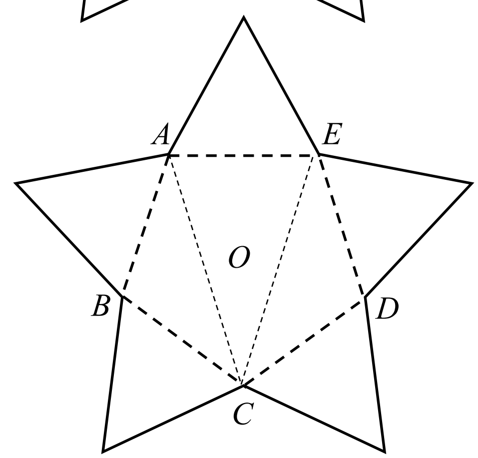
$= \frac{1}{2} \times 10 \times 10 \times \sin 60°$ $\quad\quad$ $= \frac{1}{2} \times 10 \times 5 \tan\left(\frac{\frac{3\times180}{5}}{2}\right)°$

$= 43.301$ $\quad\quad$ $= 34.410$

$Area\ of\ 5\ pointed\ star = 5 \times (43.301 + 34.410)$

$= 388.555$

$= 389\ cm^2\ (3sf)$



Method 3

$AC = 16.180\ cm$

$Area\ of\ 5\ pointed\ star$

$= 5 \times Area\ \Delta APE + 2 \times Area\ \Delta ABC + \times Area\ \Delta ACE$

$= 5 \times \left(\frac{1}{2} \times 10 \times 10 \times \sin 60°\right) + 2 \times \left(\frac{1}{2} \times 10 \times 10 \times \sin 108°\right) + \frac{1}{2} \times 16.180 \times 10 \times \sin 72°$

$= 216.506 + 95.106 + 76.940$

$= 388.552$

$= 389\ cm^2\ (3sf)$

(b)

$Density = 900 kg/m^3$
$= 0.9 g/cm^3$

Design I

$Outer\ Radius = 15.5422 + 6 = 21.5422$
$Cross\ SA = \pi(21.5422)^2 - \pi(15.5422)^2$
$= 699.024\ cm^2$
$Volume = 699.024 \times 12$
$= 8388.288\ cm^3$
$Mass\ (wood) = 8388.288 \times 0.9$
$= 7549.4592\ grams$
$Total\ Mass = 7549.4592 + 600$
$= 8149.4592\ grams$

Design II

$l = \dfrac{6}{\cos 36°}$
$= 7.4164$
$Cross\ SA$
$= 5\left[\frac{1}{2}(15.5422 + 7.4164)^2 \sin 72° - \frac{1}{2}(15.5422)^2 \sin 72°\right]$
$= 678.905\ cm^2$
$Volume = 678.905 \times 12$
$= 8146.86\ cm^3$
$Mass(wood) = 8146.86 \times 0.9$
$= 7332.174\ grams$
$Total\ Mass = 7332.174 + 600$
$= 7932.174\ grams$

Since $7932.174 < 8000 < 8149.4592\ grams$, Jack should choose **Design II**.