	EUNOIA JUNIOR COLLEGE
	JC2 Preliminary Examination 2022
と で で	General Certificate of Education Advanced Level
	Higher 1

CANDIDATE NAME	
CIVICS	INDEX NO.

MATHEMATICS

Paper 1 [100 marks]

GROUP

13 September 2022

8865/01

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE ON ANY BARCODES.

Answer all questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of 21 printed pages and 1 blank page.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Total

Section A: Pure Mathematics [40 marks]

- 1 Find the set of values of the real constant k for which the expression $kx^2 + (k-1)x + 2k + 1$ is always negative. [4]
- 2 Starluck Coffee is a distributor that brings in three types of coffee beans: Arabica, Robusta and Liberica. The selling price of Arabica coffee beans, Robusta coffee beans and Liberica coffee beans is \$4.00 per kilogram, \$1.50 per kilogram and \$3.50 per kilogram respectively.

On International Coffee Day, Starluck Coffee brought in a total of 550 kilograms of coffee beans to sell at a bazaar. At the end of the day, $\frac{4}{5}$ of the Arabica coffee beans and $\frac{4}{9}$ of the Robusta coffee beans brought in were sold. The amount of Liberica coffee beans sold was 70 kilograms less than the amount of Arabica coffee beans sold. The amount collected from selling these coffee beans was \$1315.

If all the coffee beans brought in were sold at the end of International Coffee Day, Starluck Coffee would have collected \$1685.

- By writing down three linear equations, find the mass of each type of coffee beans, in kilograms, brought in by Starluck Coffee on International Coffee Day. [4]
- (ii) Given that the cost of one kilogram of Arabica coffee beans, Robusta coffee beans and Liberica coffee beans is \$3.70, \$1.10 and \$3.20 respectively, determine if Starluck Coffee made a profit or loss on International Coffee Day. Justify your answer. [2]

3 (a) Differentiate
$$\left(3x^{\frac{2}{3}} - \frac{2}{x}\right)^5$$
 with respect to x. [2]

(b) Find the exact value of
$$\int_0^1 \frac{1}{\sqrt{3-2x}} dx$$
.

[3]

4



The diagram shows a sketch of the curve with equation $y = 3 - \ln(2 - px)$, where *p* is a constant. The point *A* is where the curve cuts the *x*-axis.

(a) Find the equation of the asymptote to the curve, giving your answer in terms of *p*. [1]

- (b) Show that the x-coordinate of A is $\frac{2-e^3}{p}$.
- (c) Find the equation of the tangent to the curve at the point A, giving your answer in the form y = mx + cwhere m is in terms of p and c is an exact constant. [4]
- 5 The curve C has equation $y=1+\frac{9}{x}$.
 - (i) Sketch the graph of *C*, stating the exact coordinates of any points where the curve crosses the *x*-axis and the equation of any asymptote(s).
 [3]
 - (ii) Find the area of the region bounded by *C*, the *x*-axis, the *y*-axis, and the lines y = 2x + 4 and x = awhere $a > \frac{3}{2}$. Give your answer in terms of *a*. [4]
- 6 Kenrus, a young entreprenuer, sets up his business to sell tablets on 1^{st} January 2022. He tries to model his cost, *C* million dollars per year, at time *t* years. The model he uses is

$$C = 2e^{0.5t-1} - 0.4t - \frac{1}{e}$$
, for $t \ge 0$.

- (i) Find the startup cost of his business.
- (ii) Use differentiation to find the minimum value of *C*, justifying that this value is a minimum. [4]
- (iii) Sketch the graph of *C* against *t*, stating the coordinates of the minimum point and any intersections with the coordinate axes.
- (iv) Use your calculator to find the value of $\int_{1}^{3} 2e^{0.5t-1} 0.4t \frac{1}{e} dt$. In the context of the question, what does this value represent? [2]

Kenrus also models his total profit, P million dollars, at time t years. He believes that

$$P = 2t^{\frac{4}{3}} + e^{0.45t} - 1.$$

(v) Find the rate of increase of his total profit on 1st June 2022.

[2]

[1]

[1]

Section B: Probability and Statistics [60 marks]

7 A dance club of 18 students consists of 4 from the School of Engineering, 6 from the School of Humanities and 8 from the School of Business.

10 students are selected at random to form a team to represent the club in a dance competition.

(i) Find the number of different teams that can be formed by the club if the team consists of 2 students from the School of Engineering, 3 students from the School of Humanities and 5 students from the School of Business.

After the competition, all the 18 students decide to watch a movie in a cinema, and they are to sit in a row of 18 seats at random. It is known that one of the students from the School of Humanities and one of the students from the School of Business are sisters.

- (ii) Find the probability that the 2 sisters sit next to each other. [2]
- (iii) Find the probability that the 4 students from the School of Engineering are all separated given that the 2 sisters sit next to each other. [3]
- 8 Two events A and B are such that P(A) = p, P(B) = 2p, P(A'|B) = 0.8 and $P(A \cup B) = 0.728$.

(a)	Explain what is meant by $P(A' B)$). [1]

- (b) Show that p = 0.28. [2]
- (c) Determine whether the events *A* and *B* are mutually exclusive. [2]
- (d) Complete the Venn diagram to show the probability in each of the four regions. [1]



9 A certain college has two year groups, Year 1 and Year 2. There are 600 students in Year 1 and 500 students in Year 2. Each student takes up exactly one co-curricular activity from Sports, Performing Arts or Clubs and Societies. The number of students in each activity are summarised in the table.

	Sports	Performing	Clubs and
		Arts	Societies
Year 1	320	146	134
Year 2	285	154	61

A student is chosen at random from the college.

(a) Fin	nd the probability that the student is in Year 1 and takes up sports.	[1]
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- (b) Find the probability that the student is either in Year 2 or takes up sports. [1]
- (c) Find the probability that the student is in Year 2 given that the student does not take up performing arts. [1]

On another occasion, 3 of these students are chosen at random, without replacement.

- (d) Find the probability that exactly 2 are in clubs and societies. [3]
- 10 In a large supply of avocados, p% are rotten. A shop sells avocados in boxes of 16 which are randomly chosen.
 - (i) Given that the mean number of rotten avocados in a box is 3.52, show that the value of p is 22. [1]

A box of avocados is rejected if there are more than 3 rotten avocados in it.

(ii) Find the probability that a randomly selected box of avocados was rejected. [2]

Fifteen boxes of avocados are selected at random.

(iii) Find the probability that fewer than 5 boxes were rejected. [2]

[2]

Six boxes of avocados are selected at random.

- (iv) Find the probability that all 6 boxes will be accepted.
- 11 Subbay Cookies bakes two types of cookies, oatmeal cookies and white chip cookies. The masses, in grams, of oatmeal cookies and white chip cookies have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Oatmeal cookies	40	1.2
White chip cookies	45	0.4

- (i) Find the probability that the mass of a randomly chosen oatmeal cookie is more than 2% above the mean mass. [2]
- (ii) Find the probability that two randomly chosen white chip cookies each weigh less than 44.9 grams. [2]

The cookies are sold by weight. The selling price of the oatmeal cookies and white chip cookies are \$4 per 100 grams and \$6 per 100 grams respectively.

(iii) Mr Eu bought 6 oatmeal cookies and 10 white chip cookies for his family. Find the probability that Mr Eu paid less than \$36.50 for these cookies. State the mean and variance of the distribution that you use.
[4]

The waiting time, *T* minutes, before a customer is served at Subbay Cookies during Sundays has a mean of 10 minutes and a standard deviation of 2.5 minutes.

- (iv) On a randomly chosen Sunday, 80 customers patronised Subbay Cookies. Estimate the probability that the average waiting time of the 80 customers exceeds the mean waiting time by at most 0.5 minutes.
 [4]
- 12 The management of a school claims that the mean studying time per week of the students is 38 hours. A teacher suspects that the management may have overstated the mean studying time per week and decides to carry out a hypothesis test. He selects a random sample of 40 students and records their studying time per week, in hours as shown below.

Studying time per week (hours)	30	32	34	38	40	42
Number of students		5	8	7	8	9

- (i) Find unbiased estimates of the population mean and variance. [3]
- (ii) Explain what it means for a sample to be random in this context. [1]
- (iii) Test at the 10% significance level whether the teacher's suspicion is supported by the data. [4]

It is now known that the studying time per week of the students have a standard deviation of σ hours. The teacher selects another random sample of 90 students and finds that the mean studying time per week of the students is 38.4 hours. Using this second sample, a test, at the 10% significance level, indicates that there is insufficient evidence to suggest that the mean studying time per week of the students differs from 38 hours.

(iv) Find the range of values of σ .

[4]

13 A manufacturer sells ice-cream. The average temperature, $x \circ C$, and its corresponding sales from ice-cream, y hundred dollars, for 8 randomly chosen days are given in the following table.

x	28.0	29.5	30.2	32.8	33.4	31.7	32.2	29.8
У	12.5	14.8	16.0	17.1	18.5	16.9	17.6	13.9

[2]

(a) Give a sketch of the scatter diagram for the data, as shown on your calculator



- (b) Find the value of the product moment correlation coefficient and comment on its value in the context of the data. [2]
- (c) Find the equation of the regression line of y on x, giving your answer in the form y = mx + c, with the values of m and c correct to 3 significant figures. Sketch this line on your scatter diagram. [2]
- (d) Use the equation of your regression line to calculate an estimate for the sales from ice-cream on a day with average temperature of 31°C. Explain why you would expect this to be a reliable estimate.
 [2]

It is given that the ice-cream sales for each of the 8 chosen days were over-reported by a fixed amount, p.

(e) Without further calculations, explain briefly what will happen to the values of *m* and *c* found in **part** (c) if the mistake was corrected. [2]